Problem Set II. - 1984

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.
The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, student assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.
In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. We have used an entire record tape to record sounds. On one side of the tape, we have recorded sounds with a speed of $4\frac{\text{cm}}{\text{s}}$. On the other side of the tape, we have recorded sound with a speed of $9\frac{\text{cm}}{\text{s}}$. The length of the entire program we have taped is 1 hour and 18 minutes. How many meters long is the ribbon in the tape? How many minutes can we tape if we record with a speed of $9\frac{\text{cm}}{\text{s}}$ on both sides?

2. In an isosceles triangle, the altitude (or height) belonging to the base is 10 units long. The altitude belonging to the other two sides are both 12 units long. Find the area of the triangle.

3. Solve the given equation over the real numbers.
$$\frac{1}{\sqrt{2+x}-\sqrt{2-x}}+\frac{1}{\sqrt{2+x}+\sqrt{2-x}}=1$$

4. Suppose that C is a circle with radius r. We draw a trapezoid around the circle so that each side of the trapezoid is tangent to the circle. One of the angles of the trapezoid is $90°$. The shortest side of the trapezoid is $\frac{3r}{2}$. Compute the area and perimeter of the trapezoid.

5. Write an equation for the BC side side in triangle ABC, where the equation of side AB is $3x-2y+1=0$, the equation for side AC is $x-y+1=0$, and the median that contains C has equation $2x-y-1=0$. (A median is a line segment connecting the midpoint of a side with the vertex opposite that side).

6. Prove that if α, β, and γ are angles in a triangle, then
$$\sin^2\gamma \geq \sin 2\alpha \cdot \sin 2\beta$$
For what triangles is the equality true?

7. A rectangular prism has a square base. The ratio of its height and side of its base is an integer. The midpoints of the four sides of the base and the midpoints of the four sides of the top form another rectangular prism. The ratio between the surface area of the second prism and six times the area of the base of the first prism is between 0.5 and 2. How many different such rectanguar prism exist if we consider similar prisms as identical?

8. Suppose that p is a positive integer. If we add the digits of p, the result is q. If we add the digits of q, the result is r. Find p if we also know that $p+q+r=60$.

