Part 1 - Multiplying Fractions

Theorem: To multiply two fractions, we multiply one numerator by the other and one denominator by the other.

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}$$

We will refer to multiplying numerator by numerator and denominator by denominator as mutiplying straight across.

Example 1. Perform each of the multiplications as indicated.

a) $\frac{3}{4}\cdot\frac{5}{12}$ b) $-2\cdot\frac{5}{4}$ c) $-3\frac{1}{3}\cdot 1\frac{1}{2}$ d) $\left(\frac{2}{5}\right)^2$

Solution: a) We multiply straight across: numerator by numerator and denominator by denominator. Then we reduce the fraction to lowest terms before presenting our final answer.

$$\frac{3}{4}\cdot\frac{5}{12}=\frac{3\cdot 5}{4\cdot 12}=\frac{15}{48}$$

The result is not in lowest terms. We reduce it before presentig it as our answer by dividing both numerator and denominator by 3.

$$\frac{15}{48}=\boxed{\frac{5}{16}}$$

We often save time and effort by simplifying the fraction before performing the multiplications. Consider the problem we just solved example. We were able to divide both numerator and denominator by 3, because the numerator in $\frac{3}{4}$ and the denominator in $\frac{5}{12}$ are both divisible by 3. So, we can divide out by that 3 before we perform the multiplication. We just re-write 12 as $3\cdot 4$ and cross out a factor of 3 from both numerator and denominator.

$$\frac{3}{4}\cdot\frac{5}{12}=\frac{3\cdot 5}{4\cdot 12}=\frac{\cancel{3}\cdot 5}{4\cdot\cancel{3}\cdot 4}=\frac{5}{16}$$

Crossing out the same factor from the products in both numerator and denominator is the same as dividing both by that factor. This is often called cancellation.

b) We can interpret integers as fractions with denominator 1. If there is a negative sign, we keep it with the numerator for all computations. Therefore, we will re-write -2 as $\frac{-2}{1}$.

$$-2\cdot\frac{5}{4}=\frac{-2}{1}\cdot\frac{5}{4}=\frac{-2\cdot 5}{1\cdot 4}=\frac{-10}{4}=\boxed{-\frac{5}{2}}$$

c) While it is possible to multiply mixed numbers, it takes a lot of work and it will not be shown here. We can simply re-write mixed numbers as improper fractions. If there is a negative sign, we keep it in the numerator during computations.

$$-3\frac{1}{3}\cdot 1\frac{1}{2}=\frac{-10}{\cancel{3}}\cdot\frac{\cancel{3}}{2}=\frac{-10}{2}=\boxed{-5}$$

d) If we square a fraction, we must use a pair of parentheses. Other wise, $\frac{2}{5}^2$ can be interpreted as division between integers $\frac{2^2}{5}$. The parentheses guarantees that we all understand that the entire fraction $\frac{2}{5}$ is being suared, and not just its denominator.

$$\left(\frac{2}{5}\right)^2 = \frac{2}{5}\cdot\frac{2}{5} = \boxed{\frac{4}{25}}$$

As we are defining operations on fractions, let us note that the notation and order of operations agreement remain the same with fractions as they were with integers. Some issues, such as the difference between -3^2 and $(-3)^2$ will also remain.

Example 2. Perform the indicated operations. $-\frac{2}{3}-\frac{4}{5}\left(-2\frac{6}{7}\right)$

Solution: We see a multiplication and a subtraction. We start with the multiplication, but first re-write the mixed number as an improper fraction. We keep the negative sign in the numerator. $2\frac{6}{7}=\frac{20}{7}$ ad so $-2\frac{6}{7}=\frac{-20}{7}$.

$$-\frac{2}{3}-\frac{4}{5}\left(-2\frac{6}{7}\right) = -\frac{2}{3}-\frac{4}{5}\left(\frac{-20}{7}\right) = -\frac{2}{3}-\frac{4(-20)}{5\cdot 7} = -\frac{2}{3}-\frac{4(-4)\cdot \cancel{5}}{\cancel{5}\cdot 7} = -\frac{2}{3}-\frac{-16}{7}$$

