Test #2 AMATYC Student Mathematics League Feb/Mar 2010

Leo Livshutz
Truman College, Chicago, Illinois

Solutions

1. Let $P(x) = x^3 - 2x^2 + 3x - 4$.Find the largest prime factor of $P(4) - P(2)$.
A 17 B 19 C 23 D 29 E 31

Solution. $P(4) = 40$; $P(2) = 2$; $P(4) - P(2) = 38 = 19 \cdot 2$

Answer - B

2. A circle of radius 2 and center E is inscribed inside the square $ABCD$. Find the area that is inside $\triangle\mathbf{ABE}$ but outside the circle.
A $\pi - 3$ B $\pi/2 - 1$ C $4 - \pi$ D $\pi - 2$ E $3 - \pi/2$

Solution. (Please draw a picture) The area *of the circle is* πr^2. The area of the square is $(2r)^2 = 4r^2$. The area that is inside the square but outside the circle is $4r^2 - \pi r^2 = r^2(4 - \pi)$. The area that is inside $\triangle\mathbf{ABE}$ but outside the circle is $\frac{1}{4}r^2(4 - \pi)$. If $r = 2$, then the last area is $4 - \pi$

Answer - C

3. The unique solution to the equation $ax + b = 10$ is $x = 2$, and the unique solution to the equation $bx + a = 8$ is $x = 3$. Find $a + b$
A $\frac{26}{5}$ B $\frac{28}{5}$ C 6 D $\frac{32}{5}$ E $\frac{24}{5}$

Solution. Substitute $x = 2$ and $x = 3$ correspondingly into the first and the the second equations to obtain the system $\begin{cases} 2a + b = 10 \\ a + 3b = 8 \end{cases}$. Add twice the first and the second equations, to obtain $5a + 5b = 28$, or, $a + b = \frac{28}{5}$

Answer - B

4. The solution to the inequality $\frac{x+1}{x-3} \geq 2$ is
A $[-1, 3)$ B $[-1, 3]$ C $(3, 7)$ D $[3, 7)$ D $(3, 7]$ E $(3, 7]$

Solution. Since $\frac{x+1}{x-3} = 1 + \frac{4}{x-3}$, then the inequality is transformed into $\frac{4}{x-3} \geq 1$, or $\frac{1}{x-3} \geq \frac{1}{4}$. From the last inequality it follows that $0 < x - 3 \leq 4$, or $3 < x \leq 7$

Answer - E

5. Let a_0, a_1, ... be an arithmentic sequence with $a_0 = 2$, $a_3 = a_1^2 - 8$, and $a_5 > 0$.Find a_3
A 4 B 6 C 8 D 10 E 12

Solution. Denote by d the sequence difference and take a_3 for the base element of the sequence. Then $2 = a_0 = a_3 - 3d$, and $d = \frac{1}{3}(a_3 - 2)$. Subsequently $a_1 = a_3 - 2d = a_3 - 2 \cdot \frac{1}{3}(a_3 - 2) = \frac{1}{3}a_3 + \frac{4}{3}$.

Substitute the last expression for a_1 into the condition $a_3 = a_1^2 - 8$, to obtain $a_3 = \left(\frac{1}{3}a_3 + \frac{4}{3}\right)^2 - 8$. Simplify the last equation to the form $a_3^2 - a_3 + 56 = 0$.The negative solution of this equation should be omitted because it leads to $a_5 < 0$.The positive solution $a_3 = 8$ is the answer to the problem

Answer - C

6. All solutions to the equation $a^3+b^3+c^2=2010$ (a,b,c positive integers) have the same value for $a+b$. Find this value of $a+b$.

A 11 B 12 C 13 D 14 E 15

Solution. The approach is a numerical verification of the equation on some restricted domain of a,b values.

The numbers a,b,c cannot all be even, because otherwise the left side of the the equation will be divisible by 4, but not the right side. Among the numbers a,b,c we cannot have only one odd value or all three odd values because otherwise the left side of the equation will be an odd number, but not the right side. So for any solution (a,b,c) of the equation two of these numbers should be odd, and one should be even.

