Problem Set II - 1985

author: Lajos Solti
translated from: KoMaL, December 1985, pages 438-439.
Pdf file: KoMaL, December 1985, pages 438-439.

In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.
The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, student assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.
In Mathematics, the same exam serves as the GED and the entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Find all positive solutions of the equation $\sqrt{x+2} = -x$.
2. Two angles in a triangle have sines 0.7431 and 0.6691. Compute the cosine of the third angle.
3. The diagonals of a complex quadrilateral split it into four triangles. The areas of three such triangles are $1\,\mathrm{cm}^2$, $2\,\mathrm{cm}^2$, and $3\,\mathrm{cm}^2$. Find the area of the fourth triangle.
4. Solve the given equation.
$$\sqrt{\sin x \cos x} \cdot \left(1 - \log\left(16 - x^2\right)\right) = 0$$
5. In a triangle, the sides around angle α have lengths $\sin\alpha$ and $\cos\alpha$. The side opposite angle α has length $1 - \sin\alpha$. Compute the sides and angles in the triangle.
6. Find the value of the parameter p for which the vector that translates the graph of $y = x^2 - 4px + 2$ into the parabola $y = x^2 + 2px - 4$ has the shortest possible length.
7. Find all (x, y) that satisfy the given equation:
$$\tan\left(\frac{2y}{x}\right) + \cot\left(\frac{2y}{x}\right) + \sqrt{4x - x^2} + \sqrt{x^2 - 10x + 16} + \sqrt{11x - x^2 - 18} = 3x$$
8. Solve the given equation for x over the real numbers.
$$\sqrt{\log_a \sqrt[4]{ax} + \log_x \sqrt[4]{ax}} + \sqrt{\log_a \sqrt[4]{\frac{x}{a}} + \log_x \sqrt[4]{\frac{a}{x}}} = a$$