Next we bring the fractions to the least common denominator and perform the subtraction of signed numbers in the numerator. The least common denominator is 21.

$$-\frac{2}{3}-\frac{-16}{7} = -\frac{14}{21}-\frac{-48}{21} = \frac{-14-(-48)}{21} = \frac{-14+48}{21} = \boxed{\frac{34}{21}}$$

Part 2 - Multiplying by One

It is a frequently occuring task in algebra to re-write an expression without changing its value. There are just about two operations that guarantee this in case of any quantity: we can always *add zero* to it or *multiply* it *by one*. (Actually, we can also subtract zero and divide by one.) In abstract algebra, an element that does not make any change under the operation to any quantity, is called an identity element. Zero is called the **additive identity**, as it does nothing in addition. The number 1 is called the **multiplicative identity** because it does nothing in multiplication.

> For every real number x, $x+0=x$. This is the **additive identity** property of zero.
> For every real number x, $x\cdot 1=x$. This is the **multiplicative identity** property of one.

Many algebraic techniques are based zero or multiplication (or division) by 1. With fractions, multiplication by 1 is extremely useful. We can even explain the fundamental property of fractions in terms of multiplication by 1. Recall the fundamental property of fractions: we can multiply both numerator and denominator by the same non-zero number and the value of the fraction would remain the same. In the addition $\frac{2}{3}+\frac{1}{2}$, we would need to re-write $\frac{2}{3}$ with a denominator of 6.

$$\frac{2}{3}=\frac{2}{3}\cdot 1=\frac{2}{3}\cdot\frac{2}{2}=\frac{2\cdot 2}{3\cdot 2}=\frac{4}{6}$$

Theorem: The fractions $-\frac{2}{3}$, $\frac{-2}{3}$, and $\frac{2}{-3}$ all have the same value.

Proof. We already knew that $\frac{-2}{3}$ and $\frac{2}{-3}$ are equivalent fractions. We can get from one to the other one by multiplying both numerator and denominator by -1. As a general habit, we should rarely tolerate a negative denominator.

$$\frac{2}{-3} = \frac{2\cdot(-1)}{-3\cdot(-1)} = \frac{-2}{3}$$

But only now can we connect $\frac{-2}{3}$ to $-\frac{2}{3}$.

$-\frac{2}{3}$ is the opposite of $\frac{2}{3}$. That means multiplication by -1, and we will express -1 as $\frac{-1}{1}$.

$$-\frac{2}{3} = -1\cdot\frac{2}{3} = \frac{-1}{1}\cdot\frac{2}{3} = \frac{-1\cdot 2}{1\cdot 3} = \frac{-2}{3} \qquad \blacksquare \text{ (End of Proof)}$$

The negative sign is not the only thing that can freely move in a fraction. Other factors can only move between being a factor in the numerator to being a multiplyer of a fraction.

Theorem: The algebraic expressions $\frac{3}{5}x$ and $\frac{3x}{5}$ are equivalent.

Proof. The key here is that we can re-write x as $\frac{x}{1}$. Division by one never changes the value of a number.

$$\frac{3}{5}x = \frac{3}{5}\cdot x = \frac{3}{5}\cdot\frac{x}{1} = \frac{3x}{5}. \qquad \blacksquare \text{ (End of Proof)}$$

Units behave the same way as x does. Kilogramm is a measure of mass, 1 kilogram is about 2.2 pounds. If we measure mass in kilograms, (denoted by kg), we can have the unit either in the numerator, or after the number. For example, $\frac{2}{3}\,\text{kg}$ is the same as $\frac{2\,\text{kg}}{3}$. Although these fractions express slightly different things, they are equaivalent, and we can freely move between the two forms. Inside computations we often prefer $\frac{2\,\text{kg}}{3}$ and we would present the final answer as $\frac{2}{3}\,\text{kg}$.

