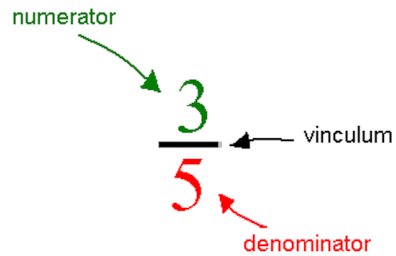
A fraction has three components as shown on the picture. The important parts are the **numerator** above and the **denominator** below the little line that is called the vinculum.

At first let us not even consider a fraction alone. We will just define a **fraction of something**.



Definition: $\frac{3}{5}$ of a quantity can be obtained as follows.

Step 1. We first divide the quantity into 5 equal shares.

Step 2. Let us take 3 such shares. That is $\frac{3}{5}$ of our quantity.

So the numerator tells us how many shares we have. The denominator tells us how big each share is.

Example 1. Find $\frac{3}{5}$ of 100 dollars.

Solution: Step 1. Divide 100 dollars into five equal shares. This means that we exchange a 100 dollar bill into five twenty-dollar bills. In other words, $\frac{1}{5}$ of 100 dollars is 20 dollars.

Step 2. To obtain $\frac{3}{5}$, we take three such shares. In this case this means taking three twenty-dollar bills. That is 60 dollars. In other words, $\frac{3}{5}$ of 100 dollars is $\boxed{60 \text{ dollars}}$.

Example 2. Compute $\frac{4}{7}$ of 42.

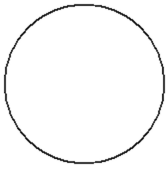
Solution: Step 1. We divide 42 into seven equal shares. $\frac{1}{7}$ of 42 is 6.

Step 2. We take four such shares. Thus $\frac{4}{7}$ of 42 is $4 \cdot 6 = 24$. The answer is $\boxed{24}$.

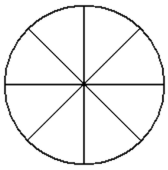
Example 2v. Find each of the following.

a) $\frac{1}{6}$ of 18 b) $\frac{5}{6}$ of 18 Solution - Youtube link

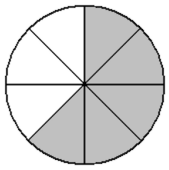
Example 3. Shade the region on the picture that corresponds to $\frac{5}{8}$ of the circle.



Solution: Step 1. Divide the circle into 8 equal parts.

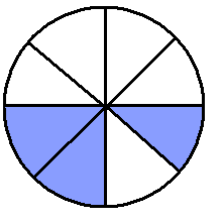


Step 2. We shade five such parts.

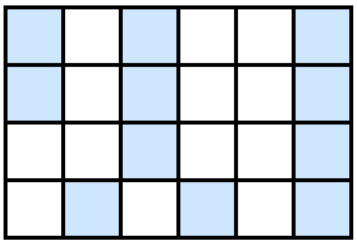


Example 3v. Express the shaded area as a fraction.

a) Solution - Youtube link



b) Solution - Youtube link



Example 4. A cake was sliced into equal slices. Amy ate 2 slices and Betsy ate 3 slices. If 2 slices were remaining, what fraction of the cake was eaten?

Solution: We need to first figure out how many slices made up the cake. If 2 were eaten by Amy and 3 by Betsy and 2 more were left, then there were all together $2+3+2=7$ slices. 5 slices were eaten which were $\frac{5}{7}$ of the cake. So the answer is that $\boxed{\frac{5}{7} \text{ of the cake was eaten}}$.

Example 4v. There are 800 students enrolled in a school. If $\frac{3}{4}$ of the students are boys, how many students are girls? Solution - Youtube link

Example 5. Compute $\frac{8}{100}$ of 2000 dollars.

Solution: We divide 2000 dollars into 100 equal shares. Then each share is 20 dollars. (We divide 2000 by 100). Then we take 8 such shares, that is $8 \cdot 20$ dollars $=$ 160 dollars. Thus $\frac{8}{100}$ of 2000 dollars is $\boxed{160 \text{ dollars}}$.

Definition: We often use fractions with 100 in the denominator. Such a fraction is also called a **percent** and is denoted by $\%$.

For example, computing 9% of a quantity is exactly the same as computing $\frac{9}{100}$ of it.

Example 5v. Find 9% of 200. Solution - Youtube link

Example 6. In 2012, there were approximately 235 000 000 people eligible to vote in the USA. If 59% of them voted in 2012, how many people voted?

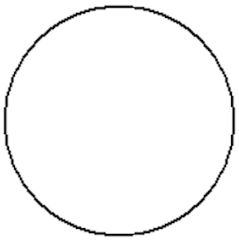
Solution: We need to compute 59% of 235 000 000. That is the same as computing $\frac{59}{100}$ of 235 000 000.

We first divide 235 000 000 by 100. Division by 100 is very easy in this case: just cut off the last two zeroes. So, 1% or $\frac{1}{100}$ of the number is 2 350 000. Now we take 59 of that, we multiply 2 350 000 by 59. The result is 138 650 000. Thus, $\boxed{\text{approximately 138 650 000 people voted}}$ in 2012.

Example 6v. A \$500 TV went on a 6% off sale. What is the sale price? Solution - Youtube link

Practice Problems

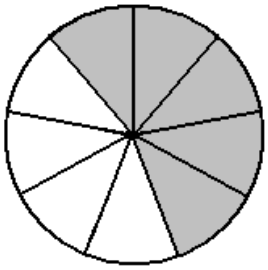
1. Compute $\frac{4}{9}$ of 63.
2. Compute $\frac{5}{6}$ of 24.
3. Shade $\frac{5}{9}$ of the circle given.



4. The price of a TV is \$400. We want to raise the price by 5%. What is the new price?
5. The price of a couch is \$700. Next week, it will go on a 15% off sale. What is the new price? (A 15% sale means that the price of the item is lowered by 15%.)
6. Find $\frac{3}{4}$ of 56.
7. We placed \$2000 into a bank account with 6% yearly interest rate. How much money do we have in the bank after one year?
8. This problem is about a method of comparing fractions.
   a) Compute $\frac{3}{7}$ of 420.
   b) Compute $\frac{4}{10}$ of 420.
   c) Based on the results of parts a) and b), which fraction is larger, $\frac{3}{7}$ or $\frac{4}{10}$?
9. Bert has made \$54 000 last year. If he has to pay 32% of his income in taxes, how much taxes does he owe and how much of his income will he keep?
10. Sally used to make \$2400 per month, but now she got a 3% raise. How much is her monthly salary now?
11. Mr. X won \$600 000 in the lottery two yers ago. By now he has spent some of the money. When he was asked what happened, he said the following. "*I didn't spend it all. I spent $\frac{1}{3}$ of the money by taking a luxury yacht trip around the world. Then I put the rest in the bank. Later I decided to buy a house I really liked. So I took $\frac{3}{4}$ of the money out of the bank and baught the house. For half of what was left, I purchased stocks that completely lost their value. Finally, I gave $\frac{2}{5}$ of what's left to my niece for her college education.*"
    How much money is left from the winnings?

Answers

Practice Problems

1. 16 2. 20
3. 

4. \$420 5. \$595 6. 42 7. \$2120
8. a) 180 b) 168 c) $\frac{3}{7}$ is larger
9. \$17 280 in taxes and will keep \$36 720.
10. \$2472 11. \$30 000

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