Part 1

1. Simplify $5-2(4b-5(b-3))$.

   A) $2b+35$ B) $35-18b$ c) $2b-25$ D) $35-2b$

   Solution:

$$\begin{aligned} 5-2(4b-5(b-3)) &= \text{distribute} \\ 5-2(4b-5b+15) &= \text{combine like terms} \\ 5-2(-b+15) &= \text{distribute} \\ 5+2b-30 &= \text{combine like terms} \\ &= 2b-25 \end{aligned}$$

2. Simplify the expression $\left(\sqrt{x}-\sqrt{2}\right)^2$

   A) $x-2\sqrt{2x}+2$ B) $x-2$ C) $x-2\sqrt{x}+2\sqrt{2}-\sqrt{x}\sqrt{2}$ D) $x-4\sqrt{x}+4$

   Solution:

$$\begin{aligned} \left(\sqrt{x}-\sqrt{2}\right)^2 &= \left(\sqrt{x}-\sqrt{2}\right)\left(\sqrt{x}-\sqrt{2}\right) \quad \text{we FOIL} \\ &= \sqrt{x}\sqrt{x}-\sqrt{x}\sqrt{2}-\sqrt{2}\sqrt{x}+\sqrt{2}\sqrt{2} \\ &= x-2\sqrt{2x}+2 \end{aligned}$$

3. Solve the equation $x^2-29=4x$ over the real numbers.

   A) There is no solution. B) $2-\sqrt{33}$ and $2+\sqrt{33}$ C) -3 and 7 D) -25

   Solution:

$$\begin{aligned} x^2-29 &= 4x && \text{subtract } 4x \\ x^2-4x-29 &= 0 && (x-2)^2 = x^2-4x+4 \\ \underbrace{x^2-4x+4}-4-29 &= 0 && \text{complete the square} \\ (x-2)^2-33 &= 0 \\ (x-2)^2-33 &= 0 \\ (x-2)^2-\left(\sqrt{33}\right)^2 &= 0 && \text{factor} \\ \left(x-2+\sqrt{33}\right)\left(x-2-\sqrt{33}\right) &= 0 \end{aligned}$$

$$x_1 = 2-\sqrt{33} \quad \text{and} \quad x_2 = 2+\sqrt{33}$$

4. Perform the indicated operations and simplify. $\dfrac{x^2-9}{x^2+7x+12} \div \dfrac{x-3}{x+5}$

   A) $\dfrac{x+5}{x+4}$ B) $\dfrac{x^2-6x+9}{9x+x^2+20}$ C) $\dfrac{x-3}{9x+x^2+20}$ D) $\dfrac{x+5}{x-4}$

   Solution: We factor everything we can and re-write the division as multiplication by the reciprocal. Finally, we cancel.

$$\frac{x^2-9}{x^2+7x+12} \div \frac{x-3}{x+5} = \frac{(x-3)(x+3)}{(x+4)(x+3)} \cdot \frac{x+5}{x-3} = \frac{x+5}{x+4}$$

5. Solve the equation $x^2 = 4x + 1$.

A) $-\frac{1}{2}, \sqrt{5}+1$ B) $2-\sqrt{5},\ 2+\sqrt{5}$ C) $2-\sqrt{10},\ 2+\sqrt{10}$ D) $2+\sqrt{20},\quad 2-\sqrt{20}$

Solution 1: We will apply the quadratic formula. First we reduce one side to zero and then identify the coefficients.

$$x^2 - 4x - 1 = 0 \qquad \Longrightarrow \quad a = 1, \quad b = -4 \quad \text{and} \quad c = -1$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)} = \frac{4 \pm \sqrt{16+4}}{2} = \frac{4 \pm \sqrt{20}}{2} =$$

$$= \frac{4 \pm 2\sqrt{5}}{2} = \begin{cases} \dfrac{4+2\sqrt{5}}{2} = \dfrac{2\left(2+\sqrt{5}\right)}{2} = 2+\sqrt{5} \\ \dfrac{4-2\sqrt{5}}{2} = \dfrac{2\left(2-\sqrt{5}\right)}{2} = 2-\sqrt{5} \end{cases} \quad \text{or} \quad 2 \pm \sqrt{5}$$

Solution 2: Complete the square.

$$\begin{aligned} x^2 &= 4x + 1 \\ x^2 - 4x - 1 &= 0 & \qquad (x-2)^2 = x^2 - 4x + 4 \\ \underbrace{x^2 - 4x + 4}\, -4 - 1 &= 0 \\ (x-2)^2 - 5 &= 0 \\ (x-2)^2 - \left(\sqrt{5}\right)^2 &= 0 \\ \left(x - 2 + \sqrt{5}\right)\left(x - 2 - \sqrt{5}\right) &= 0 \end{aligned}$$

$$x_1 = 2 - \sqrt{5} \quad \text{and} \quad x_2 = 2 + \sqrt{5}$$

6. Simplify the expression $\dfrac{1 - x^{-2}}{1 + x^{-1}}$.

A) $\dfrac{x-1}{x}$ B) $\dfrac{1-x}{x^2+1}$ C) 1 D) $-\dfrac{1}{x-1}$

Solution:

$$\frac{1 - x^{-2}}{1 + x^{-1}} = \frac{1 - \frac{1}{x^2}}{1 + \frac{1}{x}} = \frac{\frac{x^2-1}{x^2}}{\frac{x+1}{x}} = \frac{x^2-1}{x^2} \cdot \frac{x}{x+1} = \frac{(x+1)(x-1)}{x^2} \cdot \frac{x}{x+1} = \frac{x-1}{x}$$