Let us establish an upper boundary for values of a and b. We have $\max(a^3,b^3)<a^3+b^3+c^2=2010$, so that $\max(a,b)<\sqrt[3]{2010}=12.62$, or $\max(a,b)\le 12$.

The verification formula will be $c=\sqrt{2010-(a^3+b^3)}$ which in case of a solution should provide an integer value of c.

Case 1. Both numbers a,b are odd. The set of possible pairs (a,b) is $\{(11,11),(11,9),(11,7),(11,5),(11,3),(11,1),(9,9),(9,7),(9,5),(9,3),(9,1),(7,7),$
$(7,5),(7,3),(7,1),(5,5),(5,3),(5,1),(3,3),(3,1),(1,1)\}$

In this set the only solution found is $(9,5)$, with symmetric $(5,9)$, for which $a+b=14$

Case 2. The numbers a,b are of different parities. The set of possible pairs (a,b) is

$\{(12,11),(12,9),(12,7),(12,5),(12,3),(12,1),(11,10),(11,8),(11,6),(11,4),(11,2),(10,9),$
$(10,7),(10,5),(10,3),(10,1),(9,8),(9,6),(9,4),(9,2),(8,7),(8,5),(8,3),(8,1),(7,6),(7,4),$
$(7,2),(6,5),(6,3),(6,1),(5,4),(5,2),(4,3),(4,1),(3,2),(3,1),(2,1)\}$

In this set no solution found.

Answer - D

7. If $z=a+bi$ (a,b real) and $z^2=21-20i$, $|a|+|b|=$

A 7 B 8 C 9 D 10 E 11

Solution. Represent number z in polar form $z=r(\cos\theta+i\sin\theta)$.

Then $z^2=r^2(\cos 2\theta+i\sin 2\theta)=21-20i$.Thus $r^2=\sqrt{21^2+(-20)^2}=29$ and $\cos 2\theta=21/29$. By well-known formulas, $\cos\theta=\pm\sqrt{\frac{1+\cos 2\theta}{2}}=\pm\sqrt{\frac{1+21/29}{2}}=\pm\frac{5}{\sqrt{29}}$, $\sin\theta=\pm\sqrt{\frac{1-\cos 2\theta}{2}}=\pm\sqrt{\frac{1-21/29}{2}}=\pm\frac{2}{\sqrt{29}}$

It then follows that $|a|=|r\cos\theta|=\sqrt{29}\cdot\frac{5}{\sqrt{29}}=5$, $|b|=|r\sin\theta|=\sqrt{29}\cdot\frac{2}{\sqrt{29}}=2$, and $|a|+|b|=7$

Answer - A

8. A point C is chosen on the line segment AB such that $\frac{AC}{BC}=\frac{BC}{5\cdot AB}$. Find $\frac{AC}{BC}$.

A $\frac{-5+3\sqrt{5}}{10}$ B $\frac{-1+\sqrt{21}}{10}$ C $\frac{-1+\sqrt{5}}{10}$ D $\frac{-1+\sqrt{29}}{10}$ E $\frac{-1-\sqrt{5}}{10}$

Solution. Denote $AC = x, BC = y$.Then $AB = x + y$. (Please make a picture). In new notations

$\frac{AC}{BC} = \frac{BC}{5 \cdot AB} \iff \frac{x}{y} = \frac{y}{5(x+y)}$, or $\frac{x}{y} = \frac{1}{5(x/y+1)}$. Denoting $\frac{AC}{BC} \equiv \frac{x}{y} = \lambda$, we will have $\lambda = \frac{1}{5(\lambda+1)}$, which after simplifying yields $5\lambda^2 + 5\lambda - 1 = 0$. Solve this equation to obtain (positive) solution $\frac{AC}{BC} \equiv \lambda = \frac{-5+3\sqrt{5}}{10}$

Answer - A

9. Let $\lfloor x \rfloor$ represent the greatest integer $\leq x$. Find $\sum_{n=1}^{2010} \lfloor \log_5 n \rfloor$.

A 7256 B 7260 C 7262 D 7263 E 7264

Solution. Denote by M_k the smaller positive integer for which $\lfloor \log_5 M_k \rfloor = k, k \geq 0$. The values of M_k are found from the equality $\log_5 M_k = k$, or $M_k = 5^k$.

