

1. Simplify each of the following expressions.

$$(a) \frac{ab - a - b + 1}{b^2 - 1} = \frac{a - 1}{b + 1}$$

Solution: We will factor both numerator and denominator and then cancel. The numerator can be factored by grouping

$$\underbrace{ab - a}_{a(b-1)} \quad \underbrace{-b + 1}_{-1(b-1)} = a(b-1) - 1(b-1) = (a-1)(b-1)$$

The denominator factors by the difference of squares theorem.

$$b^2 - 1 = (b+1)(b-1)$$

Thus the fraction can be simplified as

$$\frac{ab - a - b + 1}{b^2 - 1} = \frac{(a-1)(b-1)}{(b+1)(b-1)} = \frac{a-1}{b+1}$$

$$(b) \frac{5x - 30}{x^2 - 36} \cdot \frac{3x + 18}{5} = 3$$

Solution: we will factor whatever we can and then cancel.

$$\frac{5x - 30}{x^2 - 36} \cdot \frac{3x + 18}{5} = \frac{5(x-6)}{(x+6)(x-6)} \cdot \frac{3(x+6)}{5} = 3$$

$$(c) \frac{px^2 - 16q - 16p + qx^2}{x^2 + 5x + 6} \cdot \frac{x^2 + 6x + 9}{4px^2 + px^3 + 4qx^2 + qx^3} = \frac{(x+3)(x-4)}{x^2(x+2)} = \frac{x^2 - x - 12}{2x^2 + x^3}$$

Solution: $x^2 + 5x + 6 = (x+2)(x+3)$ and $x^2 + 6x + 9 = (x+3)(x+3)$ are easy. With the other expressions, we start with the greatest common factor.

$$\begin{aligned} px^2 - 16q - 16p + qx^2 &= \underbrace{px^2 + qx^2}_{(p+q)x^2} - \underbrace{16p - 16q}_{16(p-q)} && \text{grouping} \\ &= x^2(p+q) - 16(p-q) && \text{factor out } p+q \\ &= (x^2 - 16)(p+q) && \text{difference of squares} \\ &= (x+4)(x-4)(p+q) \end{aligned}$$

$$\begin{aligned} 4px^2 + px^3 + 4qx^2 + qx^3 &= x^2 \left(\underbrace{4p + 4q}_{4(p+q)} + \underbrace{px + qx}_{x(p+q)} \right) && \text{grouping} \\ &= x^2(4(p+q) + x(p+q)) && \text{factor out } p+q \\ &= x^2(x+4)(p+q) \end{aligned}$$

We have all the pieces now:

$$\begin{aligned} \frac{px^2 - 16q - 16p + qx^2}{x^2 + 5x + 6} \cdot \frac{x^2 + 6x + 9}{4px^2 + px^3 + 4qx^2 + qx^3} &= \\ \frac{(x+4)(x-4)(p+q)}{(x+2)(x+3)} \cdot \frac{(x+3)(x+3)}{x^2(x+4)(p+q)} &= \frac{(x+3)(x-4)}{x^2(x+2)} = \frac{x^2 - x - 12}{2x^2 + x^3} \end{aligned}$$

$$(d) \frac{3x}{x-2} - \frac{x+4}{x-2} = 2$$

Solution: This is a subtraction of fractions. The denominators are the same, the only difficulty is that we are subtracting expressions instead of numbers. The second pair of parentheses is essential.

$$\frac{3x}{x-2} - \frac{x+4}{x-2} = \frac{(3x) - (x+4)}{x-2} = \frac{3x - x - 4}{x-2} = \frac{2x - 4}{x-2} = \frac{2(x-2)}{x-2} = 2$$

$$(e) \sqrt{125} - 3\sqrt{80} + \sqrt{45} = -4\sqrt{5}$$

Solution:

$$\begin{aligned} \sqrt{125} - 3\sqrt{80} + \sqrt{45} &= \sqrt{25 \cdot 5} - 3\sqrt{16 \cdot 5} + \sqrt{9 \cdot 5} = \sqrt{25}\sqrt{5} - 3\sqrt{16}\sqrt{5} + \sqrt{9}\sqrt{5} \\ &= 5\sqrt{5} - 3(4)\sqrt{5} + 3\sqrt{5} = (5 - 12 + 3)\sqrt{5} = -4\sqrt{5} \end{aligned}$$

$$(f) (\sqrt{7} - 2)^2 = 11 - 4\sqrt{7}$$

Solution:

$$(\sqrt{7} - 2)^2 = (\sqrt{7} - 2)(\sqrt{7} - 2) = \sqrt{7}\sqrt{7} - 2\sqrt{7} - 2\sqrt{7} + 4 = 7 - 4\sqrt{7} + 4 = 11 - 4\sqrt{7}$$

$$(g) (\sqrt{3} - 1)^3 = -10 + 6\sqrt{3}$$

Solution: We will first work out $(\sqrt{3} - 1)^2$ and then multiply that by $(\sqrt{3} - 1)$.

$$\begin{aligned} (\sqrt{3} - 1)^3 &= (\sqrt{3} - 1)(\sqrt{3} - 1)(\sqrt{3} - 1) = (\sqrt{3}\sqrt{3} - 1\sqrt{3} - 1\sqrt{3} + 1)(\sqrt{3} - 1) \\ &= (3 - 2\sqrt{3} + 1)(\sqrt{3} - 1) = (4 - 2\sqrt{3})(\sqrt{3} - 1) = 4\sqrt{3} - 4 - 2\sqrt{3}\sqrt{3} + 2\sqrt{3} \\ &= 4\sqrt{3} - 4 - 2(3) + 2\sqrt{3} = 4\sqrt{3} - 4 - 6 + 2\sqrt{3} = -10 + 6\sqrt{3} \end{aligned}$$

2. Rationalize the denominator in each of the following expressions.

$$(a) \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

Solution: To rationalize the denominator, we will multiply both the numerator and denominator by $\sqrt{5}$.

$$\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

$$(b) \frac{1}{\sqrt{10} - 3} = \sqrt{10} + 3$$

Solution: To rationalize the denominator, we will multiply both the numerator and denominator by the conjugate of the denominator, which is $\sqrt{10} + 3$.

$$\frac{1}{\sqrt{10} - 3} = \frac{1}{\sqrt{10} - 3} \cdot \frac{\sqrt{10} + 3}{\sqrt{10} + 3} = \frac{\sqrt{10} + 3}{1} = \sqrt{10} + 3$$

The denominator is 1 since

$$(\sqrt{10} - 3)(\sqrt{10} + 3) = \sqrt{10}\sqrt{10} + 3\sqrt{10} - 3\sqrt{10} - 9 = 10 - 9 = 1$$

$$(c) \frac{2}{\sqrt{7}+1} = \frac{\sqrt{7}-1}{3}$$

Solution: To rationalize the denominator, we will multiply both the numerator and denominator by the conjugate of the denominator, which is $\sqrt{7}-1$

$$\frac{2}{\sqrt{7}+1} = \frac{2}{\sqrt{7}+1} \cdot \frac{\sqrt{7}-1}{\sqrt{7}-1} = \frac{2(\sqrt{7}-1)}{7-1} = \frac{2(\sqrt{7}-1)}{6} = \frac{\sqrt{7}-1}{3}$$

3. Find the exact value of $x^2 - 4x + 6$ if $x = 2 - \sqrt{3}$. 5

Solution: We work out x^2 first.

$$\begin{aligned} x^2 &= (2 - \sqrt{3})^2 = (2 - \sqrt{3})(2 - \sqrt{3}) = 4 - 2\sqrt{3} - 2\sqrt{3} + \sqrt{3}\sqrt{3} \\ &= 4 - 4\sqrt{3} + 3 = 7 - 4\sqrt{3} \end{aligned}$$

Now we substitute $x = 2 - \sqrt{3}$ into $x^2 - 4x + 6$.

$$x^2 - 4x + 6 = (2 - \sqrt{3})^2 - 4(2 - \sqrt{3}) + 6 = 7 - 4\sqrt{3} - 8 + 4\sqrt{3} + 6 = 7 - 8 + 6 = 5$$

4. Factor $13x + 2x^2 - 24$ by completing the square. $2(x+8)\left(x - \frac{3}{2}\right) = (x+8)(2x-3)$

Solution: We rearrange the terms first and then factor out the leading coefficient.

$$\begin{aligned} 13x + 2x^2 - 24 &= 2x^2 + 13x - 24 \\ &= 2\left(x^2 + \frac{13}{2}x - 12\right) \end{aligned}$$

Half of the linear coefficient is $\frac{13}{4}$, thus we work out $\left(x + \frac{13}{4}\right)^2$ first to see what we need to smuggle in to complete the square.

$$\begin{aligned} \left(x + \frac{13}{4}\right)^2 &= \left(x + \frac{13}{4}\right)\left(x + \frac{13}{4}\right) = x^2 + \frac{13}{4}x + \frac{13}{4}x + \frac{169}{16} \\ &= x^2 + \frac{13}{2}x + \frac{169}{16} \end{aligned}$$

Thus we need to smuggle in $\frac{169}{16}$

$$\begin{aligned} 2x^2 + 13x - 24 &= 2\left(x^2 + \frac{13}{2}x - 12\right) \\ &= 2\left(\underbrace{x^2 + \frac{13}{2}x + \frac{169}{16}} - \frac{169}{16} - 12\right) \end{aligned}$$

We bring the last two numbers to the common denominator

$$\begin{aligned} 2x^2 + 13x - 24 &= 2 \left(\left(x + \frac{13}{4} \right)^2 - \frac{169}{16} - \frac{12(16)}{1(16)} \right) \\ &= 2 \left(\left(x + \frac{13}{4} \right)^2 - \frac{169}{16} - \frac{192}{16} \right) \\ &= 2 \left(\left(x + \frac{13}{4} \right)^2 - \frac{361}{16} \right) \end{aligned}$$

Since $\frac{361}{16} = \left(\frac{19}{4} \right)^2$, we factor via the difference of squares theorem.

$$\begin{aligned} 2x^2 + 13x - 24 &= 2 \left(\left(x + \frac{13}{4} \right)^2 - \left(\frac{19}{4} \right)^2 \right) \\ &= 2 \left(x + \frac{13}{4} + \frac{19}{4} \right) \left(x + \frac{13}{4} - \frac{19}{4} \right) \\ &= 2 \left(x + \frac{32}{4} \right) \left(x - \frac{6}{4} \right) \\ &= 2(x + 8) \left(x - \frac{3}{2} \right) = (x + 8)(2x - 3) \end{aligned}$$

We may distribute 2 into the second factor. Then we get

$$2x^2 + 13x - 24 = (x + 8) \left(2 \left(x - \frac{3}{2} \right) \right) = (x + 8)(2x - 3)$$

We FOIL to check:

$$\begin{aligned} (x + 8)(2x - 3) &= 2x^2 - 3x + 16x - 24 \\ &= 2x^2 + 13x - 24 \end{aligned}$$

5. Factor completely each of the following:

(a) $4a^2mn - 15abm^2 - 6abmn + 10a^2m^2 = am(2n + 5m)(2a - 3b)$

Solution:

$$\begin{aligned} 4a^2mn - 15abm^2 - 6abmn + 10a^2m^2 &= \quad \text{the GCF is } am \\ am(4an - 15bm - 6bn + 10am) &= \quad \text{rearrange} \\ am \left(\underbrace{4an - 6bn}_{+10am - 15bm} \right) &= \\ am(2n(2a - 3b) + 5m(2a - 3b)) &= am(2n + 5m)(2a - 3b) \end{aligned}$$

$$(b) \quad a^2x^3 - b^2x - a^2x + b^2x^3 = x(a^2 + b^2)(x + 1)(x - 1)$$

Solution:

$$\begin{aligned} a^2x^3 - b^2x - a^2x + b^2x^3 &= && \text{the GCF is } x \\ x(a^2x^2 - b^2 - a^2 + b^2x^2) &= && \text{rearrange} \\ x\left(\underbrace{a^2x^2 - a^2}_{+} \quad \underbrace{+b^2x^2 - b^2}_{-}\right) &= && \\ x(a^2(x^2 - 1) + b^2(x^2 - 1)) &= && x(a^2 + b^2)(x^2 - 1) \end{aligned}$$

We are not done yet since $(x^2 - 1) = (x^2 - 1^2)$ further factors via the difference of squares theorem. Thus the answer is

$$x(a^2 + b^2)(x^2 - 1) = x(a^2 + b^2)(x^2 - 1^2) = x(a^2 + b^2)(x + 1)(x - 1)$$

$$(c) \quad 162a + 162b - 2ax^4 - 2bx^4 = 2(9 + x^2)(3 + x)(3 - x)(a + b)$$

Solution:

$$\begin{aligned} 162a + 162b - 2ax^4 - 2bx^4 &= && \text{the GCF is } 2 \\ 2\left(\underbrace{81a + 81b}_{+} \quad \underbrace{-ax^4 - bx^4}_{-}\right) &= && \\ 2(81(a + b) - x^4(a + b)) &= && 2(81 - x^4)(a + b) \end{aligned}$$

We are not done yet, since $81 - x^4 = 9^2 - (x^2)^2$ further factors via the difference of squares theorem.

$$\begin{aligned} 2(81 - x^4)(a + b) &= 2(9^2 - (x^2)^2)(a + b) \\ &= 2(9 + x^2)(9 - x^2)(a + b) \end{aligned}$$

One factor still further factors: $9 - x^2 = 3^2 - x^2 = (3 + x)(3 - x)$.

$$\begin{aligned} &= 2(9 + x^2)(9 - x^2)(a + b) \\ &= 2(9 + x^2)(3^2 - x^2)(a + b) \\ &= 2(9 + x^2)(3 + x)(3 - x)(a + b) \end{aligned}$$

$$(d) \quad x^2 - 6x + 8 = (x - 2)(x - 4)$$

Solution: we will factor by grouping. First we conduct the "pq-game".

$$\begin{array}{ll} pq &= 8 && \text{1st coefficient times 3rd coefficient} \\ p + q &= -6 && \text{2nd coefficient} \end{array}$$

We start by expressing 8 as a product of two numbers. there are only two pairs, 1 with 8 and 2 with 4. Since the product pq is positive, p and q have to have the same sign. Since the sum $p + q$ is negative, they both have to be negative. We only need to consider -1 with -8 and -2 with -4 . Clearly -2 with -4 work as p and q . We use these numbers to express the second term as the sum of two terms, and then factor by grouping.

$$\begin{aligned} x^2 - 6x + 8 &= \underbrace{x^2 - 2x}_{+} \quad \underbrace{-4x + 8}_{-} \\ &= x(x - 2) - 4(x - 2) = (x - 2)(x - 4) \end{aligned}$$

We check by multiplication:

$$(x - 2)(x - 4) = x^2 - 4x - 2x + 8 = x^2 - 6x + 8$$

Thus our result is correct.

(e) $3a^2 - 5a - 2 = (a - 2)(3a + 1)$

Solution: we will factor by grouping. First we conduct the "pq-game".

$$\begin{array}{ll} pq = -6 & \text{1st coefficient times 3rd coefficient} \\ p + q = -5 & \text{2nd coefficient} \end{array}$$

We start by expressing 6 as a product of two numbers. there are only two pairs, 1 with 6 and 2 with 3. Since the product pq is negative, one number must be positive, the other one must be positive.. Since the sum $p + q$ is negative, the negative sign has to be in front of the larger number. We only need to consider 1 with -6 and 2 with -3 . Clearly 1 with -6 work as p and q . We use these these numbers to express the second term as the sum of two terms, and then factor by grouping.

$$\begin{aligned} 3a^2 - 5a - 2 &= \underbrace{3a^2 + a}_{a(3a+1)} \underbrace{-6a - 2}_{-2(3a+1)} \\ &= a(3a + 1) - 2(3a + 1) = (a - 2)(3a + 1) \end{aligned}$$

We check by multiplication:

$$(a - 2)(3a + 1) = 3a^2 + a - 6a - 2 = 3a^2 - 5a - 2$$

Thus our result is correct.

(f) $4b^2 - b - 5 = (4b - 5)(b + 1)$

Solution: we will factor by grouping. First we conduct the "pq-game".

$$\begin{array}{ll} pq = -20 & \text{1st coefficient times 3rd coefficient} \\ p + q = -1 & \text{2nd coefficient} \end{array}$$

We start by expressing 20 as a product of two numbers. the possible pairs are, 1 with 20, 2 with 10, and 4 with 5. Since the product pq is negative, one number must be positive, the other one must be positive.. Because the sum $p + q$ is negative, the negative sign has to be in front of the larger number. We only need to consider 1 with -20 , 2 with -10 , and 4 with -5 . Clearly 4 with -5 work as p and q . We use these these numbers to express the second term as the sum of two terms, and then factor by grouping.

$$\begin{aligned} 4b^2 - b - 5 &= \underbrace{4b^2 + 4b}_{4b(b+1)} \underbrace{-5b - 5}_{-5(b+1)} \\ &= 4b(b + 1) - 5(b + 1) = (4b - 5)(b + 1) \end{aligned}$$

We check by multiplication:

$$(4b - 5)(b + 1) = 4b^2 + 4b - 5b - 5 = 4b^2 - b - 5$$

Thus our result is correct.

6. Solve each of the following equations. Make sure to check your solution(s).

(a) $2x^3 = 20x^2 + 1750x$ **35, 0, and -25**

Solution: We reduce one side to zero, then factor, and then apply the zero property.

$$\begin{aligned} 2x^3 &= 20x^2 + 1750x \\ 2x^3 - 20x^2 - 1750x &= 0 && \text{factor out GCF} \\ 2x(x^2 - 10x - 875) &= 0 && \text{divide both sides by 2} \\ x(x^2 - 10x - 875) &= 0 \end{aligned}$$

We will factor by completing the square. Half of the linear coefficient is -5 , and thus we will work with

$$(x - 5)^2 = x^2 - 10x + 25$$

We smuggle in 25.

$$\begin{aligned} x(x^2 - 10x - 875) &= 0 \\ x(\underbrace{x^2 - 10x + 25}_{(x-5)^2} - 25 - 875) &= 0 \\ x((x - 5)^2 - 900) &= 0 \\ x((x - 5)^2 - 30^2) &= 0 \\ x(x - 5 + 30)(x - 5 - 30) &= 0 \\ x(x + 25)(x - 35) &= 0 \end{aligned}$$

Applying the zero property we obtain $35, 0,$ and -25 as the solutions.

(b) $\frac{3x + 17}{2} = x - 1 + \frac{x + 19}{2}$ **identity, all real numbers are solution.**

Solution:

$$\begin{aligned} \frac{3x + 17}{2} &= x - 1 + \frac{x + 19}{2} && \text{express everything as a fraction} \\ \frac{3x + 17}{2} &= \frac{x - 1}{1} + \frac{x + 19}{2} && \text{bring everything to the common denominator} \\ \frac{3x + 17}{2} &= \frac{2(x - 1)}{2} + \frac{x + 19}{2} && \text{add fractions on right hand side} \\ \frac{3x + 17}{2} &= \frac{2(x - 1) + x + 19}{2} && \text{multiply out parentheses} \\ \frac{3x + 17}{2} &= \frac{2x - 2 + x + 19}{2} && \text{combine like terms} \\ \frac{3x + 17}{2} &= \frac{3x + 17}{2} && \text{multiply by 2} \\ 3x + 17 &= 3x + 17 \end{aligned}$$

Because the left hand side is now identical to the right hand side, this equation is an identity, and all real numbers are solution.

(c) $|3 - 2x| + 2 = 5$ 0 and 3

Solution:

$$\begin{aligned}
|3 - 2x| + 2 &= 5 && \text{subtract } 2 \\
|3 - 2x| &= 3 \\
3 - 2x &= 3 && \text{or } 3 - 2x = -3 && \text{subtract } 3 \\
-2x &= 0 && \text{or } -2x = -6 && \text{divide by } -2 \\
x &= 0 && \text{or } x = 3
\end{aligned}$$

Thus the solution are 0 and 3. We check. If $x = 0$, then

$$\text{LHS} = |3 - 2(0)| + 2 = |3| + 2 = 3 + 2 = 5 = \text{RHS}$$

and if $x = 3$, then

$$\text{LHS} = |3 - 2(-3)| + 2 = |3 - 6| + 2 = |-3| + 2 = 3 + 2 = 5 = \text{RHS}$$

Thus both numbers are solutions.

(d) $\frac{2}{3}(x - 7) = \frac{4}{5}(x + 1)$ -41

Solution:

$$\begin{aligned}
\frac{2}{3}(x - 7) &= \frac{4}{5}(x + 1) \\
\frac{2}{3} \cdot \frac{x - 7}{1} &= \frac{4}{5} \cdot \frac{x + 1}{1} && \text{bring fractions to common denominator} \\
\frac{2(x - 7)}{3} &= \frac{4(x + 1)}{5} \\
\frac{5 \cdot 2(x - 7)}{15} &= \frac{3 \cdot 4(x + 1)}{15} && \text{multiply both sides by } 15 \\
10(x - 7) &= 12(x + 1) && \text{multiply out parentheses} \\
10x - 70 &= 12x + 12 && \text{subtract } 10x \\
-70 &= 2x + 12 && \text{subtract } 12 \\
-82 &= 2x && \text{divide by } 2 \\
-41 &= x
\end{aligned}$$

We check:

$$\begin{aligned}
\text{LHS} &= \frac{2}{3}(-41 - 7) = \frac{2}{3}(-48) = -32 \\
\text{RHS} &= \frac{4}{5}(-41 + 1) = \frac{4}{5}(-40) = -32
\end{aligned}$$

Thus our solution, -41 is correct.

(e) $7x^2 + (x + 3)(2x - 1) = (3x + 1)^2$ -4

Solution:

$$\begin{aligned}
 7x^2 + (x + 3)(2x - 1) &= (3x + 1)^2 && \text{multiply the polynomials on both sides} \\
 7x^2 + 2x^2 + 5x - 3 &= 9x^2 + 6x + 1 && \text{combine like terms} \\
 9x^2 + 5x - 3 &= 9x^2 + 6x + 1 && \text{subtract } 9x^2 \\
 5x - 3 &= 6x + 1 && \text{subtract } 5x \\
 -3 &= x + 1 && \text{subtract } 1 \\
 -4 &= x &&
 \end{aligned}$$

We check our result:

$$\begin{aligned}
 \text{LHS} &= 7(-4)^2 + ((-4) + 3)(2(-4) - 1) = 7 \cdot 16 + (-1)(-9) = 112 + 9 = 121 \\
 \text{RHS} &= (3(-4) + 1)^2 = (-12 + 1)^2 = (-11)^2 = 121
 \end{aligned}$$

Thus the solution, -4 is correct.

(f) $8a + 2a^2 = 42$ **$-7, 3$**

Solution: since this equation is of a higher degree than 1, our only method is to reduce one side to zero, factor, and then apply the special zero property.

$$\begin{aligned}
 8a + 2a^2 &= 42 && \text{subtract } 42, \text{ rearrange} \\
 2a^2 + 8a - 42 &= 0 && \text{the GCF is } 2 \\
 2(a^2 + 4a - 21) &= 0
 \end{aligned}$$

We will factor $a^2 + 4a - 21$ by grouping. First we conduct the "pq-game".

$$\begin{aligned}
 pq &= -21 && \text{1st coefficient times 3rd coefficient} \\
 p + q &= 4 && \text{2nd coefficient}
 \end{aligned}$$

We start by expressing 21 as a product of two numbers. the only possible pairs are, 1 with 21 and 3 with 7. Since the product pq is negative, one number must be positive, the other one must be positive.. Because the sum $p + q$ is positive, the negative sign has to be in front of the smaller number. We only need to consider -1 with 20, and -3 with 7. Clearly -3 with 7 work as p and q . We use these these numbers to express the second term as the sum of two terms, and then factor by grouping.

$$\begin{aligned}
 2(a^2 + 4a - 21) &= 0 \\
 2(\underbrace{a^2 + 7a}_{} \quad \underbrace{-3a - 21}_{}) &= 0 \\
 2(a(a + 7) - 3(a + 7)) &= 0 \\
 2(a - 3)(a + 7) &= 0
 \end{aligned}$$

Thus our equation is

$$2(a - 3)(a + 7) = 0$$

We now apply the special zero property. If this product is zero, then either $2 = 0$ or $a - 3 = 0$ or $a + 7 = 0$. We solve these equations for a .

$$\begin{array}{ccccccc}
 a - 3 = 0 & & \text{or} & & a + 7 = 0 & & \text{or} & & 2 = 0 \\
 a = 3 & & \text{or} & & a = -7 & & \text{or} & & \text{no solution here}
 \end{array}$$

We check both solutions. If $a = 3$, then

$$\text{LHS} = 8(3) + 2(3)^2 = 8 \cdot 3 + 2 \cdot 9 = 24 + 18 = 42 = \text{RHS} \quad \checkmark$$

If $a = -7$, then

$$\text{LHS} = 8(-7) + 2(-7)^2 = 8 \cdot (-7) + 2 \cdot 49 = -56 + 98 = 42 = \text{RHS} \quad \checkmark$$

Thus both solutions, -7 and 3 are correct.

(g) $8x^3 = 50x^2$ $\frac{25}{4}, 0$

Solution: since this equation is of a higher degree than 1, our only method is to reduce one side to zero, factor, and then apply the special zero property.

$$\begin{aligned} 8x^3 &= 50x^2 && \text{subtract } 50x^2 \\ 8x^3 - 50x^2 &= 0 && \text{the GCF is } 2x^2 \\ 2x^2(4x - 25) &= 0 \end{aligned}$$

We now apply the special zero property. If this product is zero, then either $2x^2 = 0$ or $4x - 25 = 0$. We solve these equations for x .

$$\begin{array}{lll} 2x^2 = 0 & \text{or} & 4x - 25 = 0 \\ 2 \cdot x \cdot x = 0 & \text{or} & 4x = 25 \\ x = 0 & \text{or} & x = \frac{25}{4} \end{array}$$

We check both solutions. If $x = 0$, then

$$\begin{aligned} \text{LHS} &= 8 \cdot 0^3 = 8 \cdot 0 = 0 \\ \text{RHS} &= 50 \cdot 0^2 = 50 \cdot 0 = 0 \end{aligned}$$

If $x = \frac{25}{4}$, then

$$\begin{aligned} \text{LHS} &= 8 \left(\frac{25}{4} \right)^3 = \frac{8}{1} \cdot \frac{15\,625}{64} = \frac{15\,625}{8} \\ \text{RHS} &= 50 \left(\frac{25}{4} \right)^2 = \frac{50}{1} \cdot \frac{625}{16} = \frac{15\,625}{8} \end{aligned}$$

Thus both solutions, 0 and $\frac{25}{4}$ are correct.

(h) $8p^3 = 50p - \frac{5}{2}$, 0 , $\frac{5}{2}$

Solution: since this equation is of a higher degree than 1, our only method is to reduce one side to zero, factor, and then apply the special zero property.

$$\begin{aligned} 8p^3 &= 50p && \text{subtract } 50p \\ 8p^3 - 50p &= 0 && \text{the GCF is } 2p \\ 2p(4p^2 - 25) &= 0 \\ 2p((2p)^2 - 5^2) &= 0 && \text{factor via difference of squares theorem} \\ 2p(2p + 5)(2p - 5) &= 0 \end{aligned}$$

We now apply the special zero property. If this product is zero, then either $2p = 0$ or $2p + 5 = 0$ or $2p - 5 = 0$. We solve these equations for p .

$$\begin{array}{rclclcl} 2p + 5 & = & 0 & \text{or} & 2p - 5 & = & 0 & \text{or} & 2p & = & 0 \\ 2p & = & -5 & \text{or} & 2p & = & 5 & \text{or} & & & p = 0 \\ p & = & -\frac{5}{2} & \text{or} & p & = & \frac{5}{2} & & & & \end{array}$$

We check all three solutions. If $p = -\frac{5}{2}$, then

$$\begin{aligned} \text{LHS} &= 8 \left(-\frac{5}{2} \right)^3 = \frac{8}{1} \cdot \frac{-125}{8} = -125 \\ \text{RHS} &= 50 \left(-\frac{5}{2} \right) = \frac{50}{1} \cdot \frac{-5}{2} = \frac{-250}{2} = -125 \end{aligned}$$

If $p = \frac{5}{2}$, then

$$\begin{aligned} \text{LHS} &= 8 \left(\frac{5}{2} \right)^3 = \frac{8}{1} \cdot \frac{125}{8} = 125 \\ \text{RHS} &= 50 \left(\frac{5}{2} \right) = \frac{50}{1} \cdot \frac{5}{2} = \frac{250}{2} = 125 \end{aligned}$$

and if $p = 0$, then

$$\begin{aligned} \text{LHS} &= 8 \cdot 0^3 = 8 \cdot 0 = 0 \\ \text{RHS} &= 50 \cdot 0 = 0 \end{aligned}$$

Thus all three solutions, $-\frac{5}{2}$, 0 , and $\frac{5}{2}$ are correct.

(i) $2 - (3 - x)(2x + 5) = (x - 1)(2x - 1)$ **7**

Solution: We have to use the FOIL method on both sides to perform the multiplications. It is very important, however, to keep the expressions in a parentheses since we are dealing with subtraction between *algebraic expressions*.

$2 - (3 - x)(2x + 5)$	$=$	$(x - 1)(2x - 1)$	FOIL
$2 - (6x + 15 - 2x^2 - 5x)$	$=$	$2x^2 - x - 2x + 1$	combine like terms
$2 - (-2x^2 + x + 15)$	$=$	$2x^2 - 3x + 1$	perform subtraction
$2 + 2x^2 - x - 15$	$=$	$2x^2 - 3x + 1$	combine like terms
$2x^2 - x - 13$	$=$	$2x^2 - 3x + 1$	subtract $2x^2$ (the equation is linear!)
$-x - 13$	$=$	$-3x + 1$	add $3x$
$2x - 13$	$=$	1	add 13
$2x$	$=$	14	divide by 2
x	$=$	7	

We check: if $x = 7$, then

$$\text{LHS} = 2 - (3 - 7)(2(7) + 5) = 2 - (-4)(14 + 5) = 2 - (-4)19 = 2 - (-76) = 78$$

$$\text{RHS} = (7 - 1)(2(7) - 1) = 6(14 - 1) = 6 \cdot 13 = 78$$

Thus our solution, 7 is correct.

(j) $x^2 = 4x + 1$. $2 + \sqrt{5}$, $2 - \sqrt{5}$

Solution : We complete the square.

$$\begin{aligned} x^2 &= 4x + 1 \\ x^2 - 4x - 1 &= 0 & (x - 2)^2 &= x^2 - 4x + 4 \\ \underbrace{x^2 - 4x + 4} - 4 - 1 &= 0 \\ (x - 2)^2 - 5 &= 0 \\ (x - 2)^2 - (\sqrt{5})^2 &= 0 \\ (x - 2 + \sqrt{5})(x - 2 - \sqrt{5}) &= 0 \\ x_1 &= 2 - \sqrt{5} \quad \text{and} \quad x_2 = 2 + \sqrt{5} \end{aligned}$$

We check: if $x = 2 - \sqrt{5}$, then

$$\text{LHS} = (2 - \sqrt{5})^2 = 4 - 2\sqrt{5} - 2\sqrt{5} + 5 = 9 - 4\sqrt{5}$$

$$\text{RHS} = 4(2 - \sqrt{5}) + 1 = 8 - 4\sqrt{5} + 1 = 9 - 4\sqrt{5}$$

If $x = 2 + \sqrt{5}$, then

$$\text{LHS} = (2 + \sqrt{5})^2 = 4 + 2\sqrt{5} + 2\sqrt{5} + 5 = 9 + 4\sqrt{5}$$

$$\text{RHS} = 4(2 + \sqrt{5}) + 1 = 8 + 4\sqrt{5} + 1 = 9 + 4\sqrt{5}$$

Thus our solution, $2 + \sqrt{5}$ and $2 - \sqrt{5}$ is correct.

(k) $4x^2 + 20x + 7 = 0$ $\frac{-5 + \sqrt{18}}{2}$, $\frac{-5 - \sqrt{18}}{2}$

Solution:

$$\begin{aligned} 4x^2 + 20x + 7 &= 0 \\ 4\left(x^2 + 5x + \frac{7}{4}\right) &= 0 \end{aligned}$$

Half of the linear coefficient is $\frac{5}{2}$. Thus the complete square is $\left(x + \frac{5}{2}\right)^2$

$$\left(x + \frac{5}{2}\right)^2 = \left(x + \frac{5}{2}\right)\left(x + \frac{5}{2}\right) = x^2 + \frac{5}{2}x + \frac{5}{2}x + \frac{25}{4} = x^2 + 5x + \frac{25}{4}$$

So we smuggle in $\frac{25}{4}$.

$$4 \left(\underbrace{x^2 + 5x + \frac{25}{4}} - \frac{25}{4} + \frac{7}{4} \right) = 0$$

$$4 \left(\left(x + \frac{5}{2} \right)^2 - \frac{18}{4} \right) = 0$$

Although $\frac{18}{4}$ can be simplified, we will keep it as it is, because the common denominator will be easier this way. We next apply the difference of squares theorem. $\frac{18}{4}$ is the square of a number - its own square root.

$$\frac{18}{4} = \left(\sqrt{\frac{18}{4}} \right)^2 = \left(\frac{\sqrt{18}}{\sqrt{4}} \right)^2 = \left(\frac{\sqrt{18}}{2} \right)^2$$

$$4 \left(\left(x + \frac{5}{2} \right)^2 - \frac{18}{4} \right) = 0$$

$$4 \left(\left(x + \frac{5}{2} \right)^2 - \left(\frac{\sqrt{18}}{2} \right)^2 \right) = 0$$

$$4 \left(x + \frac{5}{2} + \frac{\sqrt{18}}{2} \right) \left(x + \frac{5}{2} - \frac{\sqrt{18}}{2} \right) = 0$$

We solve for the zeroes of both linear factors:

$$x + \frac{5}{2} + \frac{\sqrt{18}}{2} = 0$$

$$x + \frac{\sqrt{18}}{2} = -\frac{5}{2}$$

$$x_1 = -\frac{5}{2} - \frac{\sqrt{18}}{2} = \frac{-5 - \sqrt{18}}{2}$$

$$x + \frac{5}{2} - \frac{\sqrt{18}}{2} = 0$$

$$x - \frac{\sqrt{18}}{2} = -\frac{5}{2}$$

$$x_2 = -\frac{5}{2} + \frac{\sqrt{18}}{2} = \frac{-5 + \sqrt{18}}{2}$$

We check: if $x = \frac{-5 - \sqrt{18}}{2}$, then

$$\begin{aligned}\text{LHS} &= 4x^2 + 20x + 7 = 4 \left(\frac{-5 - \sqrt{18}}{2} \right)^2 + 20 \left(\frac{-5 - \sqrt{18}}{2} \right) + 7 \\ &= \frac{4}{1} \cdot \frac{(-5 - \sqrt{18})^2}{4} + \frac{20}{1} \left(\frac{-5 - \sqrt{18}}{2} \right) + 7 = \\ &= \frac{25 + 5\sqrt{18} + 5\sqrt{18} + 18}{1} - \frac{10(-5 - \sqrt{18})}{1} + 7 = \\ &= 43 + 10\sqrt{18} - 10(-5 - \sqrt{18}) + 7 = 43 + 10\sqrt{18} - 50 - 10\sqrt{18} + 7 \\ &= -7 + 7 = 0 = \text{RHS}\end{aligned}$$

and if $x = \frac{-5 + \sqrt{18}}{2}$, then

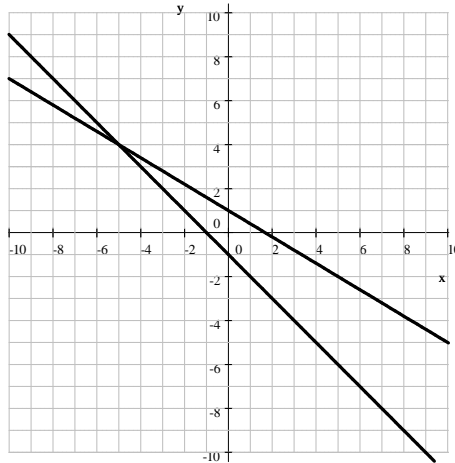
$$\begin{aligned}\text{LHS} &= 4x^2 + 20x + 7 = 4 \left(\frac{-5 + \sqrt{18}}{2} \right)^2 + 20 \left(\frac{-5 + \sqrt{18}}{2} \right) + 7 \\ &= \frac{4}{1} \cdot \frac{(-5 + \sqrt{18})^2}{4} + \frac{20}{1} \left(\frac{-5 + \sqrt{18}}{2} \right) + 7 = \\ &= \frac{25 - 5\sqrt{18} - 5\sqrt{18} + 18}{1} - \frac{10(-5 + \sqrt{18})}{1} + 7 = \\ &= 43 - 10\sqrt{18} - 10(-5 + \sqrt{18}) + 7 = 43 - 10\sqrt{18} - 50 + 10\sqrt{18} + 7 \\ &= -7 + 7 = 0 = \text{RHS}\end{aligned}$$

Thus our solution, $\frac{-5 + \sqrt{18}}{2}$ and $\frac{-5 - \sqrt{18}}{2}$ is correct.

Note: since $\sqrt{18} = 3\sqrt{2}$, the final answer $\frac{-5 + 3\sqrt{2}}{2}$ and $\frac{-5 - 3\sqrt{2}}{2}$ is equally correct.

7. Graph the straight lines $3x + 5y = 5$ and $y = -x - 1$ in the same coordinate system. Use your graph to find the coordinates of the point where the lines intersect. $(-5, 4)$

Solution: We can easily graph the lines by using the slope-intercept forms: $y = -x - 1$ and $y = -\frac{3}{5}x + 1$



8. Find an equation of the straight line that is perpendicular to $2x - 3y = -6$ and passes through the point $(-12, 5)$. $y = -\frac{3}{2}x - 13$

Solution: We obtain the slope of the line give by solving for y .

$$\begin{aligned} 2x - 3y &= -6 && \text{add } 3y \\ 2x &= 3y - 6 && \text{add } 6 \\ 2x + 6 &= 3y && \text{ivie by } 3 \\ \frac{2x + 6}{3} &= y &\implies & y = \frac{2}{3}x + \frac{6}{3} = \frac{2}{3}x + 2 \end{aligned}$$

Our line is perpendicular to one with slope $\frac{2}{3}$: thus its slope must be the negative reciprocal of $\frac{2}{3}$, which is $-\frac{3}{2}$. Using the slope-point form of the equation, we write $y - 5 = -\frac{3}{2}(x + 12)$. If we simplify the equation, we obtain the slope-intercept form, $y = -\frac{3}{2}x - 13$.

9. Find an equation of the straight line that passes through the points $(2, 7)$ and $(-2, -5)$. $y = 3x + 1$

Solution 1: We obtain the slope from the slope formula

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-5)}{2 - (-2)} = \frac{12}{4} = 3$$

For b , we substitute either point into the slope-intercept form of the equation.

$$\begin{aligned} y &= mx + b && \text{we know } m = 3 \\ y &= 3x + b && (2, 7) \text{ is on the line} \\ 7 &= 3(2) + b \\ 7 &= 6 + b \\ 1 &= b \end{aligned}$$

Thus the equation is $y = 3x + 1$. We can easily check by substituting the coordinates of the points into the equation.

Solution 2: We obtain the slope from the slope formula

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-5)}{2 - (-2)} = \frac{12}{4} = 3$$

and then apply the slope-point form (we will use $(2, 7)$)

$$y - 7 = 3(x - 2)$$

Solution 3: Write $y = mx + b$ and solve the system that we obtain by substituting the points given.

$$\begin{aligned} 7 &= m(2) + b && (2, 7) \text{ is on the line} \\ -5 &= m(-2) + b && (-2, -5) \text{ is on the line} \end{aligned}$$

Thus we get the system

$$\begin{aligned} 2m + b &= 7 \\ -2m + b &= -5 \end{aligned}$$

We eliminate m if simply add the two equations

$$\begin{aligned} 2b &= 2 \\ b &= 1 \end{aligned}$$

Now we can easily solve for m in the first equation

$$\begin{aligned} 2m + 1 &= 7 \\ 2m &= 6 \\ m &= 3 \end{aligned}$$

Thus the equation is $y = 3x + 1$. We can easily check by substituting the coordinates of the points into the equation.

10. Graph the parabola $y = 8x - x^2 - 15$. Clearly label the coordinates of five points on the parabola, including vertex and intercepts.

Solution: We obtain all forms of the equation first.

$$\begin{aligned} y &= -x^2 + 8x - 15 \implies \text{polynomial form} \\ y &= -(x^2 - 8x + 15) \qquad (x - 4)^2 = x^2 - 8x + 16 \\ y &= -\left(\underbrace{x^2 - 8x + 16}_{(x-4)^2} - 16 + 15\right) \\ y &= -\left((x - 4)^2 - 1\right) \\ y &= -(x - 4)^2 + 1 \implies \text{complete square form} \end{aligned}$$

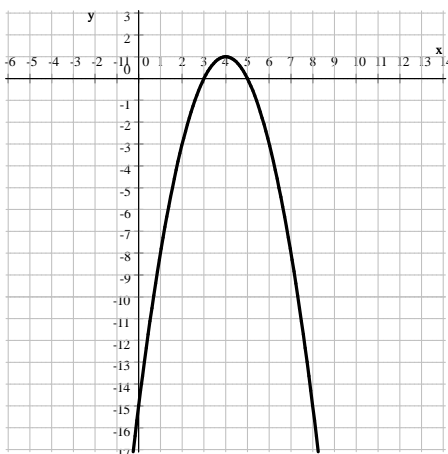
We factor via the difference of squares theorem

$$\begin{aligned} y &= -\left((x - 4)^2 - 1^2\right) \quad \text{since } 1 = 1^2 \\ y &= -(x - 4 + 1)(x - 4 - 1) \\ y &= -(x - 3)(x - 5) \implies \text{factored form} \end{aligned}$$

From the polynomial form we obtain the y -intercept, $(0, -15)$. From the complete square form, the vertex is $(4, 1)$. Finally, the factored form tells us that there are two x -intercepts, $(3, 0)$ and $(5, 0)$. The few missing points, close to the vertex can be found by substituting values for x into any of the three forms of the equations to find y . This time we will work with the polynomial form.

$$\begin{aligned} \text{if } x &= 2, \text{ then } y = -(2)^2 + 8(2) - 15 = -4 + 16 - 15 = -3 \\ \text{if } x &= 6, \text{ then } y = -(6)^2 + 8(6) - 15 = -36 + 48 - 15 = -3 \end{aligned}$$

We are ready to graph:



11. One side of a rectangle is 4 ft shorter than three times the other side. Find the sides if the perimeter is 64 ft. **9 ft and 23 ft**

Solution: Let us denote the shorter side by x . Then the other side is $3x - 4$. The equation expresses the perimeter of the rectangle.

$$\begin{aligned} 2(x) + 2(3x - 4) &= 64 && \text{multiply out parentheses} \\ 2x + 6x - 8 &= 64 && \text{combine like terms} \\ 8x - 8 &= 64 && \text{add} \\ 8x &= 72 && \text{divide by 8} \\ x &= 9 \end{aligned}$$

If the shorter side was denoted by x , we now know it is 9 ft. The longer side was denoted by $3x - 4$, so it must be $3(9 \text{ ft}) - 4 \text{ ft} = 23 \text{ ft}$. Thus the sides of the rectangle are 9 ft and 23 ft. We check: $P = 2(9 \text{ ft}) + 2(23 \text{ ft}) = 64 \text{ ft}$ and $23 \text{ ft} = 3(9 \text{ ft}) - 4 \text{ ft}$. Thus our solution is correct.

12. One side of a rectangle is 4 ft shorter than three times the other side. Find the sides if the area is 84 ft^2 . **6 ft and 14 ft**

Solution: Let us denote the shorter side by x . Then the other side is $3x - 4$. The equation expresses the area of the rectangle.

$$\begin{aligned} x(3x - 4) &= 84 && \text{multiply out parentheses} \\ 3x^2 - 4x &= 84 && \text{subtract 84} \\ 3x^2 - 4x - 84 &= 0 \end{aligned}$$

Because the equation is quadratic, we need to factor the left-hand side and then apply the zero property. We will factor by grouping. First we conduct the "pq-game". The sum of p and q has to be the linear coefficient (the number in front of x , with its sign), so it is -4 . The product of p and q has to be the product of the other coefficients, $3(-84) = -252$.

$$\begin{aligned} pq &= -252 \\ p + q &= -4 \end{aligned}$$

Now we need to find p and q . Because the product is negative, we're looking for a positive and a negative number. Because the sum is negative, the larger number must carry the negative sign. We enter $\sqrt{252} = 15.87450787$ into the calculator and get a decimal:

$$\sqrt{252} = 15.874\dots$$

So we start looking for factors of 252, starting at 15, and moving down. We soon find 14 and -18 . These are our values for p and q . We use these numbers to express the linear term:

$$-4x = 14x - 18x$$

and factor by grouping.

$$\begin{aligned} 3x^2 - 4x - 84 &= 0 \\ \underbrace{3x^2 + 14x}_{x(3x+14)} \underbrace{-18x - 84}_{-6(3x+14)} &= 0 \\ x(3x + 14) - 6(3x + 14) &= 0 \\ (x - 6)(3x + 14) &= 0 \end{aligned}$$

We now apply the zero property. Either $x - 11 = 0$ or $3x + 14 = 0$. We solve both these equations for x .

$$\begin{aligned}x - 6 &= 0 \\x &= 6\end{aligned}$$

and

$$\begin{aligned}3x + 14 &= 0 \\3x &= -14 \\x &= -\frac{14}{3}\end{aligned}$$

Since distances can not be negative, the second solution for x , $-\frac{14}{3}$ is ruled out. Thus $x = 6$. Then the longer side is $3(6 \text{ ft}) - 4 \text{ ft} = 14 \text{ ft}$, and so the rectangle's sides are 6 ft and 14 ft long. We check: $6 \text{ ft}(14 \text{ ft}) = 84 \text{ ft}^2$ and $14 \text{ ft} = 3(6 \text{ ft}) - 4 \text{ ft}$. Thus our solution is correct.

13. One side of a rectangle is 4 in shorter than 3 times the other side. Find the sides of the rectangle if its area is 319 in^2 . **11 in by 29 in**

Solution: Let us denote the shorter side by x . Then the larger side is $3x - 4$. The equation expresses the area.

$$\begin{aligned}x(3x - 4) &= 319 \\3x^2 - 4x &= 319 \\3x^2 - 4x - 319 &= 0\end{aligned}$$

We will complete the square. (It can be done by the pq-game as well)

$$\begin{aligned}3x^2 - 4x - 319 &= 0 \\3\left(x^2 - \frac{4}{3}x - \frac{319}{3}\right) &= 0\end{aligned}$$

Half of the linear coefficient is $-\frac{4}{3} \div 2 = -\frac{4}{3} \cdot \frac{1}{2} = -\frac{4}{6} = -\frac{2}{3}$, thus we work out $\left(x - \frac{2}{3}\right)^2$.

$$\begin{aligned}\left(x - \frac{2}{3}\right)^2 &= \left(x - \frac{2}{3}\right)\left(x - \frac{2}{3}\right) = x^2 - \frac{2}{3}x - \frac{2}{3}x + \frac{4}{9} \\&= x^2 - \frac{4}{3}x + \frac{4}{9}\end{aligned}$$

Thus we smuggle in $\frac{4}{9}$.

$$\begin{aligned}3\left(\underbrace{x^2 - \frac{4}{3}x + \frac{4}{9}} - \frac{4}{9} - \frac{319}{3}\right) &= 0 \\3\left(\left(x - \frac{2}{3}\right)^2 - \frac{4}{9} - \frac{319}{3}\right) &= 0\end{aligned}$$

We bring the last two numbers to the common denominator:

$$\begin{aligned} 3 \left(\left(x - \frac{2}{3} \right)^2 - \frac{4}{9} - \frac{319(3)}{3(3)} \right) &= 0 \\ 3 \left(\left(x - \frac{2}{3} \right)^2 - \frac{4}{9} - \frac{957}{9} \right) &= 0 \\ 3 \left(\left(x - \frac{2}{3} \right)^2 - \frac{961}{9} \right) &= 0 \end{aligned}$$

Since $\frac{961}{9} = \left(\frac{31}{3} \right)^2$, we factor via the difference of squares theorem.

$$\begin{aligned} 3 \left(\left(x - \frac{2}{3} \right)^2 - \left(\frac{31}{3} \right)^2 \right) &= 0 \\ 3 \left(x - \frac{2}{3} + \frac{31}{3} \right) \left(x - \frac{2}{3} - \frac{31}{3} \right) &= 0 \\ 3 \left(x + \frac{29}{3} \right) \left(x - \frac{33}{3} \right) &= 0 \\ 3 \left(x + \frac{29}{3} \right) (x - 11) &= 0 \end{aligned}$$

This equation has two solutions, $x_1 = -\frac{29}{3}$ and $x_2 = 11$. Since distances are positive, $-\frac{29}{3}$ is ruled out as a solution for the shorter side. The other solution is 11 in. This makes the longer side $3 \cdot 11 - 4 = 29$ in. We check: $3(11) - 4 = 29$ and $11(29) = 319$. Thus our solution, 11 in by 29 in is correct.

14. A bank teller has 23 more five-dollar bills than ten-dollar bills. The total value of the money is \$610. How much of each denomination of bill does he have? **33 ten-dollar bills and 56 five-dollar bills**

Solution: Let us denote the number of ten-dollar bills by x . Then we have $x + 23$ many five-dollar bills. The equation expresses the value of the bills.

$$\begin{aligned} 10x + 5(x + 23) &= 610 && \text{distribute} \\ 10x + 5x + 115 &= 610 && \text{combine like terms} \\ 15x + 115 &= 610 && \text{subtract 115} \\ 15x &= 495 && \text{divide by 15} \\ x &= 33 \end{aligned}$$

Thus we have 33 tens and $33 + 23 = 56$ fives. We check: $56 - 33 = 23$ and $33(10) + 56(5) = 610$. Thus our solution; 33 ten-dollar bills and 56 five-dollar bills; is correct.

15. The population of a town has decreased from 80 000 to 68 000. What percent of a decrease does this represent? **15% decrease**

Solution 1: We subtract 68 000 from 80 000 to determine the change. $80\,000 - 68\,000 = 12\,000$. Now the question is: 12 000 is what percent of 80 000? Then

$$\begin{aligned}(\text{is}) &= 12\,000 \\ \text{F} &= x \\ (\text{of}) &= 80\,000\end{aligned}$$

We substitute the data into the formula:

$$\begin{aligned}(\text{is}) &= \text{F} \cdot (\text{of}) \\ 12\,000 &= x \cdot 80\,000 \\ \frac{12\,000}{80\,000} &= x \\ 0.15 &= x\end{aligned}$$

Thus

$$x = 0.15 = \frac{0.15}{1} = \frac{0.15(100)}{1(100)} = \frac{15}{100} = 15\%$$

This is a 15% decrease.

Solution 2: The question may be re-phrased as: 68 000 is what percent of 80 000? Then

$$\begin{aligned}(\text{is}) &= 68\,000 \\ \text{F} &= x \\ (\text{of}) &= 80\,000\end{aligned}$$

We substitute the data into the formula:

$$\begin{aligned}(\text{is}) &= \text{F} \cdot (\text{of}) \\ 68\,000 &= x \cdot 80\,000 \\ \frac{68\,000}{80\,000} &= x \\ 0.85 &= x\end{aligned}$$

Thus

$$x = 0.85 = \frac{0.85}{1} = \frac{0.85(100)}{1(100)} = \frac{85}{100} = 85\%$$

Since the population has decreased to 85% of its previous count, this is a 15% decrease.

16. We invested \$10000 into two bank accounts. One account earns 14% per year, the other account earns 8% per year. How much did we invest into each account if the combined interest from the two accounts is \$1238 after the first year?

\$ 7300 at 14% and \$ 2700 at 8%

Solution: Let us denote the amount invested at 14% by x and the amount invested at 8% by y . The two equations express that

$$\begin{aligned} x + y &= 10000 && \text{the amounts add up to } \$10000 \\ 0.14x + 0.08y &= 1238 && \text{the interests earned add up to } \$1238 \end{aligned}$$

We solve the system of equation by elimination. But let us first make the second equation simpler:

$$\begin{aligned} 0.14x + 0.08y &= 1238 && \text{multiply by } 100 \\ 14x + 8y &= 123800 && \text{divide by } 2 \\ 7x + 4y &= 61900 \end{aligned}$$

We now have

$$\begin{aligned} x + y &= 10000 \\ 7x + 4y &= 61900 \end{aligned}$$

We will multiply the first equation by -4 to eliminate y .

$$\begin{aligned} -4x - 4y &= -40000 \\ 7x + 4y &= 61900 \end{aligned}$$

We add the equations and solve for x .

$$\begin{aligned} 3x &= 21900 && \text{divide by } 3 \\ x &= 7300 \end{aligned}$$

Thus we invested \$7300 at 14%. The other amount is then from the first equation:

$$\begin{aligned} 7300 + y &= 10000 \\ y &= 2700 \end{aligned}$$

We invested \$ 7300 at 14% and \$ 2700 at 8%. We check: the amounts add up to $\$7300 + \$2700 = \$10000$. The interest from the accounts are

$$\begin{aligned} 14\% \text{ of } 7300 \text{ is } 0.14(7300) &= 1022 \text{ and} \\ 8\% \text{ of } 2700 \text{ is } 0.08(2700) &= 216 \end{aligned}$$

Since $1022 + 216 = 1238$, our solution is correct.

17. The hypotenuse of a right triangle is 68 cm. The difference between the other two sides is 28 cm. Find the sides of the triangle. **32 cm and 60 cm**

Solution: See handout on the Pythagorean Theorem.

18. Find the distance between $(3, 8)$ and $(8, -4)$. **13 units**

Solution: See handout on the Pythagorean Theorem.