

Recall the signed numbers or integers we have introduced before. We also learned how to add them. Today, we will define subtraction.

Definition: The **opposite** of 3 is written as -3 . For any number, the sum of the number and its opposite is zero.

The opposite of 3 is -3 . The opposite of -3 is 3. The opposite of zero is zero itself.

Definition: The **absolute value of a number** is its distance from zero on the number line. We denote the absolute value of a number x by $|x|$.

Distances can never be negative. -5 is 5 units away from zero on the number line. So is 5, only it is in the other direction. So, the absolute value of 5 and -5 are both 5.

Definition: The set of all integers is the collection of all counting numbers, their opposites, and zero.

$$\dots - 3, -2, -1, 0, 1, 2, 3, \dots$$

Definition: When added to any integer, zero has no effect. Same is true for subtracting zero.

The following statement is always true, and is often extremely useful.

To subtract is to add the opposite.

Of course, we don't always use this fact. In the subtraction $10 - 3$, we would only complicate things by applying this fact. It would still get us the right result. Instead of subtracting positive 3, we add its opposite, negative 3.

$$10 - 3 = 7 \quad \text{and also,} \quad 10 + (-3) = 7$$

Consider the subtraction $100 - (-20)$. We are asked to subtract negative 20. To subtract is to add the opposite. So, instead of subtracting negative 20, we will add its opposite, positive 20.

$$100 - (-20) = 100 + 20 = 120$$

Why is $100 - (-20) = 100 + 20$? Even if we understand how to compute this, it would be nice to understand why this is correct. So here is one way to think about this.

Imagine that we have both a bank account a credit card with a bank. Suppose that at the moment, we have 150 dollars in the bank but we also owe 50 dollars to the bank on the credit card. So our net worth is 100 dollars.

Money in bank	Debt on credit card	Total Net worth
150	50	100

Suppose now that we have collected enough bonus points on the credit card to earn rewards. So the bank reduces our credit card debt by 20 dollars. (i.e. subtracts 20 debt, i.e. subtracts negative 20). We still have our 150 in cash, but now our debt is reduced to 30 dollars. So our net worth is now 120 dollars.

	Money in bank	Debt on credit card	Total Net worth
before	150	50	100
after	150	30	120

That is 20 dollars more than before. After all, reducing our debt by 20 dollars is almost the same as if someone gave us 20

dollars so that we can pay off some of our debts.

Example 1. Compute each of the following.

a) $-7 - 8$

b) $-9 - (-5)$

c) $1 - 7$

d) $6 - (-3)$

Solution: a) First, the negative sign in front of the 7 cannot denote subtraction. We are asked to subtract positive 8 from negative 7. To subtract is to add the opposite. Instead of subtracting positive 8, we will add its opposite, negative 8.

$$-7 - 8 = -7 + (-8) = -15$$

b) To subtract is to add the opposite. Instead of subtracting negative 5, we will add its opposite, positive 5.

$$-9 - (-5) = -9 + 5 = -4$$

c) To subtract is to add the opposite. Instead of subtracting positive 7, we will add its opposite, negative 7.

$$1 - 7 = 1 + (-7) = -6$$

d) To subtract is to add the opposite. Instead of subtracting negative 3, we will add its opposite, positive 3.

$$6 - (-3) = 6 + 3 = 9$$

Revisiting Changes in Notation

Recall that if there are several signs (any operation or negative) between two numbers, a pair of parentheses must separate them.

$$-2 + -6$$

+ - is not allowed

$$-5 - -3$$

- - is not allowed

For this reason, until just a few decades ago, we used to put a pair of parentheses around *every negative number*.

$$(-2) + (-6)$$

old style

$$(-5) - (-3)$$

old style

Then we stopped putting the parentheses around the first negative number in the line or inside a parentheses, because there was no risk that we would read the sign incorrectly as subtraction. Subtract from what? There is rarely an operation sign in front of the first number.

$$-2 + (-6)$$

more modern

$$-5 - (-3)$$

more modern



Practice Problems

1. Perform the indicated operations.

a) $-2 + 7$

f) $-3 - (-10)$

k) $-3 - (-3)$

p) $-5 - (1 + 7)$

b) $-7 - (-4) - 2$

g) $-20 + (4 - (-1))$

l) $|9| + (-1)$

q) $9 + |-1|$

c) $12 - ((-2) - 7)$

h) $-12 - 3 - (12 - 3)$

m) $-3 - 0$

r) $|9 + (-1)|$

d) $5 - (-3)$

i) $-4 + 7 - 6$

n) $0 - 4$

e) $-8 + 0$

j) $-6 - |-7|$

o) $-5 - 1 + 7$

2. Perform the indicated operations.

a) $2 - 7$

d) $5 - (-6)$

g) $-2 - 8$

j) $-4 - (-4)$

b) $|2 - 7|$

e) $|5 - (-6)|$

h) $|-2 - 8|$

j) $|-4 - (-4)|$

c) $|2| - |7|$

f) $|5| - |-6|$

i) $|-2| - |8|$

k) $|-4| - |-4|$



Answers

1. a) 5 b) -5 c) 21 d) 8 e) -8 f) 7 g) -15 h) -24 i) -3 j) -13 k) 0 l) 8 m) -3
n) -4 o) 1 p) -13 q) 10 r) 8
2. a) -5 b) 5 c) -5 d) 11 e) 11 f) $|5| - |-6|$ g) 8 h) 10 i) -6 j) 0 k) 0