Calculate several values of $M_k : M_0 = 1, M_1 = 5, M_2 = 25, M_3 = 125, M_4 = 625, M_5 = 3125 > 2010$. It is true that if $M_k \leq n \leq M_{k+1} - 1$, then $\lfloor \log_5 n \rfloor = k$. Really, then $\lfloor \log_5 M_k \rfloor \leq \lfloor \log_5 n \rfloor \leq \lfloor \log_5(M_{k+1} - 1) \rfloor$ .Since $\lfloor \log_5 M_k \rfloor = k$ and $\lfloor \log_5(M_{k+1} - 1) \rfloor < k + 1$, then $k \leq \lfloor \log_5 n \rfloor < k + 1$. That is $\lfloor \log_5 n \rfloor = k$

Now

$\sum_{n=1}^{2010} \lfloor \log_5 n \rfloor = \sum_{n=M_0}^{M_1-1} \lfloor \log_5 n \rfloor + \sum_{n=M_1}^{M_2-1} \lfloor \log_5 n \rfloor + \sum_{n=M_2}^{M_3-1} \lfloor \log_5 n \rfloor + \sum_{n=M_3}^{M_4-1} \lfloor \log_5 n \rfloor + \sum_{n=M_4}^{2010} \lfloor \log_5 n \rfloor$

$= \sum_{n=M_0}^{M_1-1} 0 + \sum_{n=M_1}^{M_2-1} 1 + \sum_{n=M_2}^{M_3-1} 2 + \sum_{n=M_3}^{M_4-1} 3 + \sum_{n=M_4}^{2010} 4$

$= 0(M_1 - M_0) + 1(M_2 - M_1) + 2(M_3 - M_2) + 3(M_4 - M_3) + 4(2010 - M_4)$

$= 0(5-1) + 1(25-5) + 2(125-25) + 3(625-125) + 4(2010-625)$ /* remove the parentheses*/

$= 4 \cdot 2010 - (5 + 25 + 125 + 625) = 7260$

Answer - B

The AMATYC Answer Key incorrectly indicates that the answer is E

10. If you roll three fair dice, what is the probability that the product of the three numbers rolled is prime.

A $\frac{1}{36}$ B $\frac{1}{24}$ C $\frac{1}{18}$ D $\frac{1}{8}$ E $\frac{1}{4}$

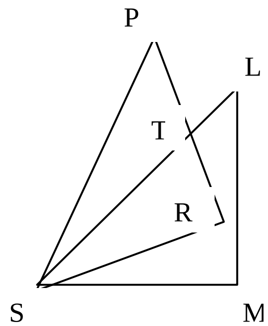
Solution. The sample space is the set of all triples $\{(n_1, n_2, n_3)\}$, where $n_j \in \{1, 2, 3, 4, 5, 6\}, j = 1.2.3$

The total number of selections from the sample space is $6 \cdot 6 \cdot 6 = 216$. The favorable outcomes are those for which $n_1 n_2 n_3 = 2$,or 3, or 5. That means that among the numbers n_1, n_2, n_3 , two should be equal to 1, and the third number should be one of 2,or 3, or 5. The number of such favorable outcomes for each of prime values $2, 3, 5$ is 3, which totals to 9. The probability we are seeking is $\frac{9}{216} = \frac{1}{24}$

Answer - B

11. Let $S = (0, 0)$, $M = (10, 0)$, and $L = (10, 10)$. and rotate $\triangle SML$ 30° counterclockwise around S.Find the area of the union of the triangles to the nearest square unit.

