1. Perform the division with remainder. $2018 \div 99$.

2. a) List all factors of 54.
   b) What is the greatest factor of 54?
   c) What is the greatest prime factor of 54?

3. Find the prime factorization of 528.

4. Use prime factorization to find the least common multiple and greatest common factor of 270 and 600.

5. Label each of the following as true or false.
   a) If the integer n is divisible by 6 and the integer m is divisible by 9, then nm is divisible by 54.
   b) If an integer n is divisible by 6 and by 9, then it is also is divisible by 54.
   c) If a right triangle's sides are integers, then at least one of them must be even.
   d) If n is an integer such that n^2 is divisible by 20, then n is divisible by 20.

   Let T be the set of all triangles, R the set of all right triangles, and S the set of all isosceles triangles. Then
   e) $R \cup S = T$ f) $R \subseteq T$ g) $R \cap S = \varnothing$

6. Prove that the repeating decimal $0.7\overline{19} = 0.719191919...$ represents a rational number by re-writing it as a quotient of two integers. You do not have to reduce the fraction.

7. Compute the sum $158 + 163 + 168 + ... + 398$

8. Perform the given operations.
   a) $\left(3\sqrt{2}-1\right)\left(5\sqrt{2}+7\right)$ c) $\left(\sqrt{5}-2\right)^7\left(\sqrt{5}+2\right)^7$ e) $\sqrt{50}-\sqrt{8}+\sqrt{18}$ g) $\left(\sqrt{x-2}\right)^2$
   b) $\left(\sqrt{5}-2\right)^3$ d) $\left(\sqrt{3}+1\right)^2-\left(\sqrt{3}-1\right)^2$ f) $\left(\sqrt{x}-2\right)^2$ h) $\sqrt[6]{x^3}$

9. Compute the exact value of $-2x^2+3x-8$ if
   a) $x=-\frac{2}{3}$ b) $x=-\sqrt{3}+1$ c) $x=5-2i$ d) $x=1-i\sqrt{2}$

10. Rationalize the denominator in each of the expressions. Simplify your answer.
    a) $\frac{3}{\sqrt{5}}$ b) $\frac{12}{\sqrt{7}-3}$ c) $\frac{12\sqrt{3}}{\sqrt{7}-\sqrt{3}}$ d) $\frac{x}{\sqrt{x}-1}$ e) $\frac{2}{\sqrt{5}-1}$

11. Simplify each of the given expressions. Assume that all variables represent positive numbers. Express your answer using only positive integer exponents.
    a) $\frac{p^{5/2}p^{2/3}}{p^{1/6}}$ b) $\sqrt[4]{\frac{x^{12}y^{18}z^{24}}{x^4y^6z^8}}$ c) $\sqrt[3]{\frac{x^{11}y^5}{x^2y^2}}$ d) $\left(\sqrt[5]{x}\right)^{30}$ e) $\sqrt{x}\sqrt[3]{x^2}\sqrt[4]{x^3}$

12. Perform the given operations on the complex numbers as indicated.
    a) $i\,(2-i)$ c) $|3-2i|$ e) $(1-i)^8$ g) $\frac{17+i}{1+3i}$ h) $\frac{1-3i}{1-i}$
    b) $(3-i)\,(2+i)$ d) $(3+4i)^2$ f) i^{99} i) $\left(3-\sqrt{5}i\right)^2\left(3+\sqrt{5}i\right)^2$

13. Expand each of the following.

a) $(3n+m)(-2a+5b)$

b) $(3x+1)(-2x+5)$

c) $\left(3\sqrt{2}+1\right)\left(-2\sqrt{2}+5\right)$

d) $\left(3\sqrt{2}+\sqrt{3}\right)\left(-2\sqrt{2}+5\sqrt{3}\right)$

e) $(3+i)(-2+5i)$

14. Expand each of the following.

a) $(a+b)\left(a^2-ab+b^2\right)$ b) $(a-b)\left(a^2+ab+b^2\right)$ c) $\left(x^2+x\sqrt{2}+1\right)\left(x^2-x\sqrt{2}+1\right)$

