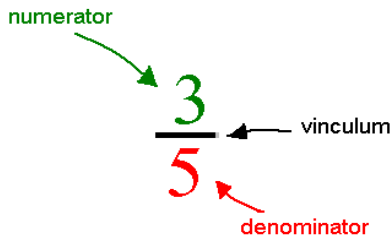


Definition of a Fraction

What is a fraction? A fraction has three components as shown on the picture below. The important parts are the numerator above and the denominator below the little line.



At first let us not even consider a fraction alone. We will just define a **fraction of something**.

Definition: $\frac{3}{5}$ of a quantity can be obtained as follows.

Step 1. We first divide the quantity into 5 equal shares.

Step 2. Let us take 3 such shares. That is $\frac{3}{5}$ of our quantity.

So the numerator tells us how many shares we have. The denominator tells us how big each share is.

Example 1. Find $\frac{3}{5}$ of \$100.

Step 1. Divide \$100 into 5 equal shares.

We break the \$100 into five twenty dollar bills. In other words, $\frac{1}{5}$ of \$100 is \$20.

Step 2. Let us take 3 such shares.

We take three twenty dollar bills, that is \$60. In other words, $\frac{3}{5}$ of \$100 is \$60.

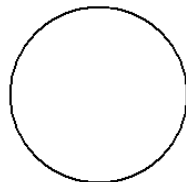
Example 2. Compute $\frac{4}{7}$ of 42.

Solution:

Step 1. We divide 42 into 7 equal shares. $\frac{1}{7}$ of 42 is 6.

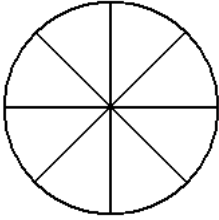
Step 2. We take 4 such shares. $\frac{4}{7}$ of 42 is $4 \cdot 6 = 24$. The answer is 24.

Example 3. Shade the region on the picture that corresponds to $\frac{5}{8}$ of the circle.

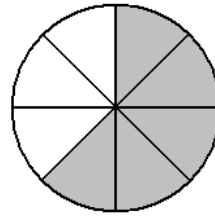


Solution:

Step 1. Divide the circle into 8 equal shares.



Step 2. Take 5 such shares.



Example 4. A cake was sliced into equal slices. Amy ate 2 slices and Betsy ate 3 slices. If 2 slices were remaining, what fraction of the cake was eaten?

Solution: We need to first figure out how many slices made up the cake. If 2 were eaten by Amy and 3 by Betsy and 2 more were left, then there were all together $2 + 3 + 2 = 7$ slices. 5 slices were eaten which were $\frac{5}{7}$ of the cake.

So the answer is $\frac{5}{7}$.

Example 5. Compute $\frac{8}{100}$ of \$2000.

Solution: We divide \$2000 into 100 equal shares. Each share is \$20. Then we take 8 such shares, that is $8 \cdot \$20 = \160 . Thus $\frac{8}{100}$ of \$2000 is \$160.

Note: We often use fractions with 100 in the denominator. These fractions also called percents and denoted by %. Thus, when we are asked to compute 8% of a quantity, that is exactly the same as $\frac{8}{100}$ of it.



Practice Problems

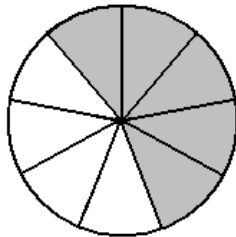
1. Compute $\frac{4}{9}$ of 63.
2. Shade $\frac{5}{9}$ of the circle below.
3. Compute $\frac{5}{6}$ of 24.
4. The price of a TV is \$400. We want to raise the price by 5%. What is the new price?
5. The price of a couch is \$700. Next week, it will go on a 15% off sale. What is the new price? (A 15% sale means that the price of the item is lowered by 15%.)
6. Find $\frac{3}{4}$ of 56.
7. We placed \$2000 into a bank account with 6% yearly interest rate. How much money do we have in the bank after one year?
8. This problem is about a method of comparing fractions.
 - a) Compute $\frac{3}{7}$ of 420.
 - b) Compute $\frac{4}{10}$ of 420.
 - c) Based on the results of parts a) and b), which fraction is larger, $\frac{3}{7}$ or $\frac{4}{10}$?
9. Bert has made \$54 000 last year. If he has to pay 32% of his income in taxes, how much taxes does he owe and how much of his income will he keep?

10. Sally used to make \$2400 per month, but now she got a 3% raise. How much is her monthly salary now?
11. Mr. X won \$600 000 in the lottery two yers ago. By now he has spent some of the money. When he was asked what happened, he said the following. "*I didn't spend it all. I spent $\frac{1}{3}$ of the money by taking a luxury yacht trip around the world. Then I put the rest in the bank. Later I decided to buy a house I really liked. So I took $\frac{3}{4}$ of the money out of the bank and baught the house. For half of what was left, I purchased stocks that completely lost their value. Finally, I gave $\frac{2}{5}$ of what's left to my niece for her college education.*"
- How much money is left from the winnings?



Answers

1. 16 2. See below



2. 20 4. \$420 5. \$595 6. 42
7. \$2120 8. a) 180 b) 168 c) $\frac{3}{7}$ is larger
9. \$17 280 in taxes and will keep \$36 720.
10. \$2472 11. \$30 000