A 79 B 81 C 83 D 85 E 87



Solution. The total area $A = Area(\triangle SML) + Area(\triangle SRP) - Area(\triangle SRT)$.

$Area(\triangle SML) = Area(\triangle SRP) = \frac{10\cdot 10}{2} = 50$. $Area(\triangle SRT) = \frac{1}{2} SR \cdot RT = \frac{1}{2} SR \cdot SR \tan(\angle RST)$

$= \frac{1}{2} SR^2 \tan(45^\circ - 30^\circ) = \frac{1}{2} SM^2 \tan(15^\circ) = \frac{1}{2} 10^2 \tan(15^\circ) \approx 13$

Total area $A \approx 50 + 50 - 13 = 87$

Answer - E

12. Three faces of a rectangular box that share a common vertex have areas 48, 50, and 54. Find the volume of the box.

A 360 B 364 C 372 D 376 E 384

Solution. Denote the lengths of the box sides by x, y, z. The $xy = 48$, $xz = 50$, $yz = 54$. Multiplying side-by-side all three equations, we get $(xy)(xz)(yz) = 48 \cdot 50 \cdot 54$, or $V = xyz = \sqrt{48 \cdot 50 \cdot 54} = 360$

Answer - A

13. *A multiplicative magic square {MMS}* is a square array of positive integers in which the product of each row, column, and long diagonal is the same. The 16 positive factors of 2010 can be formed into a 4x4 MMS. What is the common product of every row, column, and diagonal? Write your answer in the corresponding blank on the answer sheet.

Solution. Factor number $2010 = 2 \cdot 3 \cdot 5 \cdot 67$. Denote $2 = p; 3 = q; 5 = r;$ $67 = s$. Then any factor of 2010 has a form $p^a q^b r^c s^d$, where a, b, c, d take on values 0, 1. There are totally16 quadruples (a, b, c, d) with entries 0, or 1.Denote by N the product of entries in any MMS row. Then multiplying the products of all for rows, we find the product of all factors for number 2010, which on the other hand is N^4. Symbolically. this can be represented as $\prod_{\forall(a,b,c,d)} p^a q^b r^c s^d = N^4$. Simplifying the left side of the last equality.

$\prod_{\forall(a,b,c,d)} p^a q^b r^c s^d = \prod_{\forall(a,b,c,d)} p^a \prod_{\forall(a,b,c,d)} q^b \prod_{\forall(a,b,c,d)} r^c \prod_{\forall(a,b,c,d)} s^d$

Among 16 quadruples (a, b, c, d),eight have value $a = 0$, and eight have value $a = 1$. So that $\prod_{\forall(a,b,c,d)} p^a = p^8$. Similarly, $\prod_{\forall(a,b,c,d)} q^b = q^8$, $\prod_{\forall(a,b,c,d)} r^c = r^8$, and $\prod_{\forall(a,b,c,d)} s^d = s^8$. Thus,

$N^4 = \prod\limits_{\forall(a,b,c,d)} p^a q^b r^c s^d = p^8 q^8 r^8 s^8 = (pqrs)^8$ and $N = (pqrs)^2 = 2010^2 = 4040100$

Answer - the common product of the entries to every row or column must be 4040100

14. For a function $f(x)$, let $f^2(x) = f(f(x)), f^3(x) = f(f(f(x)))$, and so on. For the function $f(x) = \sqrt{\frac{x^2+1}{x^2-1}}$ on the domain $(-\infty, 1) \cup (1, \infty)$, $f^{2010}(x) =$

A x B $|x|$ C x^2 D $\frac{1}{x}$ E $\frac{1}{x^2}$

Solution. Calculate $f^2(x) = f(f(x)) = \sqrt{\left(\left(\sqrt{\frac{x^2+1}{x^2-1}}\right)^2 + 1\right) / \left(\left(\sqrt{\frac{x^2+1}{x^2-1}}\right)^2 - 1\right)} = \sqrt{x^2} = |x|$. Denote $f^2(x) \equiv g(x) = |x|$. Then, by induction, $g^n(x) = |x|$, and $f^{2n}(x) = f^2(f^{2(n-1)}(x)) = g(g^{n-1}(x)) = g^n(x) = |x|$. In particular, $f^{2010}(x) = f^{2 \cdot 1005}(x) = |x|$

Answer - B

15. You have 4 red, 4 white, and 4 blue identical dinner plates. In how many different ways can you set a square table with one plate on each side, if two settings are different only if you cannot rotate the table to make the settings match?

