

This problem set is not homework. Students can use this problem set as extra practice or study guide for quizzes.

1. Label each of the following statements as true or false.

- If  $n$  is an integer such that  $n^2$  is divisible by 12, then  $n^2$  is divisible by 36.
- If  $n$  is an integer greater than 1, then all exponents in the prime-factorization of  $n^3$  are divisible by 3.
- If  $n$  is an even number, then  $n^2$  is divisible by 4.
- If  $n$  is an odd number, then the remainder is 1 when  $n^2$  is divided by 4.

2. Find the prime factorization for each of the following.

- 18 480
- $20^{1000}$

3. Compute the least common multiple and greatest common factor for each of the following.

- 270 and 96
- $41^{59}$  and  $41^{100}$

4. Perform each of the given operations.

- $(-\infty, 7) \cup (3, 10]$
- $(-\infty, 7) \cap (3, 10]$
- $(-\infty, 1) \cap [4, \infty)$
- $(-\infty, 1) \cup [4, \infty)$

5. Compute each of the following sums.

- $141 + 146 + 151 + \dots + 516$
- $124 + 135 + 146 + \dots + 619$

6. Compute each of the following.

- $2^{-1} - 3^{-2}$
- $\frac{5^{-1} + 2^{-2}}{5^{-1} - 2^{-2}}$
- $\frac{2}{3} - \frac{4}{5} \div \left(-\frac{3}{10}\right)$
- $\left(\frac{-3a^{-2}b}{(-2ab^3)^5}\right)^0$

7. Suppose that  $A = 12\,500\,000\,000$  and  $B = 0.000\,000\,064$ . Write each of the following in scientific notation.

- $A$
- $B$
- $AB$
- $A^2B$
- $\frac{1}{B}$
- $\sqrt{10B}$

8. Simplify each of the following expressions. Present your answer using only positive integer exponents.

- $(-m)^2 \cdot (-m)^3$
- $\frac{(-a^3b^2)^3}{ab^4}$
- $\frac{(-a^{-1}b^2)^{-4}}{ab^{-7}}$
- $\left(-\frac{2x^{-3}y}{y^{-4}}\right)^{-2} \left(\frac{2x^{-3}y^2}{-8x^{-5}y^2}\right)^{-3}$
- $(m^2)^3$
- $\frac{3^{2x-1}}{9^{x-1}}$
- $\frac{(-qp^{-2})^{-3}q^5}{(-p^{-4}q^3)^{-2}q^{-4}}$
- $\sqrt[3]{x^{36}}$

9. Simplify each of the following.

- $\sqrt[5]{x^{20}}$
- $(\sqrt[5]{x})^{20}$
- $2^{99} + 2^{99}$
- $\frac{3^{2019} - 3^{2017}}{3^{2017}}$

10. Let  $M = 2^{100}$ . Write each of the following expressions in terms of  $M$ .

- $2^{100} - 2^{101} + 2^{102} - 2^{103}$
- $2^{103} - 5 \cdot 2^{102}$
- $4^{100}$
- $2^{200}$
- $2^{500}$
- $2^{99}$

11. Simplify each of the following expressions.

- $(3a^5 - 1)^2$
- $(3x^5 + 4y)(3x^5 - 4y)$
- $(x - 2)(x + 3)(x + 1)$
- $(3a^5 - 1)^3$
- $(x - 2)(x^2 + 2x + 4)$
- $(2x + 1)^2 - (2x - 1)^2$

12. Completely factor each of the following over the real numbers.

- $x^2 - 4y^{10}$
- $5x^2 + 10x$
- $2x^3 - 8x$
- $32x^4 - 2$
- $(3x^2 - 5y)^2 - (5x^2 + y)^2$

13. Completely factor each of the following **by completing the square as shown in class and the handouts.**

- $60x - 6x^2 + 2250$
- $15a^3 - 8a^4 + a^5$
- $6x^2 - 24x + 78$
- $x^2 - 16x - 192$

14. Solve each of the following equations. Make sure to check your solution(s).

a)  $2x^3 = 20x^2 + 1750x$

f)  $8x^3 = 50x^2$

b)  $\frac{3}{2}x + \frac{17}{2} = x - 1 + \frac{x}{2} + \frac{9}{2}$

g)  $8p^3 = 50p$

c)  $\frac{2}{3}(x - 7) = \frac{4}{5}(x + 1)$

h)  $2 - (3 - x)(2x + 5) = (x - 1)(2x - 1)$

d)  $7x^2 + (x + 3)(2x - 1) = (3x + 1)^2$

i)  $2x^2 = 2x$

e)  $8a + 2a^2 = 42$

j)  $2x^5 = 32x$

15. Graph each of the following.

a)  $y = -\frac{1}{2}x + 3$

b)  $3x + 2y = -12$

c)  $x + y = 4$

16. One side of a rectangle is twelve feet shorter than three times another side. Find the sides of the rectangle if we also know that its area is  $960 \text{ ft}^2$ .

17. Express a 20% increase and a subsequent 20% decrease as a single change. Is it increase or decrease? What percent?

18. We throw a small object upward from the top of a 1024 ft tall building. The vertical position (or height)  $h$  of the object, (measured in feet)  $t$  seconds after we threw it is

$$h = -16t^2 + 192t + 1024$$

a) Where is the object 2 seconds after we threw it?

b) How long does it take for the object to hit the ground?

