Consider the following problem: Find $\frac{3}{5}$ of 100.

Solution: $\frac{1}{5}$ of 100 can be obtained by splitting 100 into 5 equal shares. If we think of money, this is an easy task: 5 twenty-dollar bill make up a hundred dollar bill. In short, $\frac{1}{5}$ of 100 is 20.

We need $\frac{3}{5}$ of 100, which means that we need to take 3 shares out of the 5. This is $3\,(20) = 60$. Thus, $\frac{3}{5}$ of 100 is 60.

Notice that we obtain the same result if we simply multiply $\frac{3}{5}$ by 100.

$$\frac{3}{5} \cdot 100 = \frac{3}{5} \cdot \frac{100}{1} = \frac{300}{5} = 60$$

We will establish a languague we will use to solve word problems involving fractions. Consider the statement

$$\frac{3}{5} \text{ of } 100 \text{ is } 60.$$

These three quantities are always present in word problems involving fractions. The following definitions are not very elegant but seem to be useful. The fraction $\frac{3}{5}$ will be called F for fraction. The quantity we are splitting up (in this case, 100) will be called the (of) number, because the word *of* is always near it. The result will be called the (is) number, since the word *is* is always near it. Then all word problems involving fractions can be solved using one formula,

$$\boxed{(\text{is}) = (\text{Fraction}) \cdot (\text{of})}$$

In the following problems we are given two quantities, and are to find the third one. Consequently, there are three types of word problems involving fractions. We will call a problem type 1 if we have to find the (is) number, type 2 if we have to find F (the fraction), and type 3 if we have to find the (of) number.

Example 1. Find $\frac{2}{9}$ of 450.

Solution: The formula we use is $\boxed{(\text{is}) = (\text{Fraction}) \cdot (\text{of})}$. First we identify which two quantities are given. We will denote the third one by x.

$(\text{is}) = x$

$F = \frac{2}{9}$

$(\text{of}) = 450$

We will substitute these into the formula and solve for x. The fraction and the (of) are given, and so

$$(\text{is}) = F \cdot (\text{of}) \quad \text{becomes} \quad x = \frac{2}{9} \cdot 450 = \frac{900}{9} = 100$$

So the answer is $\boxed{100}$.

Because the fraction and the (of) numbers were given and the (is) number was to be found, this problem is type 1.

Example 2. 75 is what fraction of 400?

Solution: The formula we use is $\boxed{(\text{is}) = (\text{Fraction}) \cdot (\text{of})}$. We first write a table, listing the three quantities, is-number, fraction, and of-number. We need to identify the two quantities given, and call the third one x. In this case,

$(\text{is}) = 75$

$F = x$

$(\text{of}) = 400$

We will substitute these into the formula and solve for x.

$$\begin{aligned} (\text{is}) &= \text{F} \cdot (\text{of}) && \\ 75 &= x \cdot 400 && \text{Solve for } x \text{ by dividing both sides by } 400 \\ \frac{75}{400} &= x && \text{simplify the result} \\ x &= \frac{3}{16} && \end{aligned}$$

The answer is $\boxed{\frac{3}{16}}$. We check by computing $\frac{3}{16}$ of 400. It is indeed 75, and so our solution is correct.

Because the (is) and the (of) numbers were given and the fraction was to be found, this problem is type 2.

Example 3. $\frac{4}{11}$ of a number is 36. Find this number.

Solution: The formula we use is $\boxed{(\text{is}) = (\text{Fraction}) \cdot (\text{of})}$ We first identify the two numbers given and call the missing number x.