15. Simplify each of the following. Show all steps.

a) $\dfrac{-5^2-60\div(-4)\cdot 3}{|-21\div(-7)|-(-1)^6}$

b) $\dfrac{\frac{1}{5}+\left(-\frac{1}{2}\right)^2\cdot\left(3\frac{2}{5}\right)}{\frac{1}{5}+\frac{1}{2}}-2^{-1}$

c) $\dfrac{x^3-x}{x+1}$

d) $\dfrac{\sqrt{8}-\sqrt{5}}{\sqrt{8}+\sqrt{5}}$

e) $\dfrac{2a-\frac{1}{8a}}{4+\frac{1}{a}}$

f) $\left(-\dfrac{x^3y^0x^{-5}}{y^{-3}}\right)^{-2}$

g) $\dfrac{\left(-x^3y^{-2}\right)^4yx^{-1}\left(-2xy^{-3}x^{-2}\right)^{-1}}{x^{-3}y^0\left(-3x^{-5}y^2\right)^{-2}y^{-3}}$

h) $\dfrac{a^{2/3}\left(b^{-4/3}\right)^2}{\left(ab^2\right)^{-1/3}}$

i) $\left(\sqrt{5x}-2\right)\left(\sqrt{5x}+3\right)$

j) $2^{-1}-\left(3-2^{-1}\right)^{-1}$

k) $\dfrac{14-13i}{2+i}$

l) $(-8)^{-2/3}$

m) $-27^{-2/3}$

16. Simplify each of the following.

a) $\dfrac{x^2-10x+25}{x^2-5x+4}\left(\dfrac{x^2-2x-8}{x^2-6x+5}\div\dfrac{x-5}{x-1}\right)$ b) $\dfrac{x-5}{x+2}-\dfrac{3}{2-x}-\dfrac{14-x}{x^2-4}$

17. Completely factor each of the following expresions over the real numbers.

a) $3a^4x-48x$ b) $21x^2-18ax^2-3a^2x^2$ c) $15ax-5ay-3bx+by$

18. a) Solve the equation $9x^2-6x=1$. b) Check your solution using exact values.

19. Solve each of the following equations. Make sure to check your solution(s).

a) $\dfrac{3x+17}{2}=x-1+\dfrac{x+19}{2}$

b) $|3-2x|+2=5$

c) $2x^2+x^3=2x$

d) $\dfrac{2}{3}(x-7)=\dfrac{4}{5}(x+1)$

e) $x^5=x^4$

f) $x^5=x^3$

g) $|x-5|=3-x$

h) $\left|\dfrac{1}{2}x-2\right|=|x+1|$

i) $\sqrt{2x-1}+4=1$

j) $\sqrt{5x-4}-3=8$

k) $4-(2x-5)(x+1)=18-2x^2$

l) $x+2=\sqrt{x+10}-2$

m) $\dfrac{x}{x-8}+\dfrac{7}{x-4}=\dfrac{2x+16}{x^2-12x+32}$

n) $7x^2+(x+3)(2x-1)=(3x+1)^2$

o) $3x^2-30x+69=0$

20. Solve each of the following system of linear equations.

a) $\begin{cases}2x-y=11\\3x+2y=6\end{cases}$

b) $\begin{cases}x-y=-3\\(y-1)^2+2x=y^2\end{cases}$

c) $\begin{cases}\frac{1}{2}x-\frac{1}{5}y=-5\\\frac{1}{3}x+\frac{1}{2}y=3\end{cases}$

21. Solve each of the following the inequalites.

a) $\dfrac{1-3x}{4}-\dfrac{2x+1}{3}\geq x-17$

b) $-2(3x-1)-(x-3)<12$

c) $(x-2)^2>x^2$

22. Solve each of the following compound inequalities.

    a) $\dfrac{3-x}{-2} < 5$ or $2(x-1) \leq 3(x+2)$ b) $\dfrac{3-x}{-2} < 5$ and $2(x-1) \leq 3(x+2)$

23. Suppose that $f(x)$ is a function given by $f(x) = x^2 - 2x + 7$. Find the value of

    a) $f(0)$ b) $f(5)$ c) $f(-5)$ d) $f(2-3)$ e) $f(2) - f(3)$ f) $(f(1))^2$

24. a) Compute the perimeter and area of the parallelogram determined by the points $A(-5,-3)$, $B(2,-3)$, $C(-2,1)$ and $C(5,1)$.

25. Write an equation of the line if it

    a) passes through the points $A(-1,5)$ and $B(3,-3)$.
    b) passes through $P(-6,1)$ and is perpendicular to the line $2x - 3y = 7$.

26. Compute the exact value of the distance between the points $A(3,8)$ and $B(6,-1)$.

27. Find the greatest value of the expression $-2x^2 + 20x - 54$.

28. Graph the parabola $y = -x^2 + 6x - 5$. Clearly state the coordinates of five points on the parabola, including vertex and intercepts.

29. Express a 20% increase and a 12% decrease as a single change. (Hint: it is NOT 8% increase).

30. A citrus grower finds that if we plant 80 trees, then each tree will grow about 600 oranges. For each additional tree planted,each tree will grow 5 less oranges. What is the greatest number of oranges possible to grow on the land? How many trees should the grower plant?

31. After a 20% inrease, the budget was \$268 800. What was the budget before the increase?

32. The hypotenuse of a right triangle is 74 ft. The difference between the other two sides is 46 ft. Find the sides of the triangle.

33. There is an animal farm where chickens and cows live. All together, there are 53 heads and 174 legs. How many chickens, how many cows?

34. The area of a rectangle is $1260\,\text{m}^2$. Find the dimensions of the rectangle if we know that one side is 48 m longer than three times the other side.

35. The measure of one angle in a rectangle is one degree less than the greatest angle in the triangle, and two degrees more than twice the smallest angle. Find the angles in this triangle.

36. Suppose that a, b, and c are sides of a right triangle. Side a is 3 units longer than side b, and side c is 3 units longer than side a. Find the sides of this triangle.

37. Johanna can paint the room in 6 hours. Susan can paint the room in 4 hours. How long would it take for the two of them to paint the room?

38. We invested \$10 000 into two bank accounts. One account earns 14% per year, the other account earns 8% per year. How much did we invest into each account if the combined interest from the two accounts is \$1238 after the first year?

39. We have 150 coins in a jar, all dimes and quarters. How many of each do we have if the value of all coins is \$27. 30?

40. A 6.4 ft tall person is standing 20 ft away from a street light that is 25 ft tall. How long is his shadow?

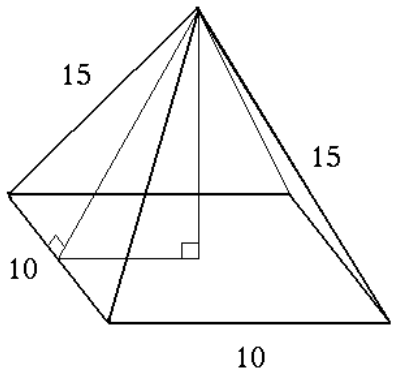
41. Ann invested \$20, 000 in two accounts. She took a 4% loss on one of the accounts and made a 12% profit on the other investment, but ended up breaking even. How much money did she lose in the first investment?

42. a) A conference had 200 attendees. If every attendee shook hands with all other attendees, how many handshakes took place?

    b) On the afternoon, people had several choices, including games and sporting events. On the men's running race, first all participants shook hands with all other participants. Later on, 5 more people joined the race. They too shook hands with all other attendees. If the number of handhsakes increased by 220 due to the new arrivals, how many men were participating in the morning?

43. The first row in the auditorium had 32 seats. The second row had 36 seats, the fourth row had 40 seats, and so on, each row had four more seats than the previous one. If the last row seats 100 people, then how many seats are there in the entire auditorium?

44. Compute the exact value of the height of the pyramid shown. Units are in meters.



45. We are standing on the top of a 896 feet tall building and launch a small object upward. The object's vertical position, measured in feet, after t seconds is

$$h(t) = -16t^2 + 160t + 896$$

    a) What is the highest point that the object reaches?
    b) How long until the object hits the ground?

Answers

1. 20 R 38
2. a) $1, 2, 3, 6, 9, 18, 27, 54$ b) 54 c) 3
3. $528 = 2^4 \cdot 3 \cdot 11$
4. $\gcd(270, 600) = 30$ $\quad \operatorname{lcm}(270, 600) = 5400$
5. a) true b) false c) true d) false e) false f) true g) false
6. $\frac{712}{990}$ 7. 13 622
8. a) $16\sqrt{2}+23$ b) $17\sqrt{5}-38$ c) 1 d) $4\sqrt{3}$ e) $6\sqrt{2}$ f) $x - 4\sqrt{x} + 4$ g) $x - 2$ h) $\sqrt{x}$
9. a) $-\frac{98}{9}$ b) $-13 + \sqrt{3}$ c) $-35 + 34i$ d) $-3 + i\sqrt{2}$
10. a) $\frac{3\sqrt{5}}{5}$ b) $-6\sqrt{7} - 18$ c) $3\sqrt{21} + 9$ d) $\frac{x(\sqrt{x}+1)}{x-1}$ e) $\frac{\sqrt{5}+1}{2}$
11. a) p^3 b) $x^2y^3z^4$ c) x^3y d) x^6 e) $\sqrt[12]{x^{23}}$
12. a) $1 + 2i$ b) $7 + i$ c) $\sqrt{13}$ d) $-7 + 24i$ e) 16 f) $-i$ g) $2 - 5i$ h) $2 - i$ i) 196
13. a) $-6an + 15bn - 2am + 5bm$ b) $-6x^2 + 13x + 5$ c) $13\sqrt{2} - 7$ d) $3 + 13\sqrt{6}$ e) $-11 + 13i$
14. a) $a^3 + b^3$ b) $a^3 - b^3$ c) $x^4 + 1$
15. a) 10 b) 1 c) $x^2 - x$ d) $\frac{13 - 4\sqrt{10}}{3}$ e) $\frac{4a-1}{8}$ f) $\frac{x^4}{y^6}$ g) $-\frac{9}{2}x^5y^3$ h) ab^2 i) $5x + \sqrt{5x} - 6$ j) $\frac{1}{10}$ k) $3 - 8i$ l) undefined m) $-\frac{1}{9}$
16. a) $\frac{x+2}{x-1}$ b) $\frac{x-1}{x+2}$
17. a) $3x\left(a^2+4\right)(a+2)(a-2)$ b) $-3x^2(a+7)(a-1)$ c) $(3x-y)(5a-b)$
18. a) $\frac{1-\sqrt{2}}{3}$ and $\frac{1+\sqrt{2}}{3}$ b) If $x = \frac{1-\sqrt{2}}{3}$, then

$$\begin{aligned} \text{LHS} &= 9\left(\frac{1-\sqrt{2}}{3}\right)^2 - 6\left(\frac{1-\sqrt{2}}{3}\right) \\ &= 9 \cdot \frac{3-2\sqrt{2}}{9} - 6 \cdot \frac{1-\sqrt{2}}{3} \\ &= 3 - 2\sqrt{2} - 2\left(1-\sqrt{2}\right) \\ &= 3 - 2\sqrt{2} - 2 + 2\sqrt{2} = 1 = \text{RHS} \end{aligned}$$

and if $x = \dfrac{1+\sqrt{2}}{3}$, then

$$\begin{aligned} \text{LHS} &= 9\left(\frac{1+\sqrt{2}}{3}\right)^2 - 6\left(\frac{1+\sqrt{2}}{3}\right) \\ &= 9\cdot\frac{3+2\sqrt{2}}{9} - 6\cdot\frac{1+\sqrt{2}}{3} \\ &= 3+2\sqrt{2} - 2\left(1+\sqrt{2}\right) \\ &= 3+2\sqrt{2}-2-2\sqrt{2} = 1 = \text{RHS} \end{aligned}$$

19. a) all real numbers b) $0, 3$
c) $-1-\sqrt{3}, 0, -1+\sqrt{3}$ d) -41 e) $0, 1$
f) $-1, 0, 1$ g) no solution h) $-6, \dfrac{2}{3}$
i) no solution j) 25 k) 3
l) -1 (-6 is extreneous) m) -9 (8 is extreneous)
n) -4 o) $5-\sqrt{2}, 5+\sqrt{2}$

20. a) $(4, -3)$ b) no solution c) $(-6, 10)$

21. a) $(-\infty, 7]$ b) $(-1, \infty)$ c) $(-\infty, 1)$

22. a) $\mathbb{R}$ b) $[-8, 13)$

23. a) 7 b) 22 c) 42 d) 10 e) -3 f) 36

24. $P = 24$ unit $A = 28$ unit^2 25. a) $y = -2x+3$
b) $y - 1 = -\dfrac{3}{2}(x+6)$ or $y = -\dfrac{3}{2}x - 8$

26. $3\sqrt{10}$ unit 27. -4 28. see below

29. 5.6% increase

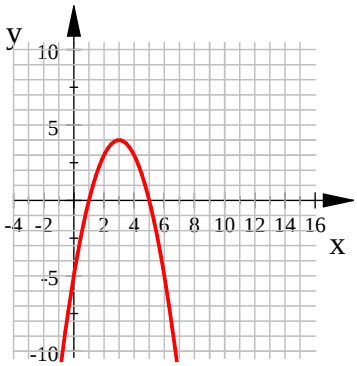
30. 50 000 oranges
if 100 trees are planted

31. \$224 000

32. 24 ft and 70 ft

33. 19 chickens and 34 cows

34. 14 m by 90 m



35. 35°, 72° and 73°

36. 9 units, 12 units, and 15 units

37. 2.4 hours (2 hours and 24 minutes)

38. \$7300 at 14% and \$2700 at 8%

39. 82 quarters and 68 dimes

40. $\dfrac{640}{93} \approx 6.881\,720$

41. \$600 42. a) 19 900 b) 42 43. $5\sqrt{7}$ m

44. 1188 45. a) 1296 ft b) 14 seconds

Last revised: December 1, 2018