7. Perform the indicated operations and simplify. $\dfrac{1}{x-y} - \dfrac{1}{x+y}$

A) 0 B) $-\dfrac{2}{x+y}$ C) $\dfrac{-2y}{y^2-x^2}$ D) $\dfrac{2x}{y^2-x^2}$

Solution:

$$\begin{aligned} \frac{1}{x-y} - \frac{1}{x+y} &= \frac{x+y}{(x-y)(x+y)} - \frac{x-y}{(x-y)(x+y)} \\ &= \frac{(x+y)-(x-y)}{(x-y)(x+y)} = \frac{x+y-x+y}{(x-y)(x+y)} = \frac{2y}{(x-y)(x+y)} \end{aligned}$$

This answer appears to be missing, but it is actually listed:

$$\frac{2y}{(x-y)(x+y)} = \frac{2y}{x^2-y^2} = \frac{2y}{x^2-y^2} \cdot 1 = \frac{2y}{x^2-y^2} \cdot \frac{-1}{-1} = \frac{-2y}{y^2-x^2}$$

which is C.

8. Simplify $\dfrac{2^{1/2}4^{-1/2}}{64^{-2/3}}$.

A) $\sqrt{2}$ B) $\dfrac{\sqrt{2}}{8}$ C) $-32\sqrt{2}$ D) $8\sqrt{2}$

Solution 1: First we get rid of the negative exponents using the rule $a^{-n} = \dfrac{1}{a^n}$.

$$\frac{2^{1/2}4^{-1/2}}{64^{-2/3}} = \frac{2^{1/2}64^{2/3}}{4^{1/2}}$$

We then interpret the fractional exponents as roots, using the rule $a^{\frac{n}{m}} = \left(\sqrt[m]{a}\right)^n$.

$$\frac{2^{1/2}64^{2/3}}{4^{1/2}} = \frac{\sqrt{2}\left(\sqrt[3]{64}\right)^2}{\sqrt{4}} = \frac{\sqrt{2}\cdot 4^2}{2} = \frac{\sqrt{2}\cdot 16}{2} = 8\sqrt{2}$$

Solution 2: We first we get rid of the negative exponents using the rule $a^{-n} = \dfrac{1}{a^n}$.

$$\frac{2^{1/2}4^{-1/2}}{64^{-2/3}} = \frac{2^{1/2}64^{2/3}}{4^{1/2}}$$

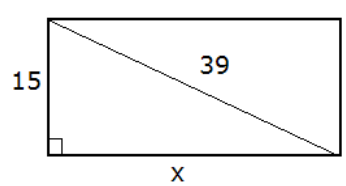
Notice now that $4 = 2^2$ and $64 = 2^6$and then we can easily apply other rules of exponents.

$$\frac{2^{1/2}64^{2/3}}{4^{1/2}} = \frac{2^{1/2}\left(2^6\right)^{2/3}}{\left(2^2\right)^{1/2}} = \frac{2^{1/2}\cdot 2^{6\cdot\left(\frac{2}{3}\right)}}{2^{2\cdot\left(\frac{1}{2}\right)}} = \frac{2^{1/2}\cdot 2^4}{2^1} = \frac{2^{4+\frac{1}{2}}}{2^1} = 2^{4+\frac{1}{2}-1} = 2^{3\frac{1}{2}} = 2^3\cdot 2^{1/2} = 8\sqrt{2}$$

9. Find the area of a rectangle if its diagonal is 39 cm long and one of its sides is 15 cm long.

A) 292.5 cm^2 B) 540 cm^2 C) 585 cm^2 D) 102 cm^2

Solution:



We have a right triangle where one leg is 15 cm and the hypotenuse is 39 cm. We find the other leg using the Pythagorean Theorem.

$$\begin{aligned}
15^2 + x^2 &= 39^2 \\
225 + x^2 &= 1521 \qquad \text{subtract 1521} \\
x^2 - 1296 &= 0 \\
x^2 - 36^2 &= 0 \\
(x+36)(x-36) &= 0 \\
x_{1,2} &= \pm 36 \implies \text{since distances can not be negative, } x = 36
\end{aligned}$$

Thus the rectangle is 15 cm by 36 cm, and thus its area is $A = 15\text{ cm }(36\text{ cm}) = 540\text{ cm}^2$, which is B).

Part 2

1. Simplify each of the following expressions. Show all work.

(a) $2^{-2}-2^{-3}=$

$$2^{-2}-2^{-3}=\frac{1}{2^2}-\frac{1}{2^3}=\frac{1}{4}-\frac{1}{8}=\frac{2-1}{8}=\frac{1}{8}$$

(b) $\dfrac{\left(x^{-2}\right)^{-2}y^3x^0\left(-2yxy^{-2}x^{-2}\right)^{-3}}{yx^5\left(y^{-2}x\right)^{-3}\left(2x^{-1}yx^3\right)^{-1}}=$

Solution: We will first omit x^0 and simplify within each parentheses. Then we apply the rules of exponents as indicated.

$$\begin{aligned}\frac{\left(x^{-2}\right)^{-2}y^3x^0\left(-2yxy^{-2}x^{-2}\right)^{-3}}{yx^5\left(y^{-2}x\right)^{-3}\left(2x^{-1}yx^3\right)^{-1}} &= \frac{\left(x^{-2}\right)^{-2}y^3\left(-2y^{-1}x^{-1}\right)^{-3}}{yx^5\left(y^{-2}x\right)^{-3}\left(2x^2y\right)^{-1}} && \text{use } (ab)^n=a^nb^n\\ &= \frac{\left(x^{-2}\right)^{-2}y^3(-2)^{-3}\left(y^{-1}\right)^{-3}\left(x^{-1}\right)^{-3}}{yx^5\left(y^{-2}\right)^{-3}x^{-3}2^{-1}\left(x^2\right)^{-1}y^{-1}} && \text{use } (a^n)^m=a^{nm}\\ &= \frac{x^4y^3(-2)^{-3}y^3x^3}{yx^5y^6x^{-3}2^{-1}x^{-2}y^{-1}}\end{aligned}$$

We apply $a^{-n}=\dfrac{1}{a^n}$ and arrange terms: first numbers, then letters, alphabetized.

$$\begin{aligned}\frac{x^4y^3(-2)^{-3}y^3x^3}{yx^5y^6x^{-3}2^{-1}x^{-2}y^{-1}} &= \frac{2^1x^4y^3y^3x^3x^3x^2y^1}{(-2)^3yx^5y^6} && \text{Now apply } a^na^m=a^{n+m}\\ &= \frac{2x^{12}y^7}{-8x^5y^7} && \text{apply } \frac{a^n}{a^m}=a^{n-m} \text{ and move } - \text{ sign upstairs}\\ &= \frac{-x^7}{4} \text{ or } -\frac{1}{4}x^7.\end{aligned}$$

(c) $\sqrt{48x^5y^3}=$

$$\sqrt{48x^5y^3}=\sqrt{16x^4y^2\cdot 3xy}=\sqrt{16x^4y^2}\sqrt{3xy}=4x^2y\sqrt{3xy}$$

(d) $\sqrt{80a^{11}}-2\sqrt{180a^{11}}+3\sqrt{245a^{11}}=$

$$\begin{aligned}\sqrt{80a^{11}}-2\sqrt{180a^{11}}+3\sqrt{245a^{11}} &=\\ \sqrt{16a^{10}\cdot 5a}-2\sqrt{36a^{10}\cdot 5a}+3\sqrt{49a^{10}\cdot 5a} &=\\ \sqrt{16a^{10}}\sqrt{5a}-2\sqrt{36a^{10}}\sqrt{5a}+3\sqrt{49a^{10}}\sqrt{5a} &=\\ 4a^5\sqrt{5a}-2\left(6a^5\right)\sqrt{5a}+3\left(7a^5\right)\sqrt{5a} &=\\ 4a^5\sqrt{5a}-12a^5\sqrt{5a}+21a^5\sqrt{5a} &=\\ (4-12+21)\,a^5\sqrt{5a} &= 13a^5\sqrt{5a}\end{aligned}$$

(e) $\sqrt[3]{56}+4\sqrt[3]{189}-\sqrt[3]{875}=$

$$\begin{aligned}\sqrt[3]{56}+4\sqrt[3]{189}-\sqrt[3]{875} &= \sqrt[3]{8\cdot 7}+4\sqrt[3]{27\cdot 7}-\sqrt[3]{125\cdot 7}\\ &= \sqrt[3]{8}\sqrt[3]{7}+4\sqrt[3]{27}\sqrt[3]{7}-\sqrt[3]{125}\sqrt[3]{7}\\ &= 2\sqrt[3]{7}+4(3)\sqrt[3]{7}-5\sqrt[3]{7}\\ &= 2\sqrt[3]{7}+12\sqrt[3]{7}-5\sqrt[3]{7}\\ &= (2+12-5)\sqrt[3]{7}=9\sqrt[3]{7}\end{aligned}$$

(f) $(2-\sqrt{x})(3+2\sqrt{x})=$

$$\left(2-\sqrt{x}\right)\left(3+2\sqrt{x}\right)=6+4\sqrt{x}-3\sqrt{x}-2\sqrt{x}\sqrt{x}=6+\sqrt{x}-2x$$

(g) $\dfrac{\sqrt{5}-1}{\sqrt{5}-2}=$

$$\frac{\sqrt{5}-1}{\sqrt{5}-2}\cdot\frac{\sqrt{5}+2}{\sqrt{5}+2}=\frac{\left(\sqrt{5}-1\right)\left(\sqrt{5}+2\right)}{\left(\sqrt{5}-2\right)\left(\sqrt{5}+2\right)}=\frac{5+2\sqrt{5}-\sqrt{5}-2}{5+2\sqrt{5}-2\sqrt{5}-4}=\frac{3+\sqrt{5}}{1}=3+\sqrt{5}$$

(h) $\dfrac{px^2-16q-16p+qx^2}{x^2+5x+6}\cdot\dfrac{x^2+6x+9}{4px^2+px^3+4qx^2+qx^3}=$

Solution: $x^2+5x+6=(x+2)(x+3)$ and $x^2+6x+9=(x+3)(x+3)$ are easy. With the other expressions, we start with the greatest common factor.

$$\begin{aligned}
px^2-16q-16p+qx^2 &= \underbrace{px^2+qx^2}\underbrace{-16p-16q} && \text{grouping}\\
&= x^2(p+q)-16(p+q) && \text{factor out } p+q\\
&= \left(x^2-16\right)(p+q) && \text{difference of squares}\\
&= (x+4)(x-4)(p+q)
\end{aligned}$$

$$\begin{aligned}
4px^2+px^3+4qx^2+qx^3 &= x^2\left(\underbrace{4p+4q}\underbrace{+px+qx}\right) && \text{grouping}\\
&= x^2\left(4(p+q)+x(p+q)\right) && \text{factor out } p+q\\
&= x^2(x+4)(p+q)
\end{aligned}$$

We have all the pieces now:

$$\begin{aligned}
&\frac{px^2-16q-16p+qx^2}{x^2+5x+6}\cdot\frac{x^2+6x+9}{4px^2+px^3+4qx^2+qx^3} =\\
&\frac{(x+4)(x-4)(p+q)}{(x+2)(x+3)}\cdot\frac{(x+3)(x+3)}{x^2(x+4)(p+q)} = \frac{(x+3)(x-4)}{x^2(x+2)}=\frac{x^2-x-12}{2x^2+x^3}
\end{aligned}$$

2. Completely factor each of the following.

(a) $357ab^2-30ab^2x-3ab^2x^2=$

$$\begin{aligned}
357ab^2-30ab^2x-3ab^2x^2 &= 3ab^2\left(119-x^2-10x\right)\\
&= -3ab^2\left(x^2+10x-119\right) && (x+5)^2=x^2+10x+25\\
&= -3ab^2\left(\underbrace{x^2+10x+25}-25-119\right)\\
&= -3ab^2\left((x+5)^2-144\right)\\
&= -3ab^2\left((x+5)^2-12^2\right)\\
&= -3ab^2(x+5+12)(x+5-12)\\
&= -3ab^2(x+17)(x-7)
\end{aligned}$$

(b) $4a^2px^5 - 2a^2qx - 4a^2px + 2a^2qx^5 =$

$$\begin{aligned}
& 4a^2px^5 - 2a^2qx - 4a^2px + 2a^2qx^5 \\
= & \ 2a^2x\left(2px^4 - q - 2p + qx^4\right) \\
= & \ 2a^2x\left(2px^4 + qx^4 - 2p - q\right) \\
= & \ 2a^2x\left(x^4(2p+q) - 1(2p+q)\right) \\
= & \ 2a^2x\left(x^4 - 1\right)(2p+q) & x^4 - 1 = \left(x^2\right)^2 - 1^2 \quad \text{factors} \\
= & \ 2a^2x\left(x^2+1\right)\left(x^2-1\right)(2p+q) & x^2 - 1 \quad \text{factors} \\
= & \ 2a^2x(x-1)(x+1)\left(x^2+1\right)(2p+q)
\end{aligned}$$

3. Factor via completing the square:

(a) $100x - x^2 - 2419 =$

$$\begin{aligned}
100x - x^2 - 2419 &= -\left(x^2 - 100x + 2419\right) & (x-50)^2 = x^2 - 100x + 2500 \\
&= -\left(\underbrace{x^2 - 100x + 2500} - 2500 + 2419\right) \\
&= -\left((x-50)^2 - 81\right) \\
&= -\left((x-50)^2 - 9^2\right) \\
&= -(x-50+9)(x-50-9) \\
&= -(x-41)(x-59)
\end{aligned}$$

(b) $x^2 - x - 462 =$

$$\begin{aligned}
x^2 - x - 462 &= & \left(x - \frac{1}{2}\right)^2 = x^2 - x + \frac{1}{4} \\
&= \underbrace{x^2 - x + \frac{1}{4}} - \frac{1}{4} - 462 \\
&= \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - \frac{1848}{4} \\
&= \left(x - \frac{1}{2}\right)^2 - \frac{1849}{4} \\
&= \left(x - \frac{1}{2}\right)^2 - \left(\frac{43}{2}\right)^2 \\
&= \left(x - \frac{1}{2} + \frac{43}{2}\right)\left(x - \frac{1}{2} - \frac{43}{2}\right) \\
&= (x+21)(x-22)
\end{aligned}$$

(c) $11x + 6x^2 - 10 =$

Solution: we first rearrange the polynomial and then factor out the leading coefficient.

$$11x + 6x^2 - 10 = 6x^2 + 11x - 10 = 6\left(x^2 + \frac{11}{6}x - \frac{5}{3}\right)$$

The magic number is $\frac{11}{6} \div 2 = \frac{11}{6}\left(\frac{1}{2}\right) = \frac{11}{12}$ and so we FOIL $\left(x+\frac{11}{12}\right)^2$

$$\begin{aligned}\left(x+\frac{11}{12}\right)^2 &= \left(x+\frac{11}{12}\right)\left(x+\frac{11}{12}\right) = x^2+\frac{11}{12}x+\frac{11}{12}x+\left(\frac{11}{12}\right)^2 \\ &= x^2+\frac{11}{6}x+\frac{121}{144} \qquad \text{thus we will smuggle in } \frac{121}{144}\end{aligned}$$

$$\begin{aligned}6\left(x^2+\frac{11}{6}x-\frac{5}{3}\right) &= 6\left(\underbrace{x^2+\frac{11}{6}x+\frac{121}{144}}-\frac{121}{144}-\frac{5}{3}\right) \\ &= 6\left(\left(x+\frac{11}{12}\right)^2-\frac{121}{144}-\frac{240}{144}\right) \\ &= 6\left(\left(x+\frac{11}{12}\right)^2-\frac{361}{144}\right) \\ &= 6\left(\left(x+\frac{11}{12}\right)^2-\left(\frac{19}{12}\right)^2\right) \\ &= 6\left(x+\frac{11}{12}+\frac{19}{12}\right)\left(x+\frac{11}{12}-\frac{19}{12}\right) \\ &= 6\left(x+\frac{30}{12}\right)\left(x-\frac{8}{12}\right) \\ &= 2\left(x+\frac{5}{2}\right)3\left(x-\frac{2}{3}\right) = (2x+5)(3x-2)\end{aligned}$$

(d) $x^2-8x+13=$

$$\begin{aligned}x^2-8x+13 &= \qquad (x-4)^2 = x^2-8x+16 \\ \underbrace{x^2-8x+16}-16+13 &= \\ (x-4)^2-3 &= \\ (x-4)^2-\left(\sqrt{3}\right)^2 &= \left(x-4+\sqrt{3}\right)\left(x-4-\sqrt{3}\right)\end{aligned}$$

(e) x^2-4x+7

$$\begin{aligned}x^2-4x+7 &= \qquad (x-2)^2 = x^2-4x+4 \\ \underbrace{x^2-4x+4}-4+7 &= (x-2)^2+3\end{aligned}$$

which does not factor over the real numbers. Over the complex numbers,

$$\begin{aligned}(x-2)^2+3 &= (x-2)^2-(-3) \\ &= (x-2)^2-\left(\sqrt{3}i\right)^2 \\ &= \left(x-2+\sqrt{3}i\right)\left(x-2-\sqrt{3}i\right)\end{aligned}$$

4. Graphing.

a) Graph the parabola $y = -2x^2 + 3x + 1$. Clearly label the coordinates of at least 5 points, including vertex and intercepts.

Solution: The y-intercept is clearly $(0, 1)$. The x-coordinate of the vertex is always $x_V = \dfrac{-b}{2a}$ where $a = -2$, $b = 3$, and $c = 1$

$$x_V = \frac{-b}{2a} = \frac{-3}{2(-2)} = \frac{3}{4}$$

For the y-coordinate of the vertex, we plug its x-coordinate into the formula:

$$y_V = -2(x_V)^2 + 3(x_V) + 1 = -2\left(\frac{3}{4}\right)^2 + 3\left(\frac{3}{4}\right) + 1 = -2\left(\frac{9}{16}\right) + \frac{9}{4} + 1 = \frac{-9}{8} + \frac{18}{8} + \frac{8}{8} = \frac{17}{8}$$

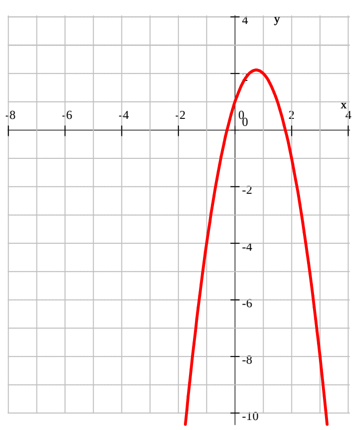
Thus the vertex is $\left(\dfrac{3}{4}, \dfrac{17}{8}\right)$. Now we know to pug in numbers close to $\dfrac{3}{4}$:

$$\begin{aligned}
\text{if } x &= -2, \text{ then } y = -2(-2)^2 + 3(-2) + 1 = -2(4) - 6 + 1 = -13 \\
\text{if } x &= -1, \text{ then } y = -2(-1)^2 + 3(-1) + 1 = -2(1) - 3 + 1 = -4 \\
\text{if } x &= 0, \text{ then } y = -2(0)^2 + 3(0) + 1 = 1 \\
\text{if } x &= 1, \text{ then } y = -2(1)^2 + 3(1) + 1 = -2(1) + 3 + 1 = 2 \\
\text{if } x &= 2, \text{ then } y = -2(2)^2 + 3(2) + 1 = -2(4) + 6 + 1 = -1
\end{aligned}$$

For the x-intercepts, we apply the quadratic formula.

$$\begin{aligned}
a &= -2, \quad b = 3 \quad \text{and} \quad c = 1 \\
x_{1,2} &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{(-3)^2 - 4(-2)(1)}}{2(-2)} = \frac{-3 \pm \sqrt{9+8}}{-4} = \\
&= \frac{-3 \pm \sqrt{17}}{-4} = \begin{cases} \dfrac{-3+\sqrt{17}}{-4} \cdot \dfrac{-1}{-1} = \dfrac{-1\left(-3+\sqrt{17}\right)}{4} = \dfrac{3-\sqrt{17}}{4} \\ \dfrac{-3-\sqrt{17}}{-4} \cdot \dfrac{-1}{-1} = \dfrac{-1\left(-3-\sqrt{17}\right)}{4} = \dfrac{3+\sqrt{17}}{4} \end{cases} \quad \text{or} \quad \frac{3 \pm \sqrt{17}}{4}
\end{aligned}$$

and so the x-intercepts are $\left(\dfrac{3-\sqrt{17}}{4}, 0\right) \simeq (-0.281, 0)$ and $\left(\dfrac{3+\sqrt{17}}{4}, 0\right) \simeq (1.781, 0)$. Now we are ready to graph:



b) Graph the parabola $y = 5x - 2x^2 + 3$ and the line $y = 5x - 5$ in the same coordinate system. Use your graph to find the coordinates of the points where they intersect.
Solution: We graph the straight line first. The equation given is already in the slope-intercept form, and so it is easy to graph it. We start at the $y-$intercept, $(0, -5)$ and move on the grid one to the right, five up. Now for the parabola: because the leading coefficient is negative, the parabola iopens downward. From the polynomial form, $y = -2x^2 + 5x + 3$, we obtain the $y-$intercept: $(0, 3)$ and the coefficients

$$a = -2,\ b = 5,\ c = 3$$

The vertex is at

$$x_V = \frac{-b}{2a} = \frac{-5}{2(-2)} = \frac{5}{4}$$

Then the $y-$coordinate of the vertex is easy. If $x = \frac{5}{4}$, then

$$y = -2\left(\frac{5}{4}\right)^2 + 5\left(\frac{5}{4}\right) + 3 = -2\left(\frac{25}{16}\right) + \frac{25}{4} + 3 = \frac{-50}{16} + \frac{25}{4} + 3 = \frac{-25}{8} + \frac{25}{4} + 3 = \frac{-25}{8} + \frac{50}{8} + \frac{24}{8} = \frac{49}{8}$$

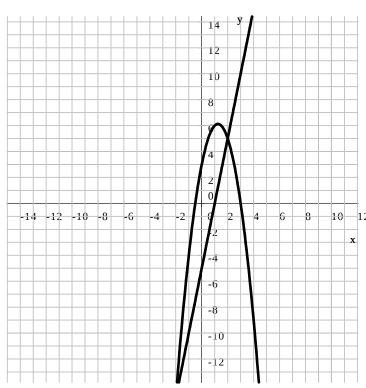
Thus the vertex is $\left(\frac{5}{4}, \frac{49}{8}\right)$. We will conduct the $5-$point dance around 1, the closest integer to $\frac{5}{4}$. Because we no longer work around the vertex, we loose the symmetry.

$$\begin{aligned}
\text{If } x &= -1, \text{ then } y = -2(-1)^2 + 5(-1) + 3 = -2(1) - 5 + 3 = -7 + 3 = -4 \implies (-1, -4) \\
&\quad \text{We have already found the } y-\text{intercept}, \ (0, 3) \\
\text{If } x &= 1, \text{ then } y = -2(1)^2 + 5(1) + 3 = -2(1) + 5 + 3 = -2 + 5 + 3 = 6 \implies (1, 6) \\
&\quad \text{We have already found the vertex}, \left(\frac{5}{4}, \frac{49}{8}\right) \\
\text{If } x &= 2, \text{ then } y = -2(2)^2 + 5(2) + 3 = -2(4) + 10 + 3 = -8 + 10 + 3 = 5 \implies (2, 5)
\end{aligned}$$

The $x-$intercepts are at $x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$$x_{1,2} = \frac{-5 \pm \sqrt{5^2 - 4(-2)3}}{2(-2)} = \frac{-5 \pm \sqrt{25 + 24}}{-4} = \frac{-5 \pm \sqrt{49}}{-4} = \frac{-5 \pm 7}{-4} = \begin{cases} \dfrac{-5+7}{-4} = \dfrac{2}{-4} = -\dfrac{1}{2} \\ \dfrac{-5-7}{-4} = \dfrac{-12}{-4} = 3 \end{cases}$$

Thus the $x-$intercepts are $\left(-\frac{1}{2}, 0\right)$ and $(3, 0)$. We compute points close to the vertex and where it appears to get closer to the line.



The intersections are $(2,5)$ and $(-2,-15)$. We can check algebraically.

5. Solve each of the following.

(a) $7-(3+4t)+2t=-5(1-t)+3-t$

$$\begin{aligned} 7-(3+4t)+2t &= -5(1-t)+3-t && \text{distribute} \\ 7-3-4t+2t &= -5+5t+3-t && \text{combine like terms} \\ -2t+4 &= 4t-2 && \text{add } 2t \\ 4 &= 6t-2 && \text{add } 2 \\ 6 &= 6t && \text{divide by } 6 \\ 1 &= t \end{aligned}$$

We check:

$$\begin{aligned} \text{LHS} &= 7-(3+4(1))+2(1)=7-(3+4)+2=7-7+2=2 \\ \text{RHS} &= -5(1-1)+3-1=-5(0)+3-1=2 \end{aligned}$$

(b) $\dfrac{2x-1}{3}-\dfrac{-3-x}{4}=x-1$

$$\begin{aligned} \frac{2x-1}{3}-\frac{-3-x}{4} &= \frac{x-1}{1} && \text{common denominator} \\ \frac{4(2x-1)}{12}-\frac{3(-3-x)}{12} &= \frac{12(x-1)}{12} && \text{multiply by } 12 \\ 4(2x-1)-3(-3-x) &= 12(x-1) && \text{distribute} \\ 8x-4+9+3x &= 12x-12 && \text{combine like terms} \\ 11x+5 &= 12x-12 && \text{subtract } 11x \\ 5 &= x-12 && \text{add } 12 \\ 17 &= x \end{aligned}$$

We check:

$$\begin{aligned} \text{LHS} &= \frac{2(17)-1}{3}-\frac{-3-17}{4}=\frac{33}{3}-\frac{-20}{4}=11-(-5)=16 \\ \text{RHS} &= 17-1=16 \end{aligned}$$

(c) $3x^3-x^2=x$

$$\begin{aligned} 3x^3-x^2 &= x \\ 3x^3-x^2-x &= 0 \\ x\left(3x^2-x-1\right) &= 0 \\ x &= 0 \quad \text{or} \quad 3x^2-x-1=0 \end{aligned}$$

We apply the quadratic formula with $a=3$, $b=-1$ and $c=-1$.

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{1\pm\sqrt{(-1)^2-4(3)(-1)}}{2(3)}=\frac{1\pm\sqrt{1+12}}{6}=\frac{1\pm\sqrt{13}}{6}$$

So the solutions are $x=0$ or $x=\dfrac{1+\sqrt{13}}{6}$ or $x=\dfrac{1-\sqrt{13}}{6}$

(d) $5-\sqrt{2x+1}=-2$

$$\begin{aligned} 5-\sqrt{2x+1} &= -2 && \text{add } \sqrt{2x+1} \\ 5 &= -2+\sqrt{2x+1} && \text{add } 2 \\ 7 &= \sqrt{2x+1} && \text{square} \\ 49 &= 2x+1 && \text{subtract 1} \\ 48 &= 2x && \text{divide by 2} \\ 24 &= x \end{aligned}$$

We check: if $x=24$, then

$$\begin{aligned} \text{LHS} &= 5-\sqrt{2(24)+1}=5-\sqrt{49}=5-7=-2 \\ \text{RHS} &= -2 \\ \text{Since LHS} &= \text{RHS, } 24 \text{ is a solution.} \end{aligned}$$

Thus $x=24$ is correct.

6. Word Problems.

a) One side of a rectangle is 16 cm longer than the other side. The area of the rectangle is 80 cm^2. Find the dimensions of the rectangle. Include units in your answer.
Solution: Let us denote the shorter side by x. Then the longer side is $x+16$. We obtain the equation for the area:

$$\begin{aligned} x(x+16) &= 80 && \text{distribute} \\ x^2+16x &= 80 && \text{subtract 80} \\ x^2+16x-80 &= 0 && \text{factor} \qquad (x+8)^2=x^2+16x+64 \\ \underbrace{x^2+16x+64}-64-80 &= 0 \\ (x+8)^2-144 &= 0 \\ (x+8)^2-12^2 &= 0 \\ (x+8+12)(x+8-12) &= 0 \\ (x+20)(x-4) &= 0 \\ x_1 &= -20 \quad \text{and} \quad x_2=4 \end{aligned}$$

Since distances are non-negative, $x=-20$ is ruled out as a solution. Thus the shorter side is 4 cm, and the longer side is $4+16=20$ cm. We check: the area is $4(20)=80$ cm^2. Thus the solution is: 4 cm by 20 cm

b) The sides of a right triangle have lengths (in centimeters) that are consecutive even integers. What are the lengths of the sides? 6 cm, 8 cm, and 10 cm
Solution: Let us denote the length of the shortest side by x. Then the other sides are $x+2$ and $x+4$ long. Clearly, $x+4$ must the length of the hypotenuse. The equation expresses the Pythagorean theorem.

$$\begin{aligned}
x^2+(x+2)^2 &= (x+4)^2 && \text{distribute} \\
x^2+x^2+4x+4 &= x^2+8x+16 && \text{combine like terms} \\
2x^2+4x+4 &= x^2+8x+16 && \text{subtract } x^2 \\
x^2+4x+4 &= 8x+16 && \text{subtract } 8x \\
x^2-4x+4 &= 16 && \text{subtract } 16 \\
x^2-4x-12 &= 0 && \text{factor by completing the square} \\
\underbrace{x^2-4x+4}-4-12 &= 0 \\
(x-2)^2-16 &= 0 \\
(x-2)^2-4^2 &= 0 \\
(x-2-4)(x-2+4) &= 0 \\
(x-6)(x+2) &= 0 && \text{apply the zero property}
\end{aligned}$$

$$x_1 = 6 \quad \text{and} \quad x_2 = -2$$

Since distances are non-negative, $x=-2$ is ruled out and so the sides of the triangle are 6 cm, 8 cm, and 10 cm long. We check: 6, 8, and 10 are indeed consecutive even numbers and work with the Pyhtagorean theorem as well, since $6^2+8^2=36+64=100=10^2$.

c) Two investments produce an annual interest income of 708. The total amount of money invested is \$8000, and the two interest rates paid are 7% and 11%. How much money is invested at each rate? \$3700 at 11% and \$4300 at 7%
Solution 1. Let us denote the amount invested at 11% by x. Then the other account must be $8000-x$ since the two accounts add to \$8000. (Remember: one information goes into labeling, the other one gives you the equation.) The equation expresses the combined interest. Since we invested

$$x \text{ at } 11\% \qquad \text{and } 8000-x \text{ at } 7\%$$

the equation is

$$\begin{aligned}
0.11x+0.07(8000-x) &= 708 && \text{multiply both sides by 100} \\
11x+7(8000-x) &= 70800 && \text{distribute} \\
11x+56000-7x &= 70800 && \text{combine like terms} \\
4x+56000 &= 70800 && \text{subtract 56000} \\
4x &= 14800 && \text{divide by 4} \\
x &= 3700
\end{aligned}$$

We invested \$3700 at 11%. The other amount is then $8000-x=8000-3700=4300$. Thus we invested \$ 3700 at 11% and \$4300 at 7%. We check: the accounts add up to $\$3700+\$4300=\$8000$. The interest from each account is

$$\begin{aligned}
11\% \text{ of } 3700 \text{ is} \quad 0.11(3700) &= 407 \text{ and} \\
7\% \text{ of } 4300 \text{ is} \quad 0.07(4300) &= 301
\end{aligned}$$

Since $407 + 301 = 708$, our solution is correct.
Solution 2: Let us denote the amount invested at 11% by x and the amount invested at 7% by y. The two equations express that

$$\begin{aligned} x + y &= 8000 && \text{the accounts add up to \$8000} \\ 0.11x + 0.07y &= 708 && \text{the combined interest is \$708} \end{aligned}$$

We solve the system of equation by elimination. Before, let us first make the second equation simpler by multiplying by 100. We now have

$$\begin{aligned} x + y &= 8000 \\ 11x + 7y &= 70800 \end{aligned}$$

We will multiply the first equation by -7 to eliminate y

1.) $-7x - 7y = -56000$
2.) $11x + 7y = 70800$ add the equations
$4x = 14800$ divide by 4
$x = 3700$

Thus we invested \$3700 at 11%. The other amount is then from the first equation:

$$\begin{aligned} 3700 + y &= 8000 && \text{subtract } 3700 \\ y &= 4300 \end{aligned}$$

We invested \$3700 at 11% and \$4300 at 7%. We check: the amounts add up to $\$3700 + \$4300 = \$8000$ ✓. The combined interest from the accounts is

$$11\% \text{ of } 3700 \text{ is } 0.11\,(3700) = 407 \quad \text{and} \quad 7\% \text{ of } 4300 \text{ is } 0.07\,(4300) = 301$$

Since $407 + 301 = 708$, our solution is correct.

d) A bank teller has 23 more five-dollar bills than ten-dollar bills. The total value of the money is \$610. How much of each denomination of bill does he have? 33 ten-dollar bills and 56 five-dollar bills
Solution: Let us denote the number of ten-dollar bills by x. Then we have $x + 23$ many five-dollar bills. The equation expresses the value of the bills.

$$\begin{aligned} 10x + 5\,(x + 23) &= 610 && \text{distribute} \\ 10x + 5x + 115 &= 610 && \text{combine like terms} \\ 15x + 115 &= 610 && \text{subtract } 115 \\ 15x &= 495 && \text{divide by } 15 \\ x &= 33 \end{aligned}$$

Thus we have 33 tens and $33 + 23 = 56$ fives. We check: $56 - 33 = 23$ and $33\,(10) + 56\,(5) = 610$. Thus our solution; 33 ten-dollar bills and 56 five-dollar bills; is correct.