$(\text{is}) = 36$

$F = \frac{4}{11}$

$(\text{of}) = x$

The fraction and the (is) are given, and so we label the (of) number as x.

$$\begin{aligned} (\text{is}) &= (\text{Fraction}) \cdot (\text{of}) && \\ 36 &= \frac{4}{11} \cdot x && \text{Divide both sides by } \frac{4}{11} \\ \frac{36}{\frac{4}{11}} &= x && \text{To divide is to multiply by the reciprocal:} \\ 99 &= x && \frac{36}{\frac{4}{11}} = 36 \cdot \frac{11}{4} = 99 \end{aligned}$$

The answer is $\boxed{99}$. We check: is it true that $\frac{4}{11}$ of 99 is 36? Since $\frac{4}{11}(99) = 36$, our solution is correct.

Because the (is) number and the fraction were given and the (of) number was to be found, this problem is type 3.

We can now solve percent problems as well, because percents are fractions with denominator 100.

Example 4. Find 15% of 400.

Solution: We first write a table, listing the three quantities, is-number, fraction, and of-number. We need to identify the two quantities given, and call the third one x. In this case,

$(\text{is}) = x$

$F = 15\% = \dfrac{15}{100}$

$(\text{of}) = 400$

We will substitute these into the formula and solve for x.

$$\begin{aligned} (\text{is}) &= \text{F} \cdot (\text{of}) \\ x &= \frac{15}{100} \cdot 400 \\ x &= 60 \end{aligned}$$

Thus 15% of 400 is $\boxed{60}$. This problem is type 1.

Example 5. 21 is what percent of 350?

Solution: We first write a table, listing the three quantities, is-number, fraction, and of-number. We need to identify the two quantities given, and call the third one x. In this case,

$(\text{is}) = 21$

$F = x$

$(\text{of}) = 350$

We will substitute these into the formula and solve for x.

$$\begin{aligned} (\text{is}) &= \text{F} \cdot (\text{of}) \\ 21 &= x \cdot 350 \qquad \text{divide by } 350 \\ \frac{21}{350} &= x \end{aligned}$$

We obtained the value of x, but not as a percent. We will convert x into a percent.

$$x = \frac{21}{350} = \frac{7 \cdot 3}{7 \cdot 50} = \frac{3}{50} = \frac{3 \cdot 2}{50 \cdot 2} = \frac{6}{100} = 6\%$$

Thus 21 is $\boxed{6\%}$ of 350. We can check by computing 6% of 350: $\dfrac{6}{100} \cdot 350 = 21$. Thus our solution, 6% is correct. This problem is type 2.

Example 6. 24% of what number is 72?

Solution: We first write a table, listing the three quantities, is-number, fraction, and of-number. We need to identify the two quantities given, and call the third one x. In this case,

$(\text{is}) = 72$

$F = 24\% = \dfrac{24}{100}$

$(\text{of}) = x$

$$\begin{aligned} (\text{is}) &= \text{F} \cdot (\text{of}) \\ 72 &= \frac{24}{100} \cdot x \qquad \text{divide by } \frac{24}{100} \\ \frac{72}{\frac{24}{100}} &= x \\ 300 &= x \end{aligned}$$

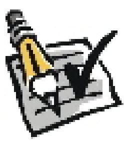
The computation is $72 \div \dfrac{24}{100} = \dfrac{72}{1} \cdot \dfrac{100}{24} = \dfrac{3 \cdot 24}{1} \cdot \dfrac{100}{24} = \dfrac{300}{1} = 300$. Thus 72 is 24% of $\boxed{300}$.

We can check by computing 24% of 300: $\dfrac{24}{100} \cdot 300 = \dfrac{24}{100} \cdot \dfrac{300}{1} = 24 \cdot 3 = 72 = 72$. Thus our solution is correct.

We have focused on the steps of solving such equations. Computations can be simplified by reducing fractions or working with decimals instead of fractions.

Sample Problems

1. Find 16% of 3600.
2. 27 is what percent of 18?
3. 120% of what number is 150?
4. What do we get if we increase 600 by 150%?
5. We placed \$ 8000 into a bank account with an annual 7% of interest rate. How much money do we have in the account a year later?
6. The population of a town has decreased from $80\,000$ to $68\,000$. What percent of a decrease does this represent?
7. Tom got a 4% raise in his job. Now he makes 2496 per month. How much was he making before the raise?
8. A TV set went on a 35% sale. The sale price is \$$312$. Find the original price.