A 21 B 24 C 27 D 30 E 36

Solution. Please sketch a unit circle and assign labels 1, 2, 3, 4 to points (1,0), (1,1),(-1,0),(-1,-1). These points represents the sides of the table. Denote by (c_1, c_2, c_3, c_4) the color settings of the table sides, where $c_j, j = 1, 2, 3, 4$, is one of the red, white, or blue colors. The total number of color settings, with regard of rotation, is $3^4 = 81$. From this number we have to subract the similar, with regard to rotation, settings. The rest should be divided by three, to count only rotation different colorings. First, we remove colorings that dublicated with rotation. We rotate counteclockwise.

Case of rotating by 90°.In this case the following colors may coincide, $c_1 = c_2, c_2 = c_3, c_3 = c_4, c_4 = c_1$.The conditions indicate that the settings with the same color should be removed. The number of the same color settings is 3 (number of colors)

Case of rotating by 180°.In this case the following colors may coincide, $c_1 = c_3, c_2 = c_4$. and the related settings should be removed. These settings are (a letter denotes a color)

$(r, r, r, r), (r, w, r, w), (r, b, r, b), (w, r, w, r), (w, w, w, w), (w, b, w, b), (b, r, b, r), (b, w, b, w), (b, b, b, b)$.Of these settings, the same color settings are already counted at the previous case. The number of new removed settings is 6.

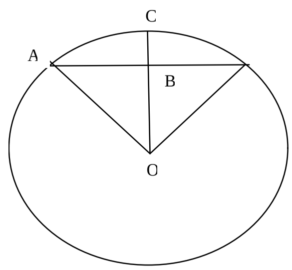
Case of rotating by 270° yields the same limitatins that case of *90°*

The total number of remaining settings is $81 - 3 - 6 = 72$. Since with any valid setting (c_1, c_2, c_3, c_4), the rotational settings (c_4, c_1, c_2, c_3) and (c_3, c_4, c_1, c_2) are also present, then the number of rotation different settings are $72/3 = 24$

Answer - B

16. A 100 m long railroad rail lies flat along level ground, fastened at both ends. Heat causes the rail to expand by 1% and rise into a circular arc. To the nearest meter, how far above the ground is the midpoint of the rail?

A 0 B 2 C 4 D 6 E 7



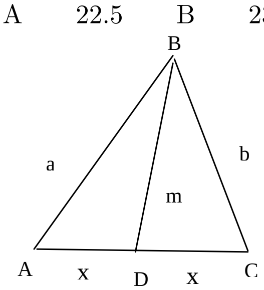
Solution. We have to find BC, see the picture. Given $AB = 50$, $\frown AC = (50)(1.01)$ Denote $OC = R$, $x = \angle AOC$ (radians). Then $\sin x = \frac{AB}{R} = \frac{50}{R}$, and $x = \frac{\frown AC}{R} = \frac{(50)(1.01)}{R}$. From the last equality it follows that $\frac{50}{R} = \frac{x}{1.01}$. Substituting, we will have $\sin x = \frac{x}{1.01}$,or $x - 1.01\sin x = 0$. Solving this equation with the *Solver* tool of TI-84 calculator, I found that $x = 0.2440966957$. From the picture, $BC = OC - OB = R - R\cos x = R(1-\cos x) = \frac{AB}{\sin x}(1-\cos x) = AB\tan\frac{x}{2} = 50\tan(0.2440966957/2) \approx 6.13$

Note. The equality $BC = AB\tan\frac{x}{2}$ follows also from $\triangle ABC$ in which $\angle CAB = \frac{x}{2}$

Answer - D

17. Two sides of a triangle have lengths 25 and 20, and the median to the third side has length 19.5 Find the length of the third side.