19. The first row in a theater has 25 seats in it. The second row has three more seats than the first row. The third row has three more seats than the second row. And so on, each row has three more seats than the row before. If the last row has 163 seats in it, how many seats are there in the entire theater?

20. The price of a TV was increased from \$120 to \$138. What percentage of an increase does this represent?

21. A total of \$20 000 is to be invested in bonds and stocks. If the amount invested in bonds is to be \$4500 more than the amount invested in stocks, how much money is invested in each category?

22. If we increase the side of a square by 5 units, its area will increase by  $95 \text{ unit}^2$ . How long are the sides of this square?

23. The tickets for the field trip were purchased yesterday for both students and instructors. Children tickets cost \$5, adult tickets cost \$12. The number of children ticket purchased was three less than four times the number of adults tickets purchased. How many of each were purchased if all of the tickets cost a total of \$209 dollars?

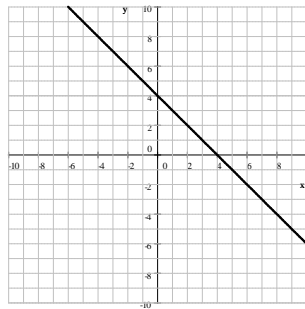
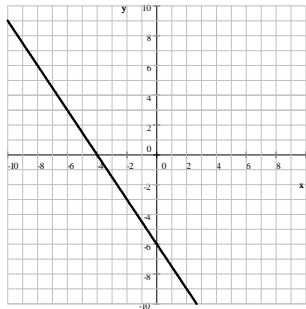
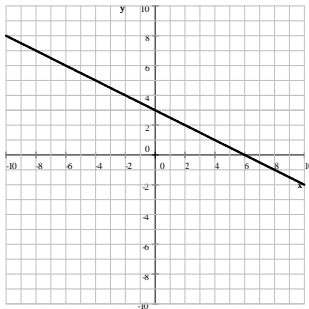
24. Sally worked 50 hours last week and made \$660 for the week. For every hour worked over 40 her job pays time and a half. What is Sally's regular hourly pay rate?

25. If we raise a number to the third power, we get nine times the number. Find this number.

26. Find the area of the triangle determined by the points  $A(-5, -2)$ ,  $B(7, -2)$ , and  $C(3, 6)$ .

## Answers

1. a) true b) true c) true d) true
2. a)  $2^4 \cdot 3 \cdot 5 \cdot 7 \cdot 11$  b)  $20^{1000} = 2^{2000} \cdot 5^{1000}$
3. a)  $\text{lcm}(270, 96) = 4320$   $\text{gcd}(270, 96) = 6$  b)  $\text{lcm} = 41^{100}$   $\text{gcd} = 41^{59}$
4. a)  $(-\infty, 10]$  b)  $(3, 7)$  c)  $\emptyset$  d)  $(-\infty, 1) \cup [4, \infty)$  (cannot be simplified)
5. a) 24966 b) 17089
6. a)  $\frac{7}{18}$  b)  $-9$  c)  $\frac{10}{3}$  d) 1
7. a)  $1.25 \cdot 10^{10}$  b)  $6.4 \cdot 10^{-8}$  c)  $8 \cdot 10^2$  d)  $1 \cdot 10^{13}$  e)  $1.5625 \cdot 10^7$  f)  $8 \cdot 10^{-4}$
8. a)  $-m^5$  b)  $m^6$  c)  $-a^8b^2$  d) 3 e)  $\frac{a^3}{b}$  f)  $-\frac{q^{12}}{p^2}$  g)  $-\frac{16}{y^{10}}$  h)  $x^{12}$
9. a)  $x^4$  b)  $x^4$  c)  $2^{100}$  d) 8
10. a)  $-5M$  b)  $-12M$  c)  $M^2$  d)  $M^2$  e)  $M^5$  f)  $\frac{M}{2}$
11. a)  $9a^{10} - 6a^5 + 1$  b)  $27a^{15} - 27a^{10} + 9a^5 - 1$  c)  $9x^{10} - 16y^2$  d)  $x^3 - 8$  e)  $x^3 + 2x^2 - 5x - 6$  f)  $8x$
12. a)  $(x - 2y^5)(x + 2y^5)$  b)  $5x(x + 2)$  c)  $2x(x + 2)(x - 2)$  d)  $2(4x^2 + 1)(2x + 1)(2x - 1)$   
e)  $-8(x^2 + 3y)(2x^2 - y)$
13. a)  $-6(x + 15)(x - 25)$  b)  $a^3(a - 3)(a - 5)$  c)  $6(x^2 - 4x + 13)$  d)  $(x + 8)(x - 24)$
14. a) 35, 0, -25 b)  $\mathbb{R}$  (all real numbers) c) -41 d) -4 e) -7, 3 f)  $0, \frac{25}{4}$  g)  $-\frac{5}{2}, 0, \frac{5}{2}$   
h) 7 i) 0, 1 j) -2, 0, 2
15. a)  $y = -\frac{1}{2}x + 3$  b)  $3x + 2y = -12$  c)  $x + y = 4$



16. 20 ft by 48 ft 17. 4% decrease 18. a) 1344 ft b) 16 seconds 19. 4418 20. 15%
21. \$7750 in stocks and \$12 250 in bonds 22. 7 units 23. 7 adult tickets and 25 children tickets
24. \$12 25. -3, 0, 3 26. 48 unit<sup>2</sup>