Practice Problems

1. What fraction of $2\frac{1}{8}$ is $\frac{1}{4}$?
2. Compute 87% of 300.
3. 130% percent of what number is 78?
4. Fifteen percent of what number is 105?
5. What percent of 450 is 288?
6. What number do we get if we increase 80 by 240%?
7. What percent of 460 is 1472?
8. 347% of what number is 2429?
9. Fifteen percent of the town's population are students. If there are 1800 students living in the town, how many people live there?
10. Paul earned \$$128$ this week in his part time job. If this was a sixty percent increase from last week, how much money did he make last week?
11. A TV went on a 14% sale. The sale price is \$$516$. Find the original price of the TV.
12. Overnight, the number of bacteria increased by one hundred sixty percent. There are now 650000 bacteria. How many was there yesterday?
13. We want to increase quantity Q by 2%. Which of the following is the correct expression for that?

    A) $1.2Q$ B) $1.12Q$ C) $1.02Q$ D) $3Q$ E) $2.2Q$
14. We want to increase quantity Q by 20%. Which of the following is a correct expression for that?

    A) $1.2Q$ B) $1.12Q$ C) $1.02Q$ D) $3Q$ E) $2.2Q$
15. We want to increase quantity Q by 12%. Which of the following is a correct expression for that?

    A) $1.2Q$ B) $1.12Q$ C) $1.02Q$ D) $3Q$ E) $2.2Q$

16. We want to increase quantity Q by 120%. Which of the following is a correct expression for that?

A) $1.2Q$ B) $1.12Q$ C) $1.02Q$ D) $3Q$ E) $2.2Q$

17. We want to increase quantity Q by 200%. Which of the following is a correct expression for that?

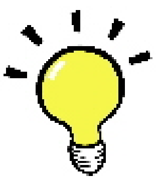
A) $1.2Q$ B) $1.12Q$ C) $1.02Q$ D) $3Q$ E) $2.2Q$

18. Last month, Randy was making $\$1800$ a month in his sales job. Based on his performance, he could get a raise at any time.

a) Two weeks ago, he got a raise of 10%. How much was he making after this increase?
b) A week ago, he got a second raise of 10%. How much is he making now?
c) Look at Randy's current pay and compare it to the original $\$1800$. What percent increase does his current pay represent?

19. The population of a town has increased 20% in the 80s. During the next ten years, the town's population further grew 15%. What percent of an increase occurred during these twenty years?

20. A stock loses 60% of its value. What must the percent of increase be to recover all of its lost value? (Hint: if no value for the stock is given, make up a few different numbers.)



Answers

Sample Problems

1. 576 2. 150% 3. 125 4. 1500 5. $\$8560$ 6. 15% decrease 7. $\$2400$ per month 8. $\$480$

Practice Problems

1. $\dfrac{2}{17}$ 2. 261 3. 60 4. 700 5. 64% 6. 272 7. 320% 8. 700 9. $12\,000$ 10. $\$80$

11. $\$600$ 12. $260\,000$ 13. C 14. A 15. B 16. E 17. D

18. a) $\$1980$ b) $\$2178$ c) 21% increase 19. 38% 20. 150%

Sample Problems Solutions

1. Find 16% of 3600.

   Solution: We first write a table, listing the three quantities, is-number, fraction, and of-number. We need to identify the two quantities given, and call the third one x. In this case,

   (is) $=$ x
   $F =$ 0.16
   (of) $=$ 3600

   We will substitute these into the formula and solve for x. $0.16 \cdot 360 = 57.6$

$$\begin{aligned} \text{(is)} &= \text{F} \cdot \text{(of)} \\ x &= 0.16 \cdot 3600 \\ x &= 576 \end{aligned}$$

   Thus 16% of 3600 is 576.