Part 3 - The Reciprocal

Example 3. Perform the multiplication $\frac{3}{8}\cdot\left(2\frac{2}{3}\right)$.

Solution: Before multiplying stratight across, we re-write the mixed number as an improper fraction.

$$\frac{3}{8}\cdot\left(2\frac{2}{3}\right)=\frac{3}{8}\cdot\frac{8}{3}=\frac{24}{24}=\boxed{1}$$

Definition: If the product of two numbers is 1, we call such a pair of numbers **reciprocal**s of each other. For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, and the reciprocal of $\frac{3}{2}$ is $\frac{2}{3}$. The reciprocal is also called the **multiplicative inverse**.

There is an obvious symmetry between $\frac{2}{3}$ and $\frac{3}{2}$. It is easy to see why those two fractions would multiply to 1. This is because the product of them is

$$\frac{a}{b}\cdot\frac{b}{a}=\frac{ab}{ab}=1.$$

So, we simply flip a fraction upside down to get to its reciprocal. However, we should always remember that the definition of the reciprocal of a number x is another number so that their product is 1.

Example 4. Find the reciprocal for each of the following.

a) $\frac{2}{5}$ b) $-\frac{3}{7}$ c) 2 d) x e) $1\frac{3}{5}$

Solution: a) To find the reciprocal of $\frac{2}{5}$, we need to flip it upside down. So the reciprocal of $\frac{2}{5}$ is $\boxed{\frac{5}{2}}$. Indeed, the product of these two fractions is $\frac{2}{5}\cdot\frac{5}{2}=\frac{10}{10}=1$.

b) How does the flipping work with the negative sign? The short story is that the reciprocal of $-\frac{3}{7}$ is $\boxed{-\frac{7}{3}}$.

The long story is that we can re-write $-\frac{3}{7}$ as $\frac{-3}{7}$. Then we flip for the reciprocal: we get from $\frac{-3}{7}$ to $\frac{7}{-3}$. Negative signs are not acceptable in the denominator, so we immediately lift it up to the numerator. (Just mutiply numerator and denominator by -1.) So the reciprocal of $\frac{-3}{7}$ is $\frac{7}{-3}$, which is $\frac{-7}{3}$. That's the same as $-\frac{7}{3}$.

$$\text{The reciprocal of } -\frac{3}{7} \text{ is the flip of } \frac{-3}{7}. \text{ That is } \frac{7}{-3}=\frac{-7}{3}=\boxed{-\frac{7}{3}}$$

This algebraic gymnastics with the negative sign works, but we can also think of reciprocals in terms of the definition. A number and its reciprocal multiply to 1. That's a positive product. Therefore, the reciprocal of a negative number is negative so that the product can be positive. For the reciprocal of $-\frac{3}{7}$, we do two things: flip the fraction $\frac{3}{7}$ and decide that the reciprocal of $-\frac{3}{7}$ must be negative. The reciprocal is then $-\frac{7}{3}$.

c) How do we take the reciprocal of something that is not necessarily a fraction? We can re-write 2 as $\frac{2}{1}$ and then we can flip. The reciprocal of 2 is the same as the reciprocal of $\frac{2}{1}$, which is $\boxed{\frac{1}{2}}$.

d) Let x be any non-zero number. We re-write it as $\frac{x}{1}$ and flip. So the reciprocal of x is $\boxed{\frac{1}{x}}$. This expression is meaningful as long as x is not zero. Therefore, every non-zero number has a unique reciprocal and is denoted by $\frac{1}{x}$. To check if x and $\frac{1}{x}$ are really reciprocals, we use the definition. The product of a number and its reciprocalis 1. Indeed,

$$x \cdot \frac{1}{x} = \frac{x}{1} \cdot \frac{1}{x} = \frac{x}{x} = 1 \qquad \text{This is true as long as } x \text{ is not zero.}$$

e) We can only find the reciprocal of a mixed number by converting it to an improper fraction and then flip it. $1\frac{3}{5} = \frac{8}{5}$, so

$$\text{the reciprocal of } 1\frac{3}{5} \text{ is the reciprocal of } \frac{8}{5}, \text{ which is } \boxed{\frac{5}{8}}.$$