A 22.5 B 23 C 23.5 D 24 E 24.5



Solution. In the picture $a = 25, b = 20, m = 19.5, x = AC/2$. We have to find the value of $AC = 2x$

Apply the Law of Cosines. For triangle $\triangle ABD$, $a^2 = m^2 + x^2 - 2mx\cos(\angle ADB)$. For triangle $\triangle DBC$, $b^2 = m^2 + x^2 - 2mx\cos(\angle BDC)$. Adding these equations, we obtain $a^2 + b^2 = 2(m^2 + x^2) - 2mx(\cos(\angle ADB) + \cos(\angle BDC))$. Since $\angle ADB + \angle BDC = 180°$, then $\cos(\angle ADB) + \cos(\angle BDC) = 0$, and $a^2 + b^2 = 2(m^2 + x^2)$. Solving this equatin for x, we get $2x = 2\sqrt{\frac{a^2+b^2}{2} - m^2} = 2\sqrt{\frac{25^2+20^2}{2} - 19.5^2} = 23$

Answer - B

18. A polindrom is a positive integer like 11,313, and 5445 which reads the same left to right and right to left. If a number is chosen at random from all four-digit polindromes, find the probability that it is divisible by 7.

A $\frac{1}{7}$ B $\frac{1}{6}$ C $\frac{2}{11}$ D $\frac{1}{5}$ E $\frac{2}{7}$

Solution. Any four-digit polindrom has a decimal form $ABBA$, where $1 \leq A \leq 9, 0 \leq B \leq 9$. There is a total of $(9)(10) = 90$ polindroms.

Numerically, $ABBA = 10^3 A + 10^2 B + 10B + A$. Since $10^3 = 143 \cdot 7 - 1$, $10^2 = 14 \cdot 7 + 2$, $10 = 7 + 3$, then

$ABBA = 7(143A + 14B + B) - A + 2B + 3B + A = 7(143A + 14B + B) + 5B$. So $ABBA$ is divisible by 7 if and only if $5B$ is divisible by 7. Thus $B = 0, 7$ and the total number of divisible by 7 polindroms is $(9)(2) = 18$. The probability that a randoms selected four-digit polindrome is divisible by 7 is $\frac{18}{90} = \frac{1}{5}$

Answer - D

Note. In the condition of the problem the number 11,313 is actually not a polindrom

19. For positive integer values of m and n, the largest value of n for which the system

$$\begin{cases} a + b = m \\ a^2 + b^2 = n \\ a^3 + b^3 = m + n \end{cases}$$ has solutions is

A 3 B 8 C 12 D 24 E 36

Solution. We will directly find all feasible pairs of (m, n) and also find solutions (a, b) for each (m, n) pair.

At the beginning, let us find the values a and b for a feasible (m, n). From the first equation it follows that $(a+b)^2 = m^2$, or $(a^2+b^2)+2ab = m^2$. Replacing here a^2+b^2 with n from the second equation, we receive $2ab = m^2 - n$. Then using the second equation we will have $(a-b)^2 = a^2 - 2ab + b^2 = n - (m^2 - n) = 2n - m^2$ and $|a - b| = \sqrt{2n - m^2}$

Case $a \geq b$. Solving the system $a + b = m, a - b = \sqrt{2n - m^2}$, we get $a = \frac{1}{2}(m + \sqrt{2n - m^2})$, $b = \frac{1}{2}(m - \sqrt{2n - m^2})$.