2. 27 is what percent of 18?

   Solution: We first write a table, listing the three quantities, is-number, fraction, and of-number. We need to identify the two quantities given, and call the third one x. Because the is-number is smaller than the of-number, we should expect a percentage larger than 100%.

   (is) $=$ 27
   $F =$ x
   (of) $=$ 18

   We will substitute these into the formula and solve for x.

$$\begin{aligned} \text{(is)} &= \text{F} \cdot \text{(of)} \\ 27 &= x \cdot 18 && \text{divide by } 18 \\ \frac{27}{18} &= x \\ x &= 1.5 = 150\% \end{aligned}$$

   Thus 27 is 150% of 18. We can check by computing 150% of 18:

$$\begin{aligned} \text{(is)} &= \text{F} \cdot \text{(of)} \\ \text{(is)} &= 1.5 \cdot 18 = 27 \end{aligned}$$

   Thus our solution, 150% is correct.

3. 120% of what number is 150?

   Solution: We first write a table, listing the three quantities, is-number, fraction, and of-number. We need to identify the two quantities given, and call the third one x. In this case,

   (is) $=$ 150
   $F =$ 1.2
   (of) $=$ x

We will substitute these into the formula and solve for x.

$$\begin{aligned} (\text{is}) &= \text{F}\cdot(\text{of}) \\ 150 &= 1.2\cdot x \qquad \text{divide by } 1.2 \\ \frac{150}{1.2} &= x \\ 125 &= x \end{aligned}$$

We can check by computing 120% of 125:

$$\begin{aligned} (\text{is}) &= \text{F}\cdot(\text{of}) \\ (\text{is}) &= 1.2\cdot 125 = 150 \end{aligned}$$

Thus our solution, 125 is correct.
The following examples all boil down to one of the basic problems shown above. One advice: before starting computations, re-write the problem to a basic question. Once we have this question, the problem is easy to solve.

4. What do we get if we increase 600 by 150%?

   Solution 1: We compute 150% of 600 and then we add it to 600. To compute 150% of 600 is a type 1 problem:

   $$\begin{aligned} (\text{is}) &= x \\ F &= \frac{150}{100} = 1.5 \\ (\text{of}) &= 600 \end{aligned}$$

   Now we use our formula to find x.

   $$\begin{aligned} (\text{is}) &= \text{F}\cdot(\text{of}) \\ x &= 1.5\cdot 600 = 900 \end{aligned}$$

   So after the increase, we have $600 + 900 = 1500$.

   Solution 2: This is a neat shortcut that will become very important in other problems. If a quantity is increased by 150%, then it "grew up" from 100% of itself to $100\% + 150\% = 250\%$ of itself. We can find the answer quickly if we simply compute 250% of 600. The basic question is: What is 250% of 600?(Type 1)

   $$\begin{aligned} (\text{is}) &= x \\ F &= 250\% = 2.5 \\ (\text{of}) &= 600 \end{aligned}$$

   We substitute the data into the formula:

   $$\begin{aligned} (\text{is}) &= \text{F}\cdot(\text{of}) \\ x &= 2.5\cdot 600 \\ x &= 1500 \end{aligned}$$

   Thus the answer is 1500.

5. We placed \$8000 into a bank account with an annual 7% of interest rate. How much money do we have in the account a year later?

   Solution 1: We compute 7% of 8000 and add the result to 8000. The basic question is: What is 7% of 8000? (Type 1)

   $$\begin{aligned} (\text{is}) &= x \\ F &= 0.07 \\ (\text{of}) &= 8000 \end{aligned}$$

We substitute the data into the formula:

$$\begin{aligned}(\text{is}) &= \text{F}\cdot(\text{of})\\ x &= 0.07\cdot 8000\\ x &= 560\end{aligned}$$

Thus we earned $\$\ 560$ in interest, and now we have $\$\ 8000+\$\ 560=\$\ 8560$.