Notice that separately flipping the integer- and fraction parts in a mixed number does not pruduce the right results: $1\frac{3}{5}$ and $1\frac{5}{3}$ are not reciprocals. Their product is

$$1\frac{3}{5} \cdot 1\frac{5}{3} = \frac{8}{5} \cdot \frac{8}{3} = \frac{64}{15} \text{ and not } 1.$$

To get the reciprocal, we *have to* convert mixed numbers to improper fractions.

Part 4 - Dividing Fractions

As it turns out, we never divide fractions. Every non-zero number has a reciprocal. We interpret division as multiplication by the reciprocal.

Theorem: To divide is to multiply by the reciprocal.

Proof. Let x be any non-zero number. Let us multiply y by $\frac{1}{x}$.

$$y \cdot \frac{1}{x} = \frac{y}{1} \cdot \frac{1}{x} = \frac{y \cdot 1}{1 \cdot x} = \frac{y}{x} \quad \text{is the same as division by } x. \ \blacksquare$$

Example 5. Perform the divisions of the fractions as indicated.

a) $\frac{5}{6} \div \frac{2}{3}$ b) $\frac{2}{5} \div (-4)$ c) $4\frac{1}{5} \div 1\frac{1}{6}$ d) $-2 \div \left(-\frac{3}{8}\right)$

Solution: a) To divide is to multiply by the reciprocal. Instead of dividing by $\frac{2}{3}$, we will multiply by its reciprocal, $\frac{3}{2}$.

$$\frac{5}{6} \div \frac{2}{3} = \frac{5}{6} \cdot \frac{3}{2} = \frac{5 \cdot 3}{6 \cdot 2} = \frac{5 \cdot \cancel{3}}{2 \cdot \cancel{3} \cdot 2} = \boxed{\frac{5}{4}}$$

b) To divide is to multiply by the reciprocal. Instead of dividing by -4, we multiply by its reciprocal. $-4 = \frac{-4}{1}$, therefore, its reciprocal is $\frac{1}{-4}$. However, we immediately lift the negative sign and as always, we keep it in the numerator. Thus, the reciprocal of -4 is $-\frac{1}{4}$ and we will write it as $\frac{-1}{4}$.

$$\frac{2}{5} \div (-4) = \frac{2}{5} \cdot \frac{-1}{4} = \frac{-2}{20} = \boxed{-\frac{1}{10}}$$

c) We re-write the subtraction with one big fraction bar and a subtraction of integers the same numerator. Then, if we can, we simplify the answer.

$$\frac{3}{7} - \frac{9}{7} = \frac{3-9}{7} = \boxed{\frac{-6}{7} \text{ or } -\frac{6}{7}}$$

d) We can not take the reciprocal of a mixed number. Therefore, we immediately convert each mixed number to an improper fraction, and instead of division, we multiply by the reciprocal.

$$4\frac{1}{5} \div 1\frac{1}{6} = \frac{21}{5} \div \frac{7}{6} = \frac{21}{5} \cdot \frac{6}{7} = \frac{(7 \cdot 3) \cdot 6}{5 \cdot 7} = \boxed{\frac{18}{5}}$$

Example 6. a) Perform the division $10 \div \frac{1}{4}$ b) How many quarters are there in a 10−dollar roll of quarters?

Solution: a) To divide is to multiply by the reciprocal. The reciprocal of $\frac{1}{4}$ is $\frac{4}{1}$ or 4.

$$10 \div \frac{1}{4} = 10 \cdot 4 = \boxed{40}$$

b) Each dollar can be exchanged for four quarters, so ten dollars would be the same as ten times four, or forty quarters.

