

1. Referring to the Declaration of Independence (1776), Lincoln introduced his Gettysburg Address in 1863 with "Four score and seven years ago..." How many years is a score of years?  
 A) 5      B) 10      C) 20      D) 25      E) 50

2. Let  $R = \{(x, y) \mid 3 \leq x^2 + y^2 \leq 5\}$ . The area of  $R$  is  
 A) 32      B) 20      C)  $4\pi$       D)  $2\pi$       E) none of these

3. At which equal sign does the error occur in the following proposed proof for  $1 = -1$ ?  
 (Note:  $i$  denotes  $\sqrt{-1}$ .)

$$1 \underset{\text{A}}{=} \sqrt{1} \underset{\text{B}}{=} \sqrt{-1}(-1) \underset{\text{C}}{=} \sqrt{-1} \cdot \sqrt{-1} = i \cdot i \underset{\text{D}}{=} i^2 \underset{\text{E}}{=} -1$$

4. Twenty years ago Jake built a house that is now half as old as Jake was when he built it. How old is Jake now?  
 A) 30      B) 40      C) 50      D) 80      E) none of these

5. Given that  $f$  is a linear function with  $f^{-1} = f$  and  $f(4) = 10$ , find  $f(9)$ .  
 A) 5      B) 6      C) 7      D) 8      E) 9

6. If a number is randomly selected from  $\{-10, -5, -3, -2, -1, 0, 1, 3, 5, 10\}$ , find the probability that one more than the square of the selected number is a solution of  $x^2 + 50 = 15x$ .  
 A) 0.1      B) 0.2      C) 0.3      D) 0.4      E) none of these

7. An angle of  $23^\circ$  is inscribed in a circle of radius 9. What is the length, to the nearest tenth, of the intercepted arc?  
 A) 7.2      B) 6.5      C) 6.1      D) 5.4      E) none of these

8. A  $3 \times 3$  magic square uses the integers 1, 2, ..., 9 once each in such a way that each column, each row, and each diagonal sums to 15. Find the value of  $n$  for the magic square, a portion of which is shown below.

8		
$n$		7

- A) 2      B) 3      C) 4      D) 5      E) 6
9. Given that  $f(x) = x^2$ ,  $-10 \leq x \leq -5$ , find  $f^{-1}(x)$ .  
 A)  $f^{-1}(x) = \sqrt{x}$ ,  $25 \leq x \leq 100$       B)  $f^{-1}(x) = \sqrt{-x}$ ,  $25 \leq x \leq 100$   
 C)  $f^{-1}(x) = \sqrt{-x}$ ,  $-100 \leq x \leq -25$       D)  $f^{-1}(x) = -\sqrt{x}$ ,  $25 \leq x \leq 100$   
 E) The inverso of  $f$  is not a function
10. Consider the points  $(5, 7)$ ,  $(-4, -4)$ ,  $(3, 2)$ ,  $(2, 2)$ ,  $(7, -6)$ ,  $(8, 1)$ , and  $(4, 4)$ . How many distinct triangles with positive area can be made by choosing any three of these points as vertices?  
 A) 34      B) 35      C) 209      D) 210      E) none of these

11. The mean of three numbers is ten more than the least of the three and fifteen less than the greatest of the three. If the median of the three numbers is 5, find their sum.  
A) 5      B) 20      C) 25      D) 30      E) none of these
12. Thirteen unit circles are arranged with their centers equally spaced on a circle with radius  $r$  in such a way that each of the unit circles is externally tangent to exactly two others. Find  $r$  to the nearest hundredth.  
A) 4.06      B) 4.14      C) 4.18      D) 8.24      E) 8.30
13. How many times does  $y = 15 \sin(180x)$  take on the value 5 on the interval  $[0, 1]$ ?  
A) 56      B) 58      C) 59      D) 60      E) none of these
14. For any positive integer  $n$ , let  $S(n)$  represent the least positive integer whose factorial is divisible by  $n$ . Find  $S(875) + S(81)$ .  
A) 16      B) 9      C) 24      D) 29      E) none of these
15. If  $a$  is a solution for  $|x - 2| = 8$  and  $b$  is a solution for  $x^3 + x^2 + 36 = 24x$ , then what is the maximum possible value for  $ab$ ?  
A) 36      B) 60      C) 30      D) 20      E) none of these
16. If three people are chosen randomly from a large population, what is the probability that they were all born on different days of the week?  
A)  $\frac{3}{5}$       B)  $\frac{27}{49}$       C)  $\frac{1}{2}$       D)  $\frac{51}{152}$       E)  $\frac{30}{49}$
17. A 4 foot pole is 10 feet from an 8 foot pole. Each pole is vertical, and the ground is level. Suppose a stake is placed in the ground aligned between the two poles. Wires are then strung from the top of each pole and fastened taut to the stake at ground level. Considering all possible positions of the stake, find the maximum angle, to the nearest tenth of a degree, between the wires.  
A)  $76.5^\circ$       B)  $77.9^\circ$       C)  $79.8^\circ$       D)  $83.4^\circ$       E)  $90.0^\circ$
18. In standard decimal notation, how many positive integers less than 10 000 have at least one 3 as a digit?  
A) 3438      B) 3439      C) 3440      D) 3441      E) none of these
19. Suppose  $f$  is a function whose domain includes the interval  $[-1, 1]$ . Then the graph of  $y = f(\sin x)$  is necessarily symmetric with respect to  
A) the  $y$ -axis      B) the vertical line  $x = \pi$       C) the origin  
D) the vertical line  $x = \frac{\pi}{2}$       E) none of these
20. Given that  $f(1) = 1$  and  $f(t) = 5 + 2f(t - 1)$  for  $t \geq 2$ , find  $f(100)$ .  
A)  $5 + 6^{99}$       B)  $5 + 6 \cdot 2^{100}$       C)  $6 \cdot 2^{99} - 5$       D)  $5 + 6^{100}$       E) none of these

## Answers

- |     |   |     |   |
|-----|---|-----|---|
| 1.  | C | 11. | D |
| 2.  | D | 12. | C |
| 3.  | C | 13. | B |
| 4.  | E | 14. | C |
| 5.  | A | 15. | A |
| 6.  | C | 16. | E |
| 7.  | A | 17. | E |
| 8.  | B | 18. | B |
| 9.  | D | 19. | D |
| 10. | A | 20. | C |