Solution 2: This is a neat shortcut that will become very important in other problems. If a quantity is increased by 7%, then it "grew up" from 100% of itself to 107% of itself. We can find the amount in the bank, if we simply compute 107% of 8000. The basic question is: What is 107% of 8000?

(is) $=$ x
$F=$ 1.07
(of) $=$ 8000

We substitute the data into the formula:

$$\begin{aligned}(\text{is}) &= \text{F}\cdot(\text{of})\\ x &= 1.07\cdot 8000\\ x &= 8560\end{aligned}$$

Thus we now have $\$8560$.

6. The population of a town has decreased from $80\,000$ to $68\,000$. What percent of a decrease does this represent?

Solution 1: We subtract $68\,000$ from $80\,000$ to determine the change. $80\,000-68\,000=12\,000$. Now the question is: $12\,000$ is what percent of $80\,000$? (Type 2)

(is) $=$ $12\,000$
$F=$ x
(of) $=$ $80\,000$

We substitute the data into the formula:

$$\begin{aligned}(\text{is}) &= \text{F}\cdot(\text{of})\\ 12\,000 &= x\cdot 80\,000\\ \frac{12\,000}{80\,000} &= x\\ 0.15 &= x\end{aligned}$$

Thus

$$x=0.15=\frac{0.15}{1}=\frac{0.15\,(100)}{1\,(100)}=\frac{15}{100}=15\%$$

This is a 15% decrease.

Solution 2: Instead of comparing the chage to the original condition, we can compare the new condition to the original condition and interpret the result. The question may be re-phrased as: $68\,000$ is what percent of $80\,000$? Then

(is) $=$ $68\,000$
$F=$ x
(of) $=$ $80\,000$

We substitute the data into the formula:

$$\begin{aligned}
(\text{is}) &= \text{F}\cdot(\text{of})\\
68\,000 &= x\cdot 80\,000\\
\frac{68\,000}{80\,000} &= x\\
0.85 &= x
\end{aligned}$$

Thus
$$x = 0.85 = \frac{0.85}{1} = \frac{0.85\,(100)}{1\,(100)} = \frac{85}{100} = 85\%$$

Since the population has decreased from 100% of itself to 85% of itself, our result reflects a 15% decrease.

7. Tom got a 4% raise in his job. Now he makes 2496 per month. How much was he making before the raise?

Solution: It would be a mistake to simply subtract 4% of 2496 from 2496, because the raise was 4% of a smaller, unknown number! The trick is to realize that Tom is now making exactly 104% of his original pay. The question is thus (Type 3) 104% of what number is 2496?

(is) = 2496
$F =$ 1.04
(of) = x

We substitute the data into the formula:

$$\begin{aligned}
(\text{is}) &= \text{F}\cdot(\text{of})\\
2496 &= 1.04\cdot x \qquad \text{divide by } 1.04\\
\frac{2496}{1.04} &= x\\
2400 &= x
\end{aligned}$$

Thus his original pay was \$ 2400 per month. We can check by taking 104% of 2400 and find that it is indeed 2496.

8. A TV set went on a 35% sale. The sale price is \$ 312. Find the original price.

Solution: This problem is similar to the previous one in the sense that it is type 3, and the method used previously is essential. It would be a mistake to simply add 35% of 312 from 312, because the deduction was 35% of a larger, unknown number! The trick is to realize that the current price is 65% of the original price. The difference is that we subtract 35% this time. The question is thus (Type 3) 65% of what number is 312?

(is) = 312
F = 0.65
(of) = x

We substitute the data into the formula:

$$\begin{aligned}
(\text{is}) &= \text{F}\cdot(\text{of})\\
312 &= 0.65\cdot x \qquad \text{divide by } 0.65\\
\frac{312}{0.65} &= x\\
480 &= x
\end{aligned}$$

Thus the original price was \$ 480. We can check by taking 65% of 480 and find that it is indeed 312.

