# Problem Set III - 1986

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translated from: KoMaL, January 1986, pages 2-3.
Pdf file: KoMaL, January 1986, pages 2-3

In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.
The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, student assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.
In Mathematics, the same exam serves as the GED and the entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The perimeter of a right triangle is 40 cm, its area is 60 $\text{cm}^2$. How long are its sides?

2. Suppose that $ABCD$ is a trapezoid with parallel sides $AB$ and $CD$. Let $P$ and $Q$ be points on $AB$ that divide $AB$ into three equal line segments. Let $S$ and $T$ be points on $CD$ that divide $CD$ into three equal line segments. We connect $P$ and $S$. The ratio between the two regions created by $PS$ is 2 to 3. What is the ratio of $AB$ to $CD$?

3. Solve the given equation.
$$\cos x + 2\tan x = \frac{7}{4\cos x}$$

4. The sum of the first six terms in a geometric sequence is three times as much as the sum of the first three terms. What is the common quotient in the sequence? (Assume that none of the terms is zero.)

5. Solve the equation.
$$\log_8\left(27^{x-1}+37\right) = \log_2\left(3^{x-1}+1\right)$$

6. Find the equation for all circles that are tangent to the line $x = 2$, contain the origin, and intersects the line $y = x$ in a chord of length of $\sqrt{2}$.

7. The angle at vertex $A$ in a rhombus measures 120°. We draw the equilateral triangles that have one vertex at $A$ and the two other vertices on the sides opposite $A$. Of these triangles, which one will have the smallest area? What is the ratio between this smallest area and the area of the rhombus?

8. Consider an arithmetic sequence consisting of different terms. The last term of the sequence is nine times the first term; the sum of the last three terms is five times the sum of the first three terms. How many terms are in the sequence?

Last revised: August 22, 2015 