From this last example we see that division still has its same old meaning: how many times can we fit $\frac{1}{4}$ into 10?

Example 7. Simplify the given expression: $\dfrac{\frac{2}{3}+\frac{3}{4}}{\frac{5}{6}-\frac{1}{3}}$

Solution: Recall that if a fraction bar (or division bar) is stretching under entire expressions, then it also serves as (invisible) parentheses. Thus

$$\dfrac{\frac{2}{3}+\frac{3}{4}}{\frac{5}{6}-\frac{1}{3}} \text{ is the same as } \left(\frac{2}{3}+\frac{3}{4}\right)\div\left(\frac{5}{6}-\frac{1}{3}\right)$$

Therefore, we will first perform the addition in the numerator and the subtraction in the denominator. We will finally divide. For additions and subtractions, we need to use a common denominator.

$$\dfrac{\frac{2}{3}+\frac{3}{4}}{\frac{5}{6}-\frac{1}{3}} = \dfrac{\frac{8}{12}+\frac{9}{12}}{\frac{5}{6}-\frac{2}{6}} = \dfrac{\frac{8+9}{12}}{\frac{5-2}{6}} = \dfrac{\frac{17}{12}}{\frac{3}{6}} = \frac{17}{12}\cdot\frac{6}{3} = \frac{17\cdot 6}{(6\cdot 2)\cdot 3} = \frac{17}{2\cdot 3} = \boxed{\frac{17}{6}}$$

Example 8. Perform each of the given divisions. a) $\dfrac{\frac{2}{3}}{5}$ b) $\dfrac{2}{\frac{3}{5}}$

Solution: a) If needed, we can always re-write an integer with a denominator of 1. $\dfrac{\frac{2}{3}}{5} = \dfrac{\frac{2}{3}}{\frac{5}{1}} = \frac{2}{3}\cdot\frac{1}{5} = \boxed{\frac{17}{6}}$

b) This time we will need to re-write 2 as a fraction. $\dfrac{2}{\frac{3}{5}} = \dfrac{\frac{2}{1}}{\frac{3}{5}} = \frac{2}{1}\cdot\frac{5}{3} = \boxed{\frac{17}{6}}$.

As the previous example shows, $\dfrac{\frac{a}{b}}{c}$ and $\dfrac{a}{\frac{b}{c}}$ are different expressions, with different values. Because of this, we have to be careful with expressions with more than one fraction bar (or division bar). Order of operations must be clearly indicated by the different sizes of the fraction bars. A fraction or division problem with more than one fraction bar is called a **complex fraction.** Complex fractions can always be simplified to a form with just one fraction bar.

Example 9. Simplify each of the given expressions. a) $\dfrac{\frac{a}{b}}{c}$ b) $\dfrac{a}{\frac{b}{c}}$

Solution:

a) In this case, we need to re-write c as $\frac{c}{1}$. Then we multiply by the reciprocal.

$$\dfrac{\frac{a}{b}}{c} = \dfrac{\frac{a}{b}}{\frac{c}{1}} = \frac{a}{b}\cdot\frac{1}{c} = \boxed{\frac{a}{bc}}$$

b) In this case, we need to re-write a as $\frac{a}{1}$. Then we multiply by the reciprocal.

$$\dfrac{a}{\frac{b}{c}} = \dfrac{\frac{a}{1}}{\frac{b}{c}} = \frac{a}{1}\cdot\frac{c}{b} = \boxed{\frac{ac}{b}}$$

Part 5 - An Application: Conversion Factors

In physics, in some applications of mathematics, and even in every day life, we dealing with numbers with units. For example, length can not be determined or communicated by a number only. 1 mile is much much longer than 1 inch. Also, 5 minutes is much shorter than 5 years.

The cancellation of factors in numerator and denominator of fractions can be used to convert a quantity from one unit to another one. Recall that we never change the value of any number if we multiply it by 1. For this reason, 1 is sometimes called the **multiplicative identity**.

Consider now the statement 1 hour $=$ 60 minutes. In physics, hour is denoted by h and minutes by min. If we divide any non-zero quantity by itself, the result is 1. Thus, using the equality $1\,\mathrm{h} = 60\,\mathrm{min}$, we can write two fractions, both of value 1.

$$\frac{1\,\mathrm{h}}{60\,\mathrm{min}} \quad \text{and} \quad \frac{60\,\mathrm{min}}{1\,\mathrm{h}}$$

These two fractions are called **conversion factors** or unit multiplyers, and we use them to converting time measurements from minutes to hours or backward, from hours to minutes.

Example 10. Convert $\frac{7}{12}$ hours to minutes.

Solution: We will re-write $\frac{7}{12}$ hours as $\frac{7\,\mathrm{h}}{12}$ and multiply it by one of the conversion factors shown above. Since we would like to get rid of *hours*, we will select the conversion factor that has *hour* in its denominator.

$$\frac{7}{12}\,\mathrm{h} = \frac{7\,\mathrm{h}}{12} \cdot 1 = \frac{7\,\mathrm{h}}{12} \cdot \frac{60\,\mathrm{min}}{1\,\mathrm{h}}$$

Now the *hour* unit is a factor in both numerator and denominator, so we can cancel it out, and we will be left with just minutes in the numerator.

$$\frac{7\,\mathrm{h}}{12} \cdot \frac{60\,\mathrm{min}}{1\,\mathrm{h}} = \frac{7 \cdot 60\,\mathrm{min}}{12} = \frac{7 \cdot (5 \cdot 12)\,\mathrm{min}}{12} = \frac{35\,\mathrm{min}}{1} = \boxed{35\,\mathrm{min}}$$

Example 11. Convert 9000 minutes to hours.

Solution: We first re-write 9000 minutes as a fraction, and keep the unit with the denominator. If needed, we can always write a denominator 1. Then we multiply it by 1.

$$9000\ \mathrm{min} = \frac{9000\ \mathrm{min}}{1} = \frac{9000\ \mathrm{min}}{1} \cdot 1$$

The fraction expressing 1 is going to be the conversion factor that has minutes in its denominator. The minutes unit is cancelled out, and we are left with hours instead.

$$9000\ \mathrm{min} = \frac{9000\ \mathrm{min}}{1} \cdot \frac{1\,\mathrm{h}}{60\,\mathrm{min}} = \frac{9000\,\mathrm{h}}{60} = \frac{150\,\mathrm{h}}{1} = \boxed{150\ \mathrm{h}}$$

Conversion factors can also be used in groups.

Example 12. Convert 10 years to seconds.

Solution: We will go from years to days, then from days to hours, then to minutes and finally to seconds. The entire computation can be done in a single line. Let us assume that a year is 365 days long.

$$10\,\mathrm{y} = \frac{10\,\mathrm{y}}{1} = \frac{10\,\mathrm{y}}{1} \cdot \frac{365\,\mathrm{d}}{1\,\mathrm{y}} \cdot \frac{24\,\mathrm{h}}{1\,\mathrm{d}} \cdot \frac{60\,\mathrm{min}}{1\,\mathrm{h}} \cdot \frac{60\,\mathrm{s}}{1\,\mathrm{min}} = \frac{10 \cdot 365 \cdot 24 \cdot 60 \cdot 60\,\mathrm{s}}{1} = \boxed{315\,360\,000\,\mathrm{s}}$$

Practice Problems

1. Perform the indicated operations. Present your answer as an integer or a reduced fraction. You do not need to convert improper numbers to mixed numbers.

   a) $-\dfrac{8}{15}\cdot\dfrac{3}{4}$

   b) $\left(-\dfrac{1}{2}\right)^2$

   c) $\dfrac{4}{15}\cdot\left(2\dfrac{1}{7}\right)$

   d) $-\left(-\dfrac{2}{3}\right)^2$

   e) $\dfrac{1}{6}-\dfrac{8}{25}\left(-1\dfrac{2}{3}\right)$

   f) $\dfrac{2}{7}-\dfrac{2}{3}\div\left(1\dfrac{2}{5}\right)$

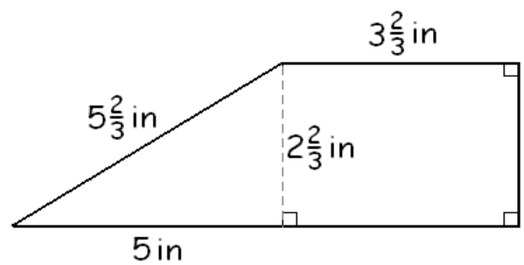
   g) $-\dfrac{5}{6}-\left(3-\dfrac{2}{5}\right)\left(-\dfrac{1}{2}\right)$

   h) $\left(3\dfrac{1}{3}\right)\div\left(2\dfrac{1}{2}\right)$

   i) $\dfrac{\dfrac{5}{4}-\left(\dfrac{1}{6}\right)^2}{\left(-1\dfrac{1}{3}\right)-\dfrac{1}{2}}$

   j) $\dfrac{1-\dfrac{2}{3}}{2-\dfrac{1}{3}}$

2. Find the perimeter and area of the object shown on the picture.



3. Evaluate the expression x^2-6x-1 if

   a) $x=-\dfrac{1}{2}$ b) $x=\dfrac{5}{2}$ c) $x=-\dfrac{2}{3}$

4. Evaluate the expression $\dfrac{-2x^2+x+3}{2x-3}$ if

   a) $x=\dfrac{1}{2}$ b) $x=\dfrac{3}{2}$ c) $x=-\dfrac{2}{3}$ d) $x=-\dfrac{5}{6}$

5. Perform each of the following conversions.

   a) 6000 inches to feet, given that $12\,\mathrm{in}=1\,\mathrm{ft}$

   b) 900 000 square-inches to square-feet. (Hint: $1\,\mathrm{ft}^2=1\,\mathrm{ft}\cdot 1\,\mathrm{ft}$)

   c) $81\,\dfrac{\mathrm{mi}}{\mathrm{h}}$ (miles per hour) to meter per second $\left(\dfrac{\mathrm{m}}{\mathrm{s}}\right)$. Hint: $1\,\mathrm{mi}\approx 1600\,\mathrm{m}$

Enrichment

1. (Enrichment) Contributed by Prof. Abdallah Shuaibi.
   Two travelers meet a third one, who is very hungry. He offers the two travelers 8 dollars for a meal. One traveler has three pieces of bread, the other one has five. So the hungry man gives them the 8 dollars, they all sit down and eat all 8 pieces of bread together. Afterwards, the two get into an argument about how to divide up the money. The one who contributed 5 pieces of bread wants to split it to 5 and 3. The other wants to divide the money evenly, 4 and 4. They go to a wise man for advice. They tell him their story and ask him to divide the money between them. The wise man gives the man who had 3 pieces of bread 1 dollar and 7 to the man with 5 pieces of bread. Is this a just or even reasonable decision?

Answers

1. a) $-\frac{2}{5}$ b) $\frac{1}{4}$ c) $\frac{4}{7}$ d) $-\frac{4}{9}$ e) $\frac{7}{10}$ f) $-\frac{4}{21}$ g) $\frac{7}{15}$ h) $\frac{4}{3}$ i) $-\frac{2}{3}$ j) $\frac{1}{5}$

1. $P = \frac{62}{3}$ in $A = \frac{148}{9}\text{ in}^2$ 3. a) $\frac{9}{4}$ b) $-\frac{39}{4}$ c) $\frac{31}{9}$

4. a) $-\frac{3}{2}$ b) undefined d) $-\frac{1}{3}$ d) $-\frac{1}{6}$ 5. a) 500 ft b) 6250 ft^2 c) $36\frac{\text{m}}{\text{s}}$

