

The first commandment of mathematics is: *"Thou shall not divide by zero. Ever..."*

The expressions  $\frac{3}{0}$  and  $\frac{0}{3}$  look very similar, and yet they are very different. How can we remember which is which?

**Method 1.** If you don't remember which is which, just ask your calculator. If we punch in  $\frac{0}{3}$ , the calculator gives us the answer 0. But if we type in  $\frac{3}{0}$ , we get some kind of an error message. Some calculator will say "domain error", others will say "error: division by zero"; but no calculator will let you go on thinking that there is a meaningful result to the operation  $\frac{3}{0}$ .

**Method 2.** You should know this one too, in case you are not allowed to use a calculator on an exam. The trick is to know that division is defined in terms of multiplication. Consider the easy division  $\frac{10}{2}$ .

$$\frac{10}{2} = 5 \quad \text{because the multiplication backwards works, i.e. } 10 = 2 \cdot 5$$

$$\frac{0}{3} = \square$$

What could we enter into the empty box so that the multiplication backwards will work? Clearly the answer is zero. Indeed,

$$\frac{0}{3} = 0 \quad \text{because } 3 \cdot 0 = 0$$

Consider now

$$\frac{3}{0} = \square$$

What could we enter into the empty box so that the multiplication backwards will work? No matter what number we would write, zero times it will be zero.

$$3 \neq 0 \cdot \square \quad \text{no matter what value we write in the box}$$

because  $0 \cdot \square = 0$ . Thus, it is impossible to enter a number into the box to make the multiplication backwards work.

Any time we are instructed to divide by zero, we need to write the final answer: undefined.

**Example 1**  $\frac{3 - 2(-2)}{3 - 2^2 + 1} = \frac{3 - (-4)}{3 - 4 + 1} = \frac{7}{-1 + 1} = \frac{7}{0} = \text{undefined}$

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