1. Simplify each of the following expressions.

   a) $\left(\dfrac{1}{2} - 3\dfrac{1}{5}\right)\left(1\dfrac{1}{2}\right)^{-2} - (-5)^{-1}$

   b) $\dfrac{(-2a^{-2})^{-2} b^3 a^0 \left(-aba^{-2}b^{-2}\right)^{-3}}{2a^2 \left(-2a^{-2}b\right)^{-2} ab^0}$

   c) $\dfrac{x^{-1} + y^{-5}}{x^{-2} - y^{-10}}$

   d) $\dfrac{1 - \dfrac{x}{x-1}}{\dfrac{x}{x+1} - 1}$

2. Completely factor each of the following.

   a) $100x - x^2 - 2419$

   b) $2p^4 - 162$

   c) $357ab^2 - 30ab^2x - 3ab^2x^2$

   d) $3a^3 - 27ab^2$

   e) $20x + 5x^3$

3. (Rational Expressions) Simplify each of the following.

   a) $\dfrac{3x - 11}{11 - 3x}$

   b) $\dfrac{495 - 3a^2 - 12a}{6a^2 - 36a - 330}$

   c) $\dfrac{x^2 - 6x - 7}{2x^2 - 98} \div \dfrac{x^2 - 4x - 5}{x + 7}$

   d) $\dfrac{2x - 13}{x - 5} - \dfrac{3}{5 - x}$

   e) $\dfrac{1}{x - 5} - \dfrac{4x - 4}{6x + x^2 - 55}$

4. (Radical Expressions) Simplify each of the following.

   a) $\sqrt{75} - \sqrt{108} + 5\sqrt{12}$

   b) $\left(2\sqrt{5} + 3\right)\left(2\sqrt{5} - 3\right)$

   c) $\left(2\sqrt{5} - 3\right)^2$

   d) $\left(2\sqrt{5} - 3\right)^3$

   e) Rationalize the denominator in $\dfrac{6}{\sqrt{11} + 3}$

   f) Rationalize the denominator in $\dfrac{4}{\sqrt{15} - 3}$

5. Let $a = \sqrt{5} - 1$. Find the exact value of each of the following expressions.

   a) $(a + 1)^2$

   b) $a^2 + a - 1$

   c) $(a - 1)^2$

   d) $a^2 + 2a - 9$

   e) $-a^2 - 7a + 1$

   f) $3a^2 - a - 4$

6. (Equations) Solve each of the following equations. Make sure to check your solutions.

   a) $3(x - 5) - 5(x - 1) = -2x + 1$

   b) $(3x)^2 - (x + 3)(5x - 3) = (5 - 2x)^2 - 16$

   c) $(x + 4)(1 - 2x) = 3x - 2(x - 3)^2$

   d) $2x^3 = 20x^2 + 1750x$

7. Solve each of the following inequalities. Graph the solution set.

   a) $\dfrac{1}{5}x - \dfrac{2}{3} < \dfrac{26}{15}$

   b) $3w - 5 \le 5(w - 2)$

   c) $7(j - 5) + 9 > 2(-2j + 5) + 5j$

8. Solve each of the following systems of linear equations.

   a) $\begin{cases} 2x - 3y = 24 \\ y = \dfrac{2}{3}x - 8 \end{cases}$

   b) $\begin{cases} 2x - y = 14 \\ 5x + 2y = -1 \end{cases}$

   c) $\begin{cases} 2x - 5y = 20 \\ x - 5 = \dfrac{5}{2}y + 1 \end{cases}$

9. Word Problems.

   (a) A bank teller has 23 more five-dollar bills than ten-dollar bills. The total value of the money is \$610. How much of each denomination of bill does he have?

   (b) We have 73 coins, all nickels and dimes. The total value of the coins is \$ 6. How many nickels and dimes do we have?

   (c) There is an animal farm where chickens and cows live. There are 106 heads and 386 legs. How many chickens, how many cows?

   (d) We throw an object upward from the top of a 1200 ft high building. The height of the object, (measured in feet) t seconds after we threw it is

$$h\left(t\right) = -16t^2 + 160t + 1200$$

   i. Where is the object 3 seconds after we threw it?
   ii. How long does it take for the object to hit the ground?

10. Graph the parabola $y = -8x + x^2 + 15$. Clearly label the coordinates of five points on the parabola, including vertex and intercepts.

11. Consider the equations $y = \dfrac{2}{3}x - 2$ and $y = -\dfrac{1}{2}x + 5$.

   (a) Graph these lines in the same coordinate system. Use your graph to find the coordinates where the points intersect.

   (b) Use algebraic methods to check your answer for part a).

Answers

1. a) -1 b) $-\dfrac{b^8}{2}$ c) $\dfrac{xy^5}{y^5 - x}$ d) $\dfrac{x+1}{x-1}$

2. a) $-(x-41)(x-59)$ b) $2\left(p^2+9\right)(p+3)(p-3)$ c) $-3ab^2(x+17)(x-7)$
   d) $3a(a+3b)(a-3b)$ e) $5x\left(x^2+4\right)$

3. a) -1 b) $-\dfrac{a+15}{2(a+5)}$ c) $\dfrac{1}{2x-10}$ d) 2 e) $-\dfrac{3}{x+11}$

4. a) $9\sqrt{3}$ b) 11 c) $29 - 12\sqrt{5}$ d) $94\sqrt{5} - 207$ e) $3\sqrt{11} - 9$ f) $\dfrac{2\sqrt{15}+6}{3}$

5. a) 5 b) $4 - \sqrt{5}$ c) $9 - 4\sqrt{5}$ d) -5 e) $2 - 5\sqrt{5}$ f) $15 - 7\sqrt{5}$

6. a) no solution   b) 0 c) 1 d) $35, 0, -25$

7. a) $x < 12$ b) $x \geq \dfrac{5}{2}$ c) $x > 6$

8. a) dependent system; there are infinitely many solutions   b) $(3, -8)$

    c) inconsistent system; there is no solution

9. a) 33 ten-dollar bills and 56 five-dollar bills   c) 19 chickens, 87 cows

    b) 26 nickels and 47 dimes   d) i) 1536 ft   ii) 15 seconds

10. $y-$intercept: $(0, 15)$.   Vertex: $(4, -1)$. $x-$intercepts: $(3, 0)$ and $(5, 0)$.
     Additional points: $(2, 3)$ and $(6, 3)$
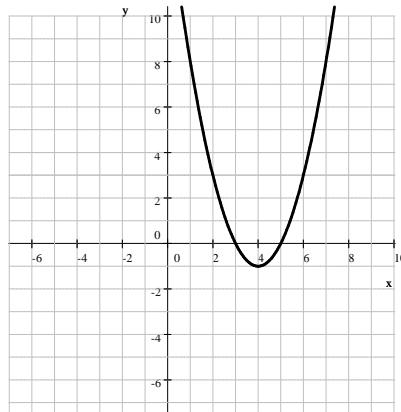
11. a) $(6, 2)$
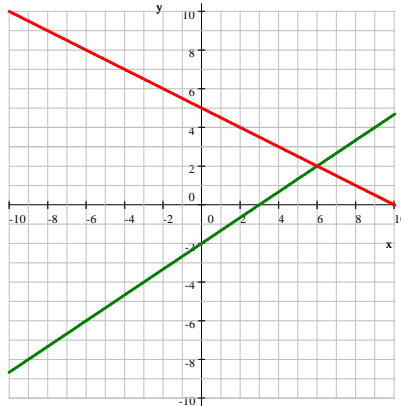
   b) Is the point $(6, 2)$ on the line $y = \dfrac{2}{3}x - 2$?

$$\text{LHS} \;=\; y = 2$$
$$\text{RHS} \;=\; \frac{2}{3}(6) - 2 = 4 - 2 = 2 \;\;\Longrightarrow\;\; \text{yes}$$

   Is the point $(6, 2)$ on the line $y = -\dfrac{1}{2}x + 5$?

$$\text{LHS} \;=\; y = 2$$
$$\text{RHS} \;=\; -\frac{1}{2}(6) + 5 = -3 + 5 = 2 \;\;\Longrightarrow\;\; \text{yes}$$

   Since the point $(6, 2)$ is on both lines, it must be the point of intersection.