1. Simplify each of the following expressions.

(a) $\dfrac{ab-a-b+1}{b^2-1} = \dfrac{a-1}{b+1}$

Solution: We will factor both numerator and denominator and then cancel. The numerator can be factored by grouping

$$\underbrace{ab-a}\ \underbrace{-b+1} = a(b-1) - 1(b-1) = (a-1)(b-1)$$

The denominaotr factors by the difference of squares theorem.

$$b^2 - 1 = (b+1)(b-1)$$

Thus the fraction can be simplified as

$$\frac{ab-a-b+1}{b^2-1} = \frac{(a-1)(b-1)}{(b+1)(b-1)} = \frac{a-1}{b+1}$$

(b) $\dfrac{5x-30}{x^2-36} \cdot \dfrac{3x+18}{5} = 3$

Solution: we will factor whatever we can and then cancel.

$$\frac{5x-30}{x^2-36} \cdot \frac{3x+18}{5} = \frac{5(x-6)}{(x+6)(x-6)} \cdot \frac{3(x+6)}{5} = 3$$

(c) $\dfrac{px^2-16q-16p+qx^2}{x^2+5x+6} \cdot \dfrac{x^2+6x+9}{4px^2+px^3+4qx^2+qx^3} = \dfrac{(x+3)(x-4)}{x^2(x+2)} = \dfrac{x^2-x-12}{2x^2+x^3}$

Solution: $x^2+5x+6 = (x+2)(x+3)$ and $x^2+6x+9 = (x+3)(x+3)$ are easy. With the other expressions, we start with the greatest common factor.

$$\begin{aligned} px^2-16q-16p+qx^2 &= \underbrace{px^2+qx^2}\underbrace{-16p-16q} && \text{grouping} \\ &= x^2(p+q) - 16(p+q) && \text{factor out } p+q \\ &= \left(x^2-16\right)(p+q) && \text{difference of squares} \\ &= (x+4)(x-4)(p+q) \end{aligned}$$

$$\begin{aligned} 4px^2+px^3+4qx^2+qx^3 &= x^2\left(\underbrace{4p+4q}\underbrace{+px+qx}\right) && \text{grouping} \\ &= x^2(4(p+q) + x(p+q)) && \text{factor out } p+q \\ &= x^2(x+4)(p+q) \end{aligned}$$

We have all the pieces now:

$$\frac{px^2-16q-16p+qx^2}{x^2+5x+6} \cdot \frac{x^2+6x+9}{4px^2+px^3+4qx^2+qx^3} =$$

$$\frac{(x+4)(x-4)(p+q)}{(x+2)(x+3)} \cdot \frac{(x+3)(x+3)}{x^2(x+4)(p+q)} = \frac{(x+3)(x-4)}{x^2(x+2)} = \frac{x^2-x-12}{2x^2+x^3}$$

(d) $\dfrac{3x}{x-2}-\dfrac{x+4}{x-2}=2$
Solution: This is a subtraction of fractions. The denominators are the same, the only difficulty is that we are subtracting expressions instead of numbers. The second pair of parentheses is essential.

$$\frac{3x}{x-2}-\frac{x+4}{x-2}=\frac{(3x)-(x+4)}{x-2}=\frac{3x-x-4}{x-2}=\frac{2x-4}{x-2}=\frac{2(x-2)}{x-2}=2$$

(e) $\sqrt{125}-3\sqrt{80}+\sqrt{45}=-4\sqrt{5}$
Solution:

$$\begin{aligned}\sqrt{125}-3\sqrt{80}+\sqrt{45} &= \sqrt{25\cdot 5}-3\sqrt{16\cdot 5}+\sqrt{9\cdot 5}=\sqrt{25}\sqrt{5}-3\sqrt{16}\sqrt{5}+\sqrt{9}\sqrt{5}\\ &= 5\sqrt{5}-3(4)\sqrt{5}+3\sqrt{5}=(5-12+3)\sqrt{5}=-4\sqrt{5}\end{aligned}$$

(f) $\left(\sqrt{7}-2\right)^2 = 11-4\sqrt{7}$
Solution:

$$\left(\sqrt{7}-2\right)^2=\left(\sqrt{7}-2\right)\left(\sqrt{7}-2\right)=\sqrt{7}\sqrt{7}-2\sqrt{7}-2\sqrt{7}+4=7-4\sqrt{7}+4=11-4\sqrt{7}$$

(g) $\left(\sqrt{3}-1\right)^3=-10+6\sqrt{3}$
Solution: We will first work out $\left(\sqrt{3}-1\right)^2$ and then multiply that by $\left(\sqrt{3}-1\right)$.

$$\begin{aligned}\left(\sqrt{3}-1\right)^3 &= \left(\sqrt{3}-1\right)\left(\sqrt{3}-1\right)\left(\sqrt{3}-1\right)=\left(\sqrt{3}\sqrt{3}-1\sqrt{3}-1\sqrt{3}+1\right)\left(\sqrt{3}-1\right)\\ &= \left(3-2\sqrt{3}+1\right)\left(\sqrt{3}-1\right)=\left(4-2\sqrt{3}\right)\left(\sqrt{3}-1\right)=4\sqrt{3}-4-2\sqrt{3}\sqrt{3}+2\sqrt{3}\\ &= 4\sqrt{3}-4-2(3)+2\sqrt{3}=4\sqrt{3}-4-6+2\sqrt{3}=-10+6\sqrt{3}\end{aligned}$$

2. Rationalize the denominator in each of the following expressions.

(a) $\dfrac{3}{\sqrt{5}}=\dfrac{3\sqrt{5}}{5}$
Solution: To rationalize the denominator, we will multiply both the numerator and denominator by $\sqrt{5}$.

$$\frac{3}{\sqrt{5}}=\frac{3}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{3\sqrt{5}}{5}$$

(b) $\dfrac{1}{\sqrt{10}-3}=\sqrt{10}+3$
Solution: To rationalize the denominator, we will multiply both the numerator and denominator by the conjugate of the denominator, which is $\sqrt{10}+3$.

$$\frac{1}{\sqrt{10}-3}=\frac{1}{\sqrt{10}-3}\cdot\frac{\sqrt{10}+3}{\sqrt{10}+3}=\frac{\sqrt{10}+3}{1}=\sqrt{10}+3$$

The denominator is 1 since

$$\left(\sqrt{10}-3\right)\left(\sqrt{10}+3\right)=\sqrt{10}\sqrt{10}+3\sqrt{10}-3\sqrt{10}-9=10-9=1$$

(c) $\dfrac{2}{\sqrt{7}+1} = \dfrac{\sqrt{7}-1}{3}$
Solution: To rationalize the denominator, we will multiply both the numerator and denominator by the conjugate of the denominator, which is $\sqrt{7}-1$

$$\frac{2}{\sqrt{7}+1} = \frac{2}{\sqrt{7}+1} \cdot \frac{\sqrt{7}-1}{\sqrt{7}-1} = \frac{2\left(\sqrt{7}-1\right)}{7-1} = \frac{2\left(\sqrt{7}-1\right)}{6} = \frac{\sqrt{7}-1}{3}$$

3. Find the exact value of x^2-4x+6 if $x = 2-\sqrt{3}$. 5
Solution: We work out x^2 first.

$$\begin{aligned} x^2 &= \left(2-\sqrt{3}\right)^2 = \left(2-\sqrt{3}\right)\left(2-\sqrt{3}\right) = 4-2\sqrt{3}-2\sqrt{3}+\sqrt{3}\sqrt{3} \\ &= 4-4\sqrt{3}+3 = 7-4\sqrt{3} \end{aligned}$$

Now we substitute $x = 2-\sqrt{3}$ into x^2-4x+6.

$$x^2-4x+6 = \left(2-\sqrt{3}\right)^2 - 4\left(2-\sqrt{3}\right)+6 = 7-4\sqrt{3}-8+4\sqrt{3}+6 = 7-8+6 = 5$$

4. Factor $13x+2x^2-24$ by completing the square. $2\left(x+8\right)\left(x-\dfrac{3}{2}\right) = \left(x+8\right)\left(2x-3\right)$
Solution: We rearrange the terms first and then factor out the leading coefficient.

$$\begin{aligned} 13x+2x^2-24 &= 2x^2+13x-24 \\ &= 2\left(x^2+\frac{13}{2}x-12\right) \end{aligned}$$

Half of the linear coefficient is $\dfrac{13}{4}$, thus we work out $\left(x+\dfrac{13}{4}\right)^2$ first to see what we need to smuggle in to complete the square.

$$\begin{aligned} \left(x+\frac{13}{4}\right)^2 &= \left(x+\frac{13}{4}\right)\left(x+\frac{13}{4}\right) = x^2+\frac{13}{4}x+\frac{13}{4}x+\frac{169}{16} \\ &= x^2+\frac{13}{2}x+\frac{169}{16} \end{aligned}$$

Thus we need to smuggle in $\dfrac{169}{16}$

$$\begin{aligned} 2x^2+13x-24 &= 2\left(x^2+\frac{13}{2}x-12\right) \\ &= 2\left(\underbrace{x^2+\frac{13}{2}x+\frac{169}{16}}-\frac{169}{16}-12\right) \end{aligned}$$

We bring the last two numbers to the common denominator

$$\begin{aligned}2x^2+13x-24 &= 2\left(\left(x+\frac{13}{4}\right)^2-\frac{169}{16}-\frac{12\,(16)}{1\,(16)}\right)\\ &= 2\left(\left(x+\frac{13}{4}\right)^2-\frac{169}{16}-\frac{192}{16}\right)\\ &= 2\left(\left(x+\frac{13}{4}\right)^2-\frac{361}{16}\right)\end{aligned}$$

Since $\dfrac{361}{16}=\left(\dfrac{19}{4}\right)^2$, we factor via the difference of squares theorem.

$$\begin{aligned}2x^2+13x-24 &= 2\left(\left(x+\frac{13}{4}\right)^2-\left(\frac{19}{4}\right)^2\right)\\ &= 2\left(x+\frac{13}{4}+\frac{19}{4}\right)\left(x+\frac{13}{4}-\frac{19}{4}\right)\\ &= 2\left(x+\frac{32}{4}\right)\left(x-\frac{6}{4}\right)\\ &= 2\,(x+8)\left(x-\frac{3}{2}\right)=(x+8)\,(2x-3)\end{aligned}$$

We may distribute 2 into the second factor. Then we get

$$2x^2+13x-24=(x+8)\left(2\left(x-\frac{3}{2}\right)\right)=(x+8)\,(2x-3)$$

We FOIL to check:

$$\begin{aligned}(x+8)\,(2x-3) &= 2x^2-3x+16x-24\\ &= 2x^2+13x-24\end{aligned}$$

5. Factor completely each of the following:

(a) $4a^2mn-15abm^2-6abmn+10a^2m^2=$ $am\,(2n+5m)\,(2a-3b)$

Solution:

$$\begin{aligned}4a^2mn-15abm^2-6abmn+10a^2m^2 &= \quad \text{the GCF is } am\\ am\,(4an-15bm-6bn+10am) &= \quad \text{rearrange}\\ am\left(\underbrace{4an-6bn}\ \underbrace{+10am-15bm}\right) &= \\ am\,(2n\,(2a-3b)+5m\,(2a-3b)) &= am\,(2n+5m)\,(2a-3b)\end{aligned}$$

(b) $a^2x^3 - b^2x - a^2x + b^2x^3 = x\left(a^2+b^2\right)(x+1)(x-1)$
Solution:

$$\begin{aligned} a^2x^3 - b^2x - a^2x + b^2x^3 &= && \text{the GCF is } x \\ x\left(a^2x^2 - b^2 - a^2 + b^2x^2\right) &= && \text{rearrange} \\ x\Big(\underbrace{a^2x^2 - a^2}\quad \underbrace{+b^2x^2 - b^2}\Big) &= && \\ x\left(a^2\left(x^2-1\right) + b^2\left(x^2-1\right)\right) &= && x\left(a^2+b^2\right)\left(x^2-1\right) \end{aligned}$$

We are not done yet since $\left(x^2-1\right) = \left(x^2-1^2\right)$ further factors via the difference of squares theorem. Thus the answer is

$$x\left(a^2+b^2\right)\left(x^2-1\right) = x\left(a^2+b^2\right)\left(x^2-1^2\right) = x\left(a^2+b^2\right)(x+1)(x-1)$$

(c) $162a + 162b - 2ax^4 - 2bx^4 = 2\left(9+x^2\right)(3+x)(3-x)(a+b)$
Solution:

$$\begin{aligned} 162a + 162b - 2ax^4 - 2bx^4 &= && \text{the GCF is } 2 \\ 2\Big(\underbrace{81a + 81b}\quad \underbrace{-ax^4 - bx^4}\Big) &= && \\ 2\left(81(a+b) - x^4(a+b)\right) &= && 2\left(81-x^4\right)(a+b) \end{aligned}$$

We are not done yet, since $81 - x^4 = 9^2 - \left(x^2\right)^2$ further factors via the difference of squares theorem.

$$\begin{aligned} 2\left(81-x^4\right)(a+b) &= 2\left(9^2 - \left(x^2\right)^2\right)(a+b) \\ &= 2\left(9+x^2\right)\left(9-x^2\right)(a+b) \end{aligned}$$

One factor still further factors: $9 - x^2 = 3^2 - x^2 = (3+x)(3-x)$.

$$\begin{aligned} &= 2\left(9+x^2\right)\left(9-x^2\right)(a+b) \\ &= 2\left(9+x^2\right)\left(3^2-x^2\right)(a+b) \\ &= 2\left(9+x^2\right)(3+x)(3-x)(a+b) \end{aligned}$$

(d) $x^2 - 6x + 8 = (x-2)(x-4)$
Solution: we will factor by grouping. First we conduct the "$pq-$game".

$$\begin{aligned} pq &= 8 && \text{1st coefficient times 3rd coefficient} \\ p+q &= -6 && \text{2nd coefficient} \end{aligned}$$

We start by expressing 8 as a product of two numbers. there are only two pairs, 1 with 8 and 2 with 4. Since the product pq is positive, p and q have to have the same sign. Since the sum $p+q$ is negative, they both have to be negative. We only need to consider -1 with -8 and -2 with -4. Clearly -2 with -4 work as p and q. We use these numbers to express the second term as the sum of two terms, and then factor by grouping.

$$\begin{aligned} x^2-6x+8 &= \underbrace{x^2-2x}\ \ \underbrace{-4x+8} \\ &= x(x-2)-4(x-2)=(x-2)(x-4) \end{aligned}$$

We check by multiplication:

$$(x-2)(x-4)=x^2-4x-2x+8=x^2-6x+8$$

Thus our result is correct.

(e) $3a^2-5a-2=(a-2)(3a+1)$
Solution: we will factor by grouping. First we conduct the "$pq-$game".

$$\begin{aligned} pq &= -6 \qquad \text{1st coefficient times 3rd coefficient} \\ p+q &= -5 \qquad \text{2nd coefficient} \end{aligned}$$

We start by expressing 6 as a product of two numbers. there are only two pairs, 1 with 6 and 2 with 3. Since the product pq is negative, one number must be positive, the other one must be positive.. Since the sum $p+q$ is negative, the negative sign has to be in front of the larger number. We only need to consider 1 with -6 and 2 with -3. Clearly 1 with -6 work as p and q. We use these these numbers to express the second term as the sum of two terms, and then factor by grouping.

$$\begin{aligned} 3a^2-5a-2 &= \underbrace{3a^2+a}\ \ \underbrace{-6a-2} \\ &= a(3a+1)-2(3a+1)=(a-2)(3a+1) \end{aligned}$$

We check by multiplication:

$$(a-2)(3a+1)=3a^2+a-6a-2=3a^2-5a-2$$

Thus our result is correct.

(f) $4b^2-b-5=\ (4b-5)(b+1)$
Solution: we will factor by grouping. First we conduct the "$pq-$game".

$$\begin{aligned} pq &= -20 \qquad \text{1st coefficient times 3rd coefficient} \\ p+q &= -1 \qquad \text{2nd coefficient} \end{aligned}$$

We start by expressing 20 as a product of two numbers. the possible pairs are, 1 with 20, 2 with 10, and 4 with 5. Since the product pq is negative, one number must be positive, the other one must be positive.. Because the sum $p+q$ is negative, the negative sign has to be in front of the larger number. We only need to consider 1 with -20, 2 with -10, and 4 with -5. Clearly 4 with -5 work as p and q. We use these these numbers to express the second term as the sum of two terms, and then factor by grouping.

$$\begin{aligned} 4b^2-b-5 &= \underbrace{4b^2+4b}\ \ \underbrace{-5b-5} \\ &= 4b(b+1)-5(b+1)=(4b-5)(b+1) \end{aligned}$$

We check by multiplication:

$$(4b-5)(b+1) = 4b^2+4b-5b-5 = 4b^2-b-5$$

Thus our result is correct.

6. Solve each of the following equations. Make sure to check your solution(s).

(a) $2x^3 = 20x^2 + 1750x$ $35, 0,$ and -25
Solution: We reduce one side to zero, then factor, and then apply the zero property.

$$\begin{aligned} 2x^3 &= 20x^2+1750x \\ 2x^3-20x^2-1750x &= 0 && \text{factor out GCF} \\ 2x\left(x^2-10x-875\right) &= 0 && \text{divide both sides by 2} \\ x\left(x^2-10x-875\right) &= 0 \end{aligned}$$

We will factor by completing the square. Half of the linear coefficient is -5, and thus we will work with

$$(x-5)^2 = x^2-10x+25$$

We smuggle in 25.

$$\begin{aligned} x\left(x^2-10x-875\right) &= 0 \\ x\left(\underbrace{x^2-10x+25}-25-875\right) &= 0 \\ x\left((x-5)^2-900\right) &= 0 \\ x\left((x-5)^2-30^2\right) &= 0 \\ x(x-5+30)(x-5-30) &= 0 \\ x(x+25)(x-35) &= 0 \end{aligned}$$

Applying the zero property we obtain $35, 0,$ and -25 as the solutions.

(b) $\dfrac{3x+17}{2} = x-1+\dfrac{x+19}{2}$ identity, all real numbers are solution.
Solution:

$$\begin{aligned} \frac{3x+17}{2} &= x-1+\frac{x+19}{2} && \text{express everything as a fraction} \\ \frac{3x+17}{2} &= \frac{x-1}{1}+\frac{x+19}{2} && \text{bring everything to the common denominator} \\ \frac{3x+17}{2} &= \frac{2(x-1)}{2}+\frac{x+19}{2} && \text{add fractions on right hand side} \\ \frac{3x+17}{2} &= \frac{2(x-1)+x+19}{2} && \text{multiply out parentheses} \\ \frac{3x+17}{2} &= \frac{2x-2+x+19}{2} && \text{combine like terms} \\ \frac{3x+17}{2} &= \frac{3x+17}{2} && \text{multiply by 2} \\ 3x+17 &= 3x+17 \end{aligned}$$

Because the left hand side is now identical to the right hand side, this equation is an identity, and all real numbers are solution.

(c) $|3-2x|+2=5$ 0 and 3
Solution:

$$\begin{aligned} |3-2x|+2 &= 5 \quad \text{subtract } 2 \\ |3-2x| &= 3 \\ 3-2x &= 3 \quad \text{or} \quad 3-2x=-3 \quad \text{subtract } 3 \\ -2x &= 0 \quad \text{or} \quad -2x=-6 \quad \text{divide by } -2 \\ x &= 0 \quad \text{or} \quad x=3 \end{aligned}$$

Thus the solution are 0 and 3. We check. If $x=0$, then

$$\text{LHS} = |3-2(0)|+2 = |3|+2 = 3+2 = 5 = \text{RHS}$$

and if $x=3$, then

$$\text{LHS} = |3-2(-3)|+2 = |3-6|+2 = |-3|+2 = 3+2 = 5 = \text{RHS}$$

Thus both numbers are solutions.

(d) $\frac{2}{3}(x-7) = \frac{4}{5}(x+1)$ -41
Solution:

$$\begin{aligned} \frac{2}{3}(x-7) &= \frac{4}{5}(x+1) \\ \frac{2}{3}\cdot\frac{x-7}{1} &= \frac{4}{5}\cdot\frac{x+1}{1} \\ \frac{2(x-7)}{3} &= \frac{4(x+1)}{5} \quad \text{bring fractions to common denominator} \\ \frac{5\cdot 2(x-7)}{15} &= \frac{3\cdot 4(x+1)}{15} \quad \text{multiply both sides by 15} \\ 10(x-7) &= 12(x+1) \quad \text{multiplty out parentheses} \\ 10x-70 &= 12x+12 \quad \text{subtract } 10x \\ -70 &= 2x+12 \quad \text{subtract 12} \\ -82 &= 2x \quad \text{divide by 2} \\ -41 &= x \end{aligned}$$

We check:

$$\begin{aligned} \text{LHS} &= \frac{2}{3}(-41-7) = \frac{2}{3}(-48) = -32 \\ \text{RHS} &= \frac{4}{5}(-41+1) = \frac{4}{5}(-40) = -32 \end{aligned}$$

Thus our solution, -41 is correct.

(e) $7x^2 + (x+3)(2x-1) = (3x+1)^2$ -4
Solution:

$$\begin{aligned} 7x^2 + (x+3)(2x-1) &= (3x+1)^2 && \text{multiply the polynomials on both sides} \\ 7x^2 + 2x^2 + 5x - 3 &= 9x^2 + 6x + 1 && \text{combine like terms} \\ 9x^2 + 5x - 3 &= 9x^2 + 6x + 1 && \text{subtract } 9x^2 \\ 5x - 3 &= 6x + 1 && \text{subtract } 5x \\ -3 &= x + 1 && \text{subtract } 1 \\ -4 &= x \end{aligned}$$

We check our result:

$$\begin{aligned} \text{LHS} &= 7(-4)^2 + ((-4)+3)(2(-4)-1) = 7 \cdot 16 + (-1)(-9) = 112 + 9 = 121 \\ \text{RHS} &= (3(-4)+1)^2 = (-12+1)^2 = (-11)^2 = 121 \end{aligned}$$

Thus the solution, -4 is correct.

(f) $8a + 2a^2 = 42$ $-7, 3$
Solution: since this equation is of a higher degree than 1, our only method is to reduce one side to zero, factor, and then apply the special zero property.

$$\begin{aligned} 8a + 2a^2 &= 42 && \text{subtract 42, rearrange} \\ 2a^2 + 8a - 42 &= 0 && \text{the GCF is } 2 \\ 2\left(a^2 + 4a - 21\right) &= 0 \end{aligned}$$

We will factor $a^2 + 4a - 21$ by grouping. First we conduct the "$pq-$game".

$$\begin{aligned} pq &= -21 && \text{1st coefficient times 3rd coefficient} \\ p + q &= 4 && \text{2nd coefficient} \end{aligned}$$

We start by expressing 21 as a product of two numbers. the only possible pairs are, 1 with 21 and 3 with 7. Since the product pq is negative, one number must be positive, the other one must be positive.. Because the sum $p+q$ is positive, the negative sign has to be in front of the smaller number. We only need to consider -1 with 20, and -3 with 7. Clearly -3 with 7 work as p and q. We use these these numbers to express the second term as the sum of two terms, and then factor by grouping.

$$\begin{aligned} 2\left(a^2 + 4a - 21\right) &= 0 \\ 2\left(\underbrace{a^2 + 7a}\ \underbrace{-3a - 21}\right) &= 0 \\ 2(a(a+7) - 3(a+7)) &= 0 \\ 2(a-3)(a+7) &= 0 \end{aligned}$$

Thus our equation is

$$2(a-3)(a+7) = 0$$

We now apply the special zero property. If this product is zero, then either $2=0$ or $a-3=0$ or $a+7=0$. We solve these equations for a.

$$\begin{array}{rclcccc} a-3 & = & 0 & \text{or} & a+7=0 & \text{or} & 2=0 \\ a & = & 3 & \text{or} & a=-7 & \text{or} & \text{no solution here} \end{array}$$

We check both solutions. If $a=3$, then

$$\text{LHS} = 8(3) + 2(3)^2 = 8\cdot 3 + 2\cdot 9 = 24 + 18 = 42 = \text{RHS} \quad \checkmark$$

If $a=-7$, then

$$\text{LHS} = 8(-7) + 2(-7)^2 = 8\cdot(-7) + 2\cdot 49 = -56 + 98 = 42 = \text{RHS} \quad \checkmark$$

Thus both solutions, -7 and 3 are correct.

(g) $8x^3 = 50x^2$ $\frac{25}{4}, 0$

Solution: since this equation is of a higher degree than 1, our only method is to reduce one side to zero, factor, and then apply the special zero property.

$$\begin{array}{rcll} 8x^3 & = & 50x^2 & \text{subtract } 50x^2 \\ 8x^3 - 50x^2 & = & 0 & \text{the GCF is } 2x^2 \\ 2x^2(4x-25) & = & 0 & \end{array}$$

We now apply the special zero property. If this product is zero, then either $2x^2=0$ or $4x-25=0$. We solve these equations for x.

$$\begin{array}{rclcr} 2x^2 & = & 0 & \text{or} & 4x-25=0 \\ 2\cdot x\cdot x & = & 0 & \text{or} & 4x=25 \\ x & = & 0 & \text{or} & x=\dfrac{25}{4} \end{array}$$

We check both solutions. If $x=0$, then

$$\begin{array}{rcl} \text{LHS} & = & 8\cdot 0^3 = 8\cdot 0 = 0 \\ \text{RHS} & = & 50\cdot 0^2 = 50\cdot 0 = 0 \end{array}$$

If $x=\frac{25}{4}$, then

$$\begin{array}{rcl} \text{LHS} & = & 8\left(\dfrac{25}{4}\right)^3 = \dfrac{8}{1}\cdot\dfrac{15\,625}{64} = \dfrac{15\,625}{8} \\ \text{RHS} & = & 50\left(\dfrac{25}{4}\right)^2 = \dfrac{50}{1}\cdot\dfrac{625}{16} = \dfrac{15\,625}{8} \end{array}$$

Thus both solutions, 0 and $\frac{25}{4}$ are correct.

(h) $8p^3 = 50p$ $-\frac{5}{2},\ 0,\ \frac{5}{2}$
Solution: since this equation is of a higher degree than 1, our only method is to reduce one side to zero, factor, and then apply the special zero property.

$$\begin{aligned} 8p^3 &= 50p && \text{subtract } 50p \\ 8p^3 - 50p &= 0 && \text{the GCF is } 2p \\ 2p\left(4p^2 - 25\right) &= 0 \\ 2p\left((2p)^2 - 5^2\right) &= 0 && \text{factor via difference of squares theorem} \\ 2p\,(2p+5)\,(2p-5) &= 0 \end{aligned}$$

We now apply the special zero property. If this product is zero, then either $2p = 0$ or $2p + 5 = 0$ or $2p - 5 = 0$. We solve these equations for p.

$$\begin{aligned} 2p + 5 &= 0 && \text{or} && 2p - 5 = 0 && \text{or} && 2p = 0 \\ 2p &= -5 && \text{or} && 2p = 5 && \text{or} && p = 0 \\ p &= -\frac{5}{2} && \text{or} && p = \frac{5}{2} \end{aligned}$$

We check all three solutions. If $p = -\frac{5}{2}$, then

$$\begin{aligned} \text{LHS} &= 8\left(-\frac{5}{2}\right)^3 = \frac{8}{1} \cdot \frac{-125}{8} = -125 \\ \text{RHS} &= 50\left(-\frac{5}{2}\right) = \frac{50}{1} \cdot \frac{-5}{2} = \frac{-250}{2} = -125 \end{aligned}$$

If $p = \frac{5}{2}$, then

$$\begin{aligned} \text{LHS} &= 8\left(\frac{5}{2}\right)^3 = \frac{8}{1} \cdot \frac{125}{8} = 125 \\ \text{RHS} &= 50\left(\frac{5}{2}\right) = \frac{50}{1} \cdot \frac{5}{2} = \frac{250}{2} = 125 \end{aligned}$$

and if $p = 0$, then

$$\begin{aligned} \text{LHS} &= 8 \cdot 0^3 = 8 \cdot 0 = 0 \\ \text{RHS} &= 50 \cdot 0 = 0 \end{aligned}$$

Thus all three solutions, $-\frac{5}{2}$, 0, and $\frac{5}{2}$ are correct.

(i) $2-(3-x)(2x+5)=(x-1)(2x-1)$ 7
Solution: We have to use the FOIL method on both sides to perform the multiplications. It is very important, however, to keep the expressions in a parentheses since we are dealing with subtraction between *algebraic expressions*.

$$\begin{aligned}
2-(3-x)(2x+5) &= (x-1)(2x-1) && \text{FOIL}\\
2-\left(6x+15-2x^2-5x\right) &= 2x^2-x-2x+1 && \text{combine like terms}\\
2-\left(-2x^2+x+15\right) &= 2x^2-3x+1 && \text{perform subtraction}\\
2+2x^2-x-15 &= 2x^2-3x+1 && \text{combine like terms}\\
2x^2-x-13 &= 2x^2-3x+1 && \text{subtract } 2x^2 \quad \text{(the equation is linear!)}\\
-x-13 &= -3x+1 && \text{add } 3x\\
2x-13 &= 1 && \text{add 13}\\
2x &= 14 && \text{divide by 2}\\
x &= 7
\end{aligned}$$

We check: if $x=7$, then

$$\begin{aligned}
\text{LHS} &= 2-(3-7)(2(7)+5)=2-(-4)(14+5)=2-(-4)19=2-(-76)=78\\
\text{RHS} &= (7-1)(2(7)-1)=6(14-1)=6\cdot 13=78
\end{aligned}$$

Thus our solution, 7 is correct.

(j) $x^2=4x+1$. $2+\sqrt{5},\ 2-\sqrt{5}$
Solution : We complete the square.

$$\begin{aligned}
x^2 &= 4x+1\\
x^2-4x-1 &= 0 && (x-2)^2=x^2-4x+4\\
\underbrace{x^2-4x+4}-4-1 &= 0\\
(x-2)^2-5 &= 0\\
(x-2)^2-\left(\sqrt{5}\right)^2 &= 0\\
\left(x-2+\sqrt{5}\right)\left(x-2-\sqrt{5}\right) &= 0\\
x_1 &= 2-\sqrt{5} \quad \text{and} \quad x_2=2+\sqrt{5}
\end{aligned}$$

We check: if $x=2-\sqrt{5}$, then

$$\begin{aligned}
\text{LHS} &= \left(2-\sqrt{5}\right)^2=4-2\sqrt{5}-2\sqrt{5}+5=9-4\sqrt{5}\\
\text{RHS} &= 4\left(2-\sqrt{5}\right)+1=8-4\sqrt{5}+1=9-4\sqrt{5}
\end{aligned}$$

If $x=2+\sqrt{5}$, then

$$\begin{aligned}
\text{LHS} &= \left(2+\sqrt{5}\right)^2=4+2\sqrt{5}+2\sqrt{5}+5=9+4\sqrt{5}\\
\text{RHS} &= 4\left(2+\sqrt{5}\right)+1=8+4\sqrt{5}+1=9+4\sqrt{5}
\end{aligned}$$

Thus our solution, $2+\sqrt{5}$ and $2-\sqrt{5}$ is correct.

(k) $4x^2+20x+7=0$ $\dfrac{-5+\sqrt{18}}{2}$, $\dfrac{-5-\sqrt{18}}{2}$
Solution:

$$\begin{aligned} 4x^2+20x+7 &= 0 \\ 4\left(x^2+5x+\frac{7}{4}\right) &= 0 \end{aligned}$$

Half of the linear coefficient is $\dfrac{5}{2}$. Thus the complete square is $\left(x+\dfrac{5}{2}\right)^2$

$$\left(x+\frac{5}{2}\right)^2=\left(x+\frac{5}{2}\right)\left(x+\frac{5}{2}\right)=x^2+\frac{5}{2}x+\frac{5}{2}x+\frac{25}{4}=x^2+5x+\frac{25}{4}$$

So we smuggle in $\dfrac{25}{4}$.

$$\begin{aligned} 4\left(\underbrace{x^2+5x+\frac{25}{4}}-\frac{25}{4}+\frac{7}{4}\right) &= 0 \\ 4\left(\left(x+\frac{5}{2}\right)^2-\frac{18}{4}\right) &= 0 \end{aligned}$$

Although $\dfrac{18}{4}$ can be simplified, we will keep it as it is, because the common denominator will be easier this way. We next apply the difference of squares theorem. $\dfrac{18}{4}$ is the square of a number - its own square root.

$$\frac{18}{4}=\left(\sqrt{\frac{18}{4}}\right)^2=\left(\frac{\sqrt{18}}{\sqrt{4}}\right)^2=\left(\frac{\sqrt{18}}{2}\right)^2$$

$$\begin{aligned} 4\left(\left(x+\frac{5}{2}\right)^2-\frac{18}{4}\right) &= 0 \\ 4\left(\left(x+\frac{5}{2}\right)^2-\left(\frac{\sqrt{18}}{2}\right)^2\right) &= 0 \\ 4\left(x+\frac{5}{2}+\frac{\sqrt{18}}{2}\right)\left(x+\frac{5}{2}-\frac{\sqrt{18}}{2}\right) &= 0 \end{aligned}$$

We solve for the zeroes of both linear factors:

$$\begin{aligned} x+\frac{5}{2}+\frac{\sqrt{18}}{2} &= 0 \\ x+\frac{\sqrt{18}}{2} &= -\frac{5}{2} \\ x_1 &= -\frac{5}{2}-\frac{\sqrt{18}}{2}=\frac{-5-\sqrt{18}}{2} \end{aligned}$$

$$
\begin{aligned}
x + \frac{5}{2} - \frac{\sqrt{18}}{2} &= 0 \\
x - \frac{\sqrt{18}}{2} &= -\frac{5}{2} \\
x_2 &= -\frac{5}{2} + \frac{\sqrt{18}}{2} = \frac{-5+\sqrt{18}}{2}
\end{aligned}
$$

We check: if $x = \dfrac{-5-\sqrt{18}}{2}$, then

$$
\begin{aligned}
\text{LHS} &= 4x^2 + 20x + 7 = 4\left(\frac{-5-\sqrt{18}}{2}\right)^2 + 20\left(\frac{-5-\sqrt{18}}{2}\right) + 7 \\
&= \frac{4}{1} \cdot \frac{\left(-5-\sqrt{18}\right)^2}{4} + \frac{20}{1}\left(\frac{-5-\sqrt{18}}{2}\right) + 7 = \\
&= \frac{25 + 5\sqrt{18} + 5\sqrt{18} + 18}{1} - \frac{10\left(-5-\sqrt{18}\right)}{1} + 7 = \\
&= 43 + 10\sqrt{18} - 10\left(-5-\sqrt{18}\right) + 7 = 43 + 10\sqrt{18} - 50 - 10\sqrt{18} + 7 \\
&= -7 + 7 = 0 = \text{RHS}
\end{aligned}
$$

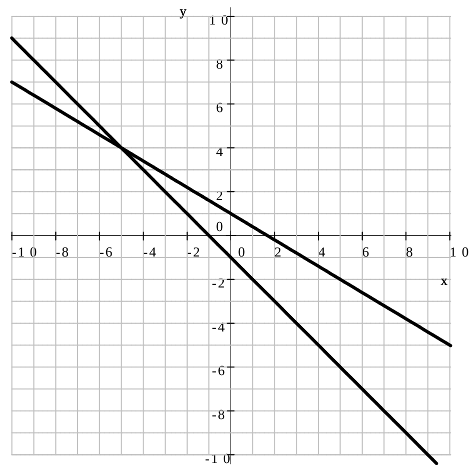
and if $x = \dfrac{-5+\sqrt{18}}{2}$, then

$$
\begin{aligned}
\text{LHS} &= 4x^2 + 20x + 7 = 4\left(\frac{-5+\sqrt{18}}{2}\right)^2 + 20\left(\frac{-5+\sqrt{18}}{2}\right) + 7 \\
&= \frac{4}{1} \cdot \frac{\left(-5+\sqrt{18}\right)^2}{4} + \frac{20}{1}\left(\frac{-5+\sqrt{18}}{2}\right) + 7 = \\
&= \frac{25 - 5\sqrt{18} - 5\sqrt{18} + 18}{1} - \frac{10\left(-5+\sqrt{18}\right)}{1} + 7 = \\
&= 43 - 10\sqrt{18} - 10\left(-5+\sqrt{18}\right) + 7 = 43 - 10\sqrt{18} - 50 + 10\sqrt{18} + 7 \\
&= -7 + 7 = 0 = \text{RHS}
\end{aligned}
$$

Thus our solution, $\dfrac{-5+\sqrt{18}}{2}$ and $\dfrac{-5-\sqrt{18}}{2}$ is correct.

Note: since $\sqrt{18} = 3\sqrt{2}$, the final answer $\dfrac{-5+3\sqrt{2}}{2}$ and $\dfrac{-5-3\sqrt{2}}{2}$ is equally correct.

7. Graph the straight lines $3x + 5y = 5$ and $y = -x - 1$ in the same coordinate system. Use your graph to find the coordinates of the point where the lines intersect. $(-5, 4)$
Solution: We can easily graph the lines by using the slope-intercept forms: $y = -x - 1$ and $y = -\frac{3}{5}x + 1$



8. Find an equation of the straight line that is parpendicular to $2x - 3y = -6$ and passes through the point $(-12, 5)$. $y = -\frac{3}{2}x - 13$
Solution: We obtain the slope of the line give by solving for y.

$$\begin{aligned} 2x - 3y &= -6 && \text{add } 3y \\ 2x &= 3y - 6 && \text{add } 6 \\ 2x + 6 &= 3y && \text{ivie by } 3 \\ \frac{2x+6}{3} &= y && \Longrightarrow \quad y = \frac{2}{3}x + \frac{6}{3} = \frac{2}{3}x + 2 \end{aligned}$$

Our line is perpenicular to one with slope $\frac{2}{3}$: thus its slope must be the negative reciprocal of $\frac{2}{3}$, which is $-\frac{3}{2}$.
We are looking for the slope-intecept form of the line:

$$\begin{aligned} y &= mx + b && \text{we know } m = -\frac{3}{2} \\ y &= -\frac{3}{2}x + b \end{aligned}$$

We will find b using the fact that the line passes through $(-12, 5)$

$$\begin{aligned} 5 &= -\frac{3}{2}(-12) + b \\ 5 &= 18 + b \\ -13 &= b \end{aligned}$$

Thus the equation is $y = -\frac{3}{2}x - 13$

9. Find an equation of the straight line that passes through the points $(2,7)$ and $(-2,-5)$.
$y = 3x + 1$
Solution 1: We obtain the slope from the slope formula

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-(-5)}{2-(-2)} = \frac{12}{4} = 3$$

For b, we substitute either point into the slope-intercept form of the equation.

$$\begin{aligned} y &= mx + b && \text{we know } m = 3 \\ y &= 3x + b && (2,7) \text{ is on the line} \\ 7 &= 3(2) + b \\ 7 &= 6 + b \\ 1 &= b \end{aligned}$$

Thus the equation is $y = 3x + 1$. We can easily check by substituting the coordinates of the points into the equation.
Solution 2: Write $y = mx + b$ and solve the system that we obtain by substituting the points given.

$$\begin{aligned} 7 &= m(2) + b && (2,7) \text{ is on the line} \\ -5 &= m(-2) + b && (-2,-5) \text{ is on the line} \end{aligned}$$

Thus we get the system

$$\begin{aligned} 2m + b &= 7 \\ -2m + b &= -5 \end{aligned}$$

We eliminate m if simply add the two equations

$$\begin{aligned} 2b &= 2 \\ b &= 1 \end{aligned}$$

Now we can easily solve for m in the first equation

$$\begin{aligned} 2m + 1 &= 7 \\ 2m &= 6 \\ m &= 3 \end{aligned}$$

Thus the equation is $y = 3x + 1$. We can easily check by substituting the coordinates of the points into the equation.

10. Graph the parabola $y = -8x + x^2 + 15$. Clearly label the coordinates of five points on the parabola, including vertex and intercepts.
Solution: We obtain all forms of the equation first.

$$y = x^2 - 8x + 15 \implies \text{polynomial form}$$

Half of the linear coefficient is -4, thus we will work with $(x-4)^2 = x^2 - 8x + 16$

$$\begin{aligned} y &= x^2 - 8x + 15 \\ y &= \underbrace{x^2 - 8x + 16}-16 + 15 \\ y &= (x-4)^2 - 1 \implies \text{complete square form} \end{aligned}$$

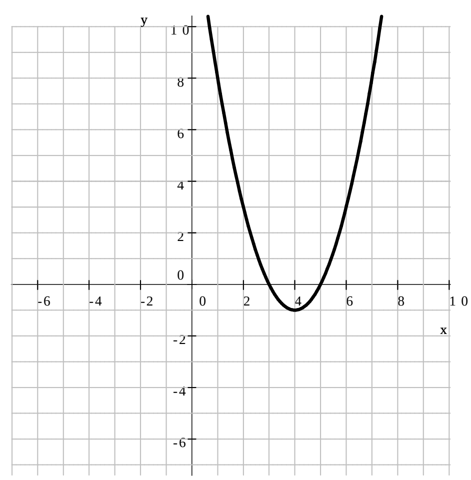
We factor via the difference of squares theorem

$$\begin{aligned} y &= (x-4)^2 - 1^2 \quad \text{since } 1 = 1^2 \\ y &= (x-4+1)(x-4-1) \\ y &= (x-3)(x-5) \implies \text{factored form} \end{aligned}$$

From the polynomial form we obtain the $y-$intercept, $(0,15)$. From the complete square form, the vertex is $(4,-1)$. Finally, the factored form tells us that there are two $x-$intercepts, $(3,0)$ and $(5,0)$. The few missing points, close to the vertex can be found by substituting values for x into any of the three forms of the equations to find y. This time we will work with the polynomial form.

$$\begin{aligned} \text{if } x &= 2, \text{ then } y = (2)^2 - 8(2) + 15 = 4 - 16 + 15 = 3 \\ \text{if } x &= 6, \text{ then } y = (6)^2 - 8(6) + 15 = 36 - 48 + 15 = 3 \end{aligned}$$

We are ready to graph:



11. One side of a rectangle is 4 ft shorter than three times the other side. Find the sides if the perimeter is 64 ft. 9 ft and 23 ft
Solution: Let us denote the shorter side by x. Then the other side is $3x-4$. The equation expresses the perimeter of the rectangle.

$$\begin{aligned} 2(x)+2(3x-4) &= 64 && \text{multiply out parentheses}\\ 2x+6x-8 &= 64 && \text{combine like terms}\\ 8x-8 &= 64 && \text{add}\\ 8x &= 72 && \text{divide by 8}\\ x &= 9 \end{aligned}$$

If the shorter side was denoted by x, we now know it is 9 ft. The longer side was denoted by $3x-4$, so it must be $3(9 \text{ ft})-4 \text{ ft} = 23 \text{ ft}$. Thus the sides of the rectangle are 9 ft and 23 ft. We check: $P = 2(9 \text{ ft}) + 2(23 \text{ ft}) = 64 \text{ ft}$ and $23 \text{ ft} = 3(9 \text{ ft}) - 4 \text{ ft}$. Thus our solution is correct.

12. One side of a rectangle is 4 ft shorter than three times the other side. Find the sides if the area is 84 ft^2. 6 ft and 14 ft
Solution: Let us denote the shorter side by x. Then the other side is $3x-4$. The equation expresses the area of the rectangle.

$$\begin{aligned} x(3x-4) &= 84 && \text{multiply out parentheses}\\ 3x^2-4x &= 84 && \text{subtract 84}\\ 3x^2-4x-84 &= 0 \end{aligned}$$

Because the equation is quadratic, we need to factor the left-hand side and then apply the zero property. We will factor by grouping. First we conduct the "$pq-$game". The sum of p and q has to be the linear coefficient (the number in front of x, with its sign), so it is -4. The product of p and q has to be the product of the other coefficients, $3(-84) = -252$.

$$\begin{aligned} pq &= -252\\ p+q &= -4 \end{aligned}$$

Now we need to find p and q. Because the product is negative, we re looking for a positive and a negative number. Becuse the sum is negtive, the larger number must carry the negative sign. We enter $\sqrt{252} = 15.874\,507\,87$ into the calculator and get a decimal:

$$\sqrt{252} = 15.874....$$

So we start looking for factors of 252, starting at 15, and moving down. We soon find 14 and -18. These are our values for p and q. We use these numbers to express the linear term:

$$-4x = 14x - 18x$$

and factor by grouping.

$$\begin{aligned} 3x^2-4x-84 &= 0\\ \underbrace{3x^2+14x}\underbrace{-18x-84} &= 0\\ x(3x+14)-6(3x+14) &= 0\\ (x-6)(3x+14) &= 0 \end{aligned}$$

We now apply the zero property. Either $x-11=0$ or $3x+14=0$. We solve both these equations for x.

$$\begin{aligned} x-6 &= 0 \\ x &= 6 \end{aligned}$$

and

$$\begin{aligned} 3x+14 &= 0 \\ 3x &= -14 \\ x &= -\frac{14}{3} \end{aligned}$$

Since distances can not be negative, the second solution for x, $-\frac{14}{3}$ is ruled out. Thus $x=6$. Then the longer side is $3\,(6\ \text{ft})-4\ \text{ft}=14\ \text{ft}$, and so the rectangle's sides are 6 ft and 14 ft long. We check: $6\ \text{ft}\,(14\ \text{ft})=84\ \text{ft}^2$ and $14\ \text{ft}=3\,(6\ \text{ft})-4\ \text{ft}$. Thus our solution is correct.

13. One side of a rectangle is 4 in shorter than 3 times the other side. Find the sides of the rectangle if its area is $319\ \text{in}^2$. 11 in by 29 in
Solution: Let us denote the shorter side by x. Then the larger side is $3x-4$. The equation expresses the area.

$$\begin{aligned} x\,(3x-4) &= 319 \\ 3x^2-4x &= 319 \\ 3x^2-4x-319 &= 0 \end{aligned}$$

We will complete the square. (It can be done by the pq-game as well)

$$\begin{aligned} 3x^2-4x-319 &= 0 \\ 3\left(x^2-\frac{4}{3}x-\frac{319}{3}\right) &= 0 \end{aligned}$$

Half of the linear coefficient is $-\frac{4}{3}\div 2=-\frac{4}{3}\cdot\frac{1}{2}=-\frac{4}{6}=-\frac{2}{3}$, thus we work out $\left(x-\frac{2}{3}\right)^2$.

$$\begin{aligned} \left(x-\frac{2}{3}\right)^2 &= \left(x-\frac{2}{3}\right)\left(x-\frac{2}{3}\right)=x^2-\frac{2}{3}x-\frac{2}{3}x+\frac{4}{9} \\ &= x^2-\frac{4}{3}x+\frac{4}{9} \end{aligned}$$

Thus we smuggle in $\frac{4}{9}$.

$$\begin{aligned} 3\left(\underbrace{x^2-\frac{4}{3}x+\frac{4}{9}}-\frac{4}{9}-\frac{319}{3}\right) &= 0 \\ 3\left(\left(x-\frac{2}{3}\right)^2-\frac{4}{9}-\frac{319}{3}\right) &= 0 \end{aligned}$$

We bring the last two numbers to the common denominator:

$$\begin{aligned}
3\left(\left(x-\frac{2}{3}\right)^2-\frac{4}{9}-\frac{319\,(3)}{3\,(3)}\right) &= 0\\
3\left(\left(x-\frac{2}{3}\right)^2-\frac{4}{9}-\frac{957}{9}\right) &= 0\\
3\left(\left(x-\frac{2}{3}\right)^2-\frac{961}{9}\right) &= 0
\end{aligned}$$

Since $\frac{961}{9}=\left(\frac{31}{3}\right)^2$, we factor via the difference of squares theorem.

$$\begin{aligned}
3\left(\left(x-\frac{2}{3}\right)^2-\left(\frac{31}{3}\right)^2\right) &= 0\\
3\left(x-\frac{2}{3}+\frac{31}{3}\right)\left(x-\frac{2}{3}-\frac{31}{3}\right) &= 0\\
3\left(x+\frac{29}{3}\right)\left(x-\frac{33}{3}\right) &= 0\\
3\left(x+\frac{29}{3}\right)(x-11) &= 0
\end{aligned}$$

This equation has two solutions, $x_1=-\frac{29}{3}$ and $x_2=11$. Since distances are positive, $-\frac{29}{3}$ is ruled out as a solution for the shorter side. The other solution is 11 in. This makes the longer side $3\cdot 11-4=29$ in. We check: $3\,(11)-4=29$ and $11\,(29)=319$. Thus our solution, 11 in by 29 in is correct.

14. A bank teller has 23 more five-dollar bills than ten-dollar bills. The total value of the money is \$610. How much of each denomination of bill does he have? 33 ten-dollar bills and 56 five-dollar bills
Solution: Let us denote the number of ten-dollar bills by x. Then we have $x+23$ many five-dollar bills. The equation expresses the value of the bills.

$$\begin{aligned}
10x+5\,(x+23) &= 610 && \text{distribute}\\
10x+5x+115 &= 610 && \text{combine like terms}\\
15x+115 &= 610 && \text{subtract 115}\\
15x &= 495 && \text{divide by 15}\\
x &= 33
\end{aligned}$$

Thus we have 33 tens and $33+23=56$ fives. We check: $56-33=23$ and $33\,(10)+56\,(5)=610$. Thus our solution; 33 ten-dollar bills and 56 five-dollar bills; is correct.

15. The population of a town has decreased from 80 000 to 68 000. What percent of a decrease does this represent? 15% decrease

Solution 1: We subtract 68 000 from 80 000 to determine the change. $80\,000 - 68\,000 = 12\,000$. Now the question is: 12 000 is what percent of 80 000? Then

$$\begin{aligned} \text{(is)} &= 12\,000 \\ \text{F} &= x \\ \text{(of)} &= 80\,000 \end{aligned}$$

We substitute the data into the formula:

$$\begin{aligned} \text{(is)} &= \text{F} \cdot \text{(of)} \\ 12\,000 &= x \cdot 80\,000 \\ \frac{12\,000}{80\,000} &= x \\ 0.15 &= x \end{aligned}$$

Thus

$$x = 0.15 = \frac{0.15}{1} = \frac{0.15\,(100)}{1\,(100)} = \frac{15}{100} = 15\%$$

This is a 15% decrease.

Solution 2: The question may be re-phrased as: 68 000 is what percent of 80 000? Then

$$\begin{aligned} \text{(is)} &= 68\,000 \\ \text{F} &= x \\ \text{(of)} &= 80\,000 \end{aligned}$$

We substitute the data into the formula:

$$\begin{aligned} \text{(is)} &= \text{F} \cdot \text{(of)} \\ 68\,000 &= x \cdot 80\,000 \\ \frac{68\,000}{80\,000} &= x \\ 0.85 &= x \end{aligned}$$

Thus

$$x = 0.85 = \frac{0.85}{1} = \frac{0.85\,(100)}{1\,(100)} = \frac{85}{100} = 85\%$$

Since the population has decreased to 85% of its previous count, this is a 15% decrease.

16. We invested \$10000 into two bank accounts. One account earns 14% per year, the other account earns 8% per year. How much did we invest into each account if the combined interest from the two accounts is \$1238 after the first year?
\$ 7300 at 14% and \$ 2700 at 8%
Solution: Let us denote the amount invested at 14% by x and the amount invested at 8% by y. The two equations express that

$$\begin{aligned} x + y &= 10000 && \text{the amounts add up to \$10000} \\ 0.14x + 0.08y &= 1238 && \text{the interests earned add up to \$1238} \end{aligned}$$

We solve the system of equation by elimination. But let us first make the second equation simpler:

$$\begin{aligned} 0.14x + 0.08y &= 1238 && \text{multiply by 100} \\ 14x + 8y &= 123800 && \text{divide by 2} \\ 7x + 4y &= 61900 \end{aligned}$$

We now have

$$\begin{aligned} x + y &= 10000 \\ 7x + 4y &= 61900 \end{aligned}$$

We will multiply the first equation by -4 to eliminate y.

$$\begin{aligned} -4x - 4y &= -40000 \\ 7x + 4y &= 61900 \end{aligned}$$

We add the equations and solve for x.

$$\begin{aligned} 3x &= 21900 && \text{divide by 3} \\ x &= 7300 \end{aligned}$$

Thus we invested \$7300 at 14%. The other amount is then from the first equation:

$$\begin{aligned} 7300 + y &= 10000 \\ y &= 2700 \end{aligned}$$

We invested \$ 7300 at 14% and \$ 2700 at 8%. We check: the amounts add up to $\$7300 + \$2700 = \$10000$. The interest from the accounts are

$$\begin{aligned} 14\% \text{ of } 7300 \text{ is } 0.14\,(7300) &= 1022 \text{ and} \\ 8\% \text{ of } 2700 \text{ is } 0.08\,(2700) &= 216 \end{aligned}$$

Since $1022 + 216 = 1238$, our solution is correct.

17. The hypotenuse of a right triangle is 68 cm. The difference between the other two sides is 28. cm. Find the sides of the triangle. 32 cm and 60 cm
Solution: Let x denote the shorter leg. Then the other leg is $x+28$ cm long. We state the Pythagorean theorem for the triangle, and solve the quadratic equation for x.

$$\begin{aligned} x^2+(x+28)^2 &= 68^2 && \text{FOIL out } (x+28)^2 \\ x^2+x^2+56x+784 &= 4624 && \text{combine like terms} \\ 2x^2+56x+784 &= 4624 && \text{subtract 4624} \\ 2x^2+56x-3840 &= 0 && \text{factor out 2} \\ 2\left(x^2+28x-1920\right) &= 0 && \text{divide by 2} \\ x^2+28x-1920 &= 0 \end{aligned}$$

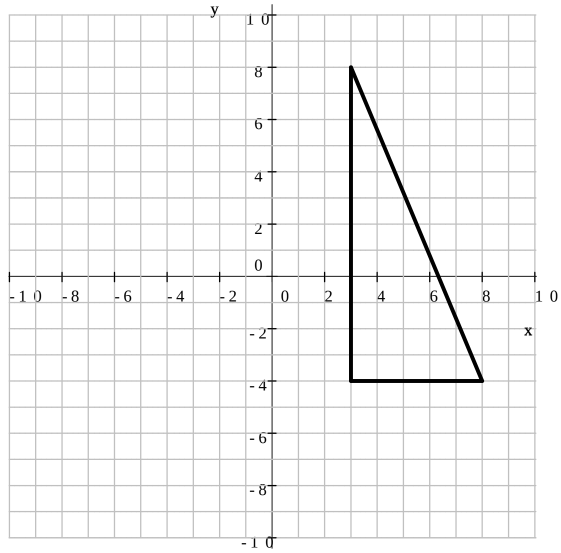
We factor by completing the square. Since half of the linear coefficient is 14, we will work with $(x+14)^2 = x^2+28x+196$

$$\begin{aligned} \underbrace{x^2+28x+196}-196-1920 &= 0 \\ (x+14)^2-2116 &= 0 \\ (x+14)^2-46^2 &= 0 \\ (x+14+46)(x+14-46) &= 0 \\ (x+60)(x-32) &= 0 \\ x_1 &= -60 \quad \text{and} \quad x_2 = 32 \end{aligned}$$

Since distances are never negative, -60 is ruled out. If the shortest side is 32 cm, the other side is $32\,\text{cm} + 28\,\text{cm} = 60\,\text{cm}$. Thus the solution is 32 cm and 60 cm. We check:

$$\begin{aligned} 60-32 &= 28 \text{ and} \\ 60^2+32^2 &= 3600+1024 = 4624 = 68^2 \end{aligned}$$

18. Find the distance between $(3, 8)$ and $(8, -4)$. 13 units
Solution: We graph the points, they determine a right triangle as shown below. The legs are 5 and 12 units long, and we need to find the hypotenuse.



$$\begin{aligned}
5^2 + 12^2 &= x^2 \\
25 + 144 &= x^2 \\
169 &= x^2 \\
0 &= x^2 - 13^2 \\
0 &= (x + 13)(x - 13) \\
x_1 &= -13 \quad \text{and} \quad x_2 = 13
\end{aligned}$$

Since distances are never negative, the answer is 13 units.