Case $a \leq b$ is resolved similarly and yields $a = \frac{1}{2}(m - \sqrt{2n - m^2}), b = \frac{1}{2}(m + \sqrt{2n - m^2})$

Totally, for any feasible (m, n), the solutions are $(a, b) = \left(\frac{1}{2}(m \pm \sqrt{2n - m^2}), \frac{1}{2}(m \pm (-\sqrt{2n - m^2})\right)$

To find possible (m, n), copy from above $2ab = m^2 - n$. Then from the equations of the problem, $m+n = a^3+b^3 = (a+b)(a^2-ab+b^2) = m(n - \frac{m^2-n}{2})$, or $m(3n - m^2) = 2m + 2n$, $m \geq 0$, $n \geq 0$

This is the condition on possible (m, n),

Solve the condition equality for n to obtain $n = \frac{m^3+2m}{3m-2}$. Multiply both sides of this equation by 3^3 to obtain $27n = \frac{(3m)^3+18(3m)}{3m-2}$. Denote $3m = x$, so that the right side of the last equation can be written as

$\frac{(3m)^3+18(3m)}{3m-2} = \frac{x^3+18x}{x-2} = x^2 + 2x + 22 + \frac{44}{x-2} \equiv Q(x)$.

Since x and $Q(x)$, which is equal to $27n$, are nonnegative integers, then $\frac{44}{x-2}$ is a positive integer. The factors of 44 are $\pm1, \pm2, \pm4, \pm11, \pm22 \pm 44$, so that all possible values of nonnegative x are being $x = 2 + \{\pm 1, \pm 2, 4, 11, 22, 44\} = \{3, 1, 4, 0, 6, 13, 24, 46\}$. Since $x = 3m$, then feasible values of x are $0, 3, 6, 24$ and related $m = 0, 1, 2, 8$. The feasible values of $Q(x)$ are $Q(0) = 0$, $Q(3) = 81$, $Q(6) = 81$, $Q(24) = 648$ and related $n = Q(0)/27$, $Q(3)/27$, $Q(6)/27$, $Q(24)/27$ are 0, 3, 3, 24. The solutions (m, n) are $(0, 0)$, $(1, 3)$, $(2, 3)$, $(8, 24)$

The corresponding (a, b) solutions are, from above,

$(m, n) = (0, 0)\,,\ (a, b) = (0.0)\,,$

$(m, n) = (1, 3)\,,\ (a, b) = \left(\frac{1}{2}(1 \pm \sqrt{5}), \frac{1}{2}(1 \pm (-\sqrt{5})\right)$

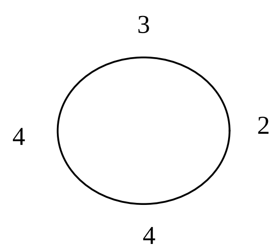
$(m, n) = (2, 3)\,,\ (a, b) = \left(\frac{1}{2}(2 \pm \sqrt{2}), \frac{1}{2}(2 \pm (-\sqrt{2})\right)$

$(m, n) = (8, 24)\,,\ (a, b) = (4 \pm 2i, 4 \pm (-2)2i)$

Answer - D

20. A game is played using the 4 piles of chips shown. A move consists of choosing two adjacent piles around the circle and removing at least one chip from at least one of the piles. The winner is the player who removes the last chip. If 2W 3N means remove 2 chips from the west pile and 3 chips from the north pile, which move quarantees a win for the current player?

A 2W 1S B 2W 1N C 1W 2S D 2E 0S E 1W 4S



Solution. Denote by n_N, n_W, n_S, n_E the number of chips at correspondingly north, west, south, east piles at some time instance. Call the chips layout axes-symmetric if $n_N = n_S$, $n_W = n_E$ A winning strategy for the current player would be to have after each of his/her moves the axes-symmetric chips layout. By the rules of the game, any move following axes-symmetric layout cannot lead to axes-symmetric layout. As all zeros outcome is axes-symmetric, then it can be obtained only at a move of current player.

To implement the strategy, the current player has at the first move to go $2W, 1S$, and at each next move to match the action of the opponent but with the opposite piles, thus every time setting an axes-symmetric disposition.

Answer - A