Prime numbers are a fascinating study within mathematics. Let us first recall the definition. Given a number n, we can find all of its divisors. For example, $n = 20$ has six divisors: $1, 2, 4, 5, 10$, and 20. Prime numbers have a very short list of divisors: only the trivial divisors, 1 and the number itself. For example, 7 is a prime number. 6 is not a prime number since it has divisors other than 1 and 6.

Definition: An integer greater than 1 is a **prime number** if it has exactly two divisions.

It is important to notice that 1 is not a prime number. The first few prime numbers are: $2, 3, 5, 7, 11, 13, 17, 19, ...$ With respect to multiplication, prime numbers are the basic building blocks of numbers.

Theorem: (*Fundamental Theorem of Arithmetic*) Every integer greater than 1 can be written as a product of prime numbers, and this decomposition is unique up to order of factors.

For example, $20 = 2 \cdot 2 \cdot 5$. According to the fundamental theorem of arithmetic, there is no other way to write 20 as a product of prime numbers. We usually use exponential notation: $20 = 2^2 \cdot 5$.

Example 1 Find the prime factorization of 300.

We start with the first prime number, 2. Is our number, 300 divisible by 2? If yes, we divide 300 by 2. Since $300 = 2 \cdot 150$, we now have one prime factor, 2 and we must find the prime factorization of 150.

300	2
150	2
75	3
25	5
5	5
1	

We ask next: is the number 150 divisible by 2? If yes, we divide 150 by 2. So now $300 = 2 \cdot 2 \cdot 75$ and we are looking for the prime factorization of 75.

We ask next: is the number 75 divisible by 2? This time, the answer is no. We have exhausