

- Suppose that $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 4, 9\}$, and $B = \{2, 4, 6, 7\}$. Find each of the following.
 - $A \cap B$
 - $A \cup B$
 - $P = \{x \in U : x > 4 \text{ or } x \leq 7\}$
 - $Q = \{x \in U : x > 4 \text{ and } x \leq 7\}$
- Label each of the following statements as true or false.
 - For all sets A , $\emptyset \subseteq A$.
 - For all sets A , $A \cap \emptyset = A$.
 - For all sets A and B , $A \cap B \subseteq A$.
 - For all sets A and B , $A \subseteq A \cup B$.
 - For all sets A and B , if $A \subseteq B$, then $A \cup B = B$.
 - $\mathbb{N} \cup \mathbb{Z} = \mathbb{N}$
 - $\mathbb{N} \cap \mathbb{Z} = \mathbb{N}$
- Suppose that S is the set of all squares and R is the set of all rectangles. Label each of the following statements as true or false.
 - $S \subseteq R$
 - $R \subseteq S$
 - $R \cup S = S$
 - $R \cup S = R$
 - $R \cap S = S$
 - $R \cap S = R$
- Suppose that F is the set of all integers divisible by four, S is the set of all integers divisible by six, and T is the set of all integers divisible by three. Label each of the following statements as true or false.
 - $S \subseteq T$
 - $F \subseteq S$
 - $F \cap T = S$
 - $F \cap T \subseteq S$
 - $F \subseteq S \cup T$
- Suppose that $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 4, 5, 7, 10\}$, $B = \{1, 4, 6, 7, 8\}$, and $C = \{1, 4, 5, 6, 9\}$.
 - Draw a Venn diagram depicting these sets.
 - Find each of the following sets.
 - $(A \cup B) \cap C$
 - $A \cup (B \cap C)$
 - $(A \cap B) \cup C$
 - $A \cap (B \cap C)$
 - $A \cap (B \cup C)$
- The freshman class had 35 students. 22 students took Mathematics, 19 students took English, and 14 student took both of these subjects. How many students took neither Mathematics, nor English?
- Perform the division with remainder: $2017 \div 17$
- List all factors of 84.
- Consider the following numbers: 2011, 11 060 904, 321, 3106. Select all the numbers from the list that are divisible
 - by 2
 - by 3
 - by 6
- Which of the following numbers is a prime? 2007, 143, 151, 91
- Find the prime factorization for each of the following.
 - 270
 - 40^{50}
 - $3 \cdot 4 \cdot 5 \cdot 6$
- Which is greater, 10^{100} or 100^{10} ? Explain your answer.
- Label each of the following statements as true or false.
 - If an integer n is divisible by 6, then it is also divisible by 3.
 - If integer a is divisible by 4 and integer b is divisible by 6, then the product ab is divisible by 24.
 - If an integer n is divisible by 4 and also by 6, then is also divisible by 24.
 - If the product ab is divisible by 7, then a is divisible by 7 or b is divisible by 7.
 - If the product ab is divisible by 6, then a is divisible by 2 or b is divisible by 3, or a is divisible by 3 and b is divisible by 2.
- Find the smallest positive integer that is divisible by 2, 3, 4, 5, and 6.

15. Let $S = \{x \in \mathbb{N} : x \geq 2\}$ and $T = \{x \in \mathbb{N} : x < 10\}$. Find $S \cap T$ and $S \cup T$.
16. Let $A = \{n \in \mathbb{N} : n \text{ is divisible by } 2\}$, $B = \{n \in \mathbb{N} : n \text{ is divisible by } 6\}$. Which (if any) of the following is true?
 $A \subseteq B$ $B \subseteq A$
17. Let E be the set of all even integers and O be the set of all odd integers. Find $E \cap O$ and $E \cup O$.
18. Let $P = \{n \in \mathbb{N} : n \text{ is divisible by } 2\}$, $Q = \{n \in \mathbb{N} : n \text{ is divisible by } 5\}$. What is $P \cap Q$?
19. Simplify each of the following.
- a) -3^2 c) $12 \div 3 \cdot 2$ e) $12 - 2(7 - 4 \cdot 3)$ g) $-\sqrt{49}$ i) -2^2
b) $-|-6|$ d) $15 - 3 + 2$ f) $|-8 + 5|$ h) $\sqrt{-49}$ j) $(-2)^2$
20. Simplify each of the following.
- a) $|-3^3 - 2|-5 - 2(-4)|$ f) $\sqrt{6^2 - 5\sqrt{16}}$
b) $\sqrt{-4^2 - (-1)^4 + 2 \cdot 3^2 \div 2 \cdot 6 - 1}$ g) $\frac{24 - (-2)^2 + 12 - 3 + 1}{-3^2 - 12 \div 3(-2)}$
c) $-3^2 - |-12 + 2 \cdot 5| - 2 + 1$ h) $|3 - 2 - |8 - 10||$
d) $-2^2 - 5(-2)$ i) $|3 - |2 - 8| - 10|$
e) $-3^2 - 2(4 - 5^2 + 3(10 - 7 + 2))$ j) $|3 - 2|-8 - 10||$
21. a) Find $\frac{3}{8}$ of 400. d) Express 30% as a reduced fraction.
b) Simplify $\frac{80}{120} = \frac{2}{3}$ e) Re-write $\frac{5}{12}$ with a denominator of 60.
c) Express $\frac{2}{5}$ as a percent f) Re-write $\frac{5}{12}$ with a numerator of 60.
22. Compute each of the given sums. a) $52 + 59 + 66 + \dots + 619$ b) $105 + 116 + 127 + \dots + 2019$
23. a) Re-write 6320 000 000 in scientific notation.
b) Re-write $3.508 \cdot 10^{11}$ using regular notation.
24. Suppose that $x = 3 \cdot 10^{11}$ and $y = 1.2 \cdot 10^{18}$. Perform the indicated operations. Present your answer in scientific notation.
a) x^2 b) xy^2 c) $\frac{y}{x}$ d) \sqrt{xy}
25. Simplify each of the following.
- a) $-\sqrt[4]{16}$ c) $\sqrt[4]{-16}$ e) $\sqrt[7]{-1}$ g) $(\sqrt[4]{2})^{20}$ i) $\sqrt[10]{7^{100}}$
b) $-\sqrt[3]{-8}$ d) $\sqrt[6]{-1}$ f) $(\sqrt[10]{3})^{20}$ h) $\sqrt{7^{100}}$ j) $\sqrt[3]{10^{36}}$
26. Consider the inequality $x^2 + 3x \leq x + 24$. Which of the given numbers are solutions of the inequality? The given numbers: 5, 6, 0, -10, 3, and 4
27. Simplify each of the following.
- a) $(3x + 8) + (2x - 8)$ c) $4(3x + 8) - 5(2x - 8)$ e) $3(4(5(2x - 1) - 7x) - 10(x - 2) - 2x)$
b) $(3x + 8) - (2x - 8)$ d) $3(x - 4(2x - 1) - 3x + 4)$ f) $3(2x - 7) - 2(2x - 1) - (x - 4)$
28. Let us denote the shorter side of a rectangle by x . The longer side is 4 feet longer than twice the shorter side. Express the perimeter of the rectangle as an algebraic expression in terms of x . Simplify the expression.

29. Simplify each of the following.

a) $(-x)^3(-x)^4$ b) $(-x^3)(-x^4)$ c) $(-x^4)^3$ d) $\frac{(-2x^3y^2x)^4(-x^5y^3)}{(-2x^5y)^3(-xy^2)}$

30. Suppose that $3^{400} = A$. Express each of the following in terms of A .

a) 3^{402} b) 3^{399} c) 3^{200} d) 9^{400} e) $3^{400} + 3^{401} + 4^{402}$

31. Solve each of the given equations. Make sure to check your solutions.

a) $3x - 7 = -22$

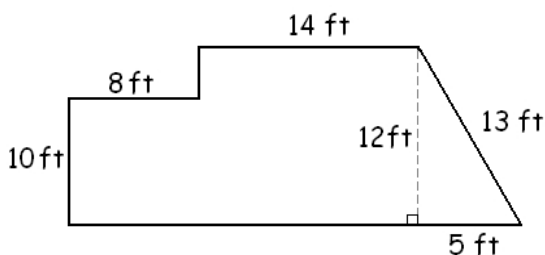
c) $\frac{x - 5}{4} = -2$

e) $\frac{5x + 1}{5} - 3 + 1 = 4$

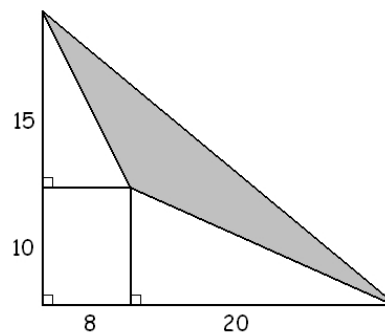
b) $3(x - 7) = 21$

d) $\frac{x}{4} - 5 = -2$

32. Compute the perimeter and area of the figure shown. Angles that look like right angles are right angles. Include units in your computation and answer.



33. Compute the area of the shaded region shown on the picture. Units are in meters. Include units in your computation and answer.



34. What is the last digit of the number 3^{99} ?

35. Solve each of the given word problems. In each case, set up an equation and solve it.

a) Suppose that X is three less than twice the opposite of Y . Find the value of Y if we know that $X = 13$.

b) The longer side of a rectangle is 5 feet less than six times the length of another side. Find the length of the shorter side if the longer side is 13 feet long.

c) The difference between four times a number and the sum of the number and 7 is 23. Find this number.

d) The sum of three consecutive numbers is 51. Find these numbers.

e) Three times the difference of x and 2 is one less than 100. Find x .

f) Suppose that Peter's age is ten less than twice the age of his younger brother, David. If Peter is 14 years old, how old is David?

g) Suppose that a cab-fare for one person in Chicago is 3 dollars for the first mile and 2 dollars for each additional mile. We take a cab and pay 55 dollars. What distance did we travel?

h) The first night in a hotel cost \$45. All additional nights cost \$35. How many nights did we stay in the hotel if the cost was \$360? $45 + 35x = 360$, Solution is: 9

36. Given that $A \cap B = \{1, 2, 4, 5\}$ and $A \cup B = \{1, 2, 3, 4, 5, 6\}$, how many different sets are possible for A ?

37. The conference was attended by 220 people.
- If everyone shook hands with everyone, how many handshakes took place?
 - 120 of the attendees were men and 100 women. How many handshakes took place if men only shook hands with men and women only with women?
 - 120 of the attendees were men and 100 women. How many handshakes took place if men only shook hands with women?
 - Out of the 220 people, 200 were married couples, the other 20 were single men. How many handshakes took place if everyone shook hands with everyone else, except for their spouse?
38. List all 2-element subsets of $S = \{1, 2, 3, 4, 5, 6, 7\}$
39. List all three-digit numbers that can be formed using the numbers 1, 3, 6 and 7 if repetition of digits is not allowed.

Answers

- $\{4\}$
 - $\{1, 2, 4, 6, 7, 9\}$
 - $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
 - $\{5, 6, 7\}$
 - true
 - false
 - true
 - true
 - true
 - false
 - true
 - true
 - false
 - true
 - true
 - false
 - true
 - true
 - false
 - true
 - false
 - false
 - true
 - false
 - - $\{1, 4, 5, 6\}$
 - $\{1, 2, 4, 5, 6, 7, 10\}$
 - $\{1, 4, 5, 6, 7, 9\}$
 - $\{4\}$
 - $\{4, 5, 7\}$
 - 6
 - 8
 - 118
 - R
 - 11
 - 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84
 - 11 060 904, 3106
 - 11 060 904, 321
 - 11 060 904
 - 151
 - $2 \cdot 3^3 \cdot 5$
 - $2^{150} \cdot 5^{50}$
 - $2^3 \cdot 3^2 \cdot 5$
 - 10^{100} is greater, because $100^{10} = (10^2)^{10} = 10^{20}$ and $10^{100} > 10^{20}$.
 - true
 - true
 - false
 - true
 - false
 - 60
15. $S \cap T = \{2, 3, 4, 5, 6, 7, 8, 9\}$ and $S \cup T = \mathbb{N}$ (set of all natural numbers)
16. $B \subseteq A$
17. $E \cap O = \emptyset$ and $E \cup O = \mathbb{Z}$
18. $\{n \in \mathbb{N} : n \text{ is divisible by } 10\} = \{10, 20, 30, 40, \dots\}$
- 9
 - 6
 - 8
 - 14
 - 22
 - 3
 - 7
 - undefined
 - 4
 - 4
 - 33
 - 6
 - 12
 - 6
 - 3
 - 4
 - 30
 - 1
 - 13
 - 33
 - 150
 - $\frac{2}{3}$
 - 40%
 - $\frac{3}{10}$
 - $\frac{25}{60}$
 - $\frac{60}{144}$
 - 27 511
 - 185 745
 - $6.32 \cdot 10^9$
 - 350 800 000 000
 - $9 \cdot 10^{22}$
 - $4.32 \cdot 10^{47}$
 - $4 \cdot 10^6$
 - $6 \cdot 10^{14}$
 - 0, 3, and 4
 - $5x$
 - $x + 16$
 - $2x + 72$
 - $-30x + 24$
 - 0
 - $x - 15$
 - $P = 6x + 8$
 - $-x^7$
 - x^7
 - $-x^{12}$
 - $-2x^5y^6$
 - 4
 - 2
 - undefined
 - undefined
 - 1
 - 9
 - 32
 - 7^{50}
 - 7^{10}
 - 10^{12}

30. a) $9A$ b) $\frac{A}{3}$ c) \sqrt{A} d) A^2 e) $13A$ 31. a) -5 b) 14 c) -3 d) 12 e) 7

32. $P = 74 \text{ ft}$ $A = 278 \text{ ft}^2$ 33. 460 m^2 34. 7

35. a) -8 b) 3 ft c) 10 d) $16, 17, 18$ e) 35 f) 12 years g) 27 miles h) 10 nights 36. 4

37. a) $24\,090$ b) $12\,090$ c) $12\,000$ d) $23\,990$

38. $\{1, 2\}$		39. 136	316	613	713
$\{1, 3\}$	$\{2, 3\}$	137	317	617	716
$\{1, 4\}$	$\{2, 4\}$	163	361	631	731
$\{1, 5\}$	$\{2, 5\}$	167	367	637	736
$\{1, 6\}$	$\{2, 6\}$	173	371	671	761
$\{1, 7\}$	$\{2, 7\}$	176	376	673	763
	$\{3, 4\}$				
	$\{3, 5\}$				
	$\{3, 6\}$				
	$\{3, 7\}$				
	$\{4, 5\}$				
	$\{4, 6\}$				
	$\{4, 7\}$				
	$\{5, 6\}$				
	$\{5, 7\}$				
	$\{6, 7\}$				

Last revised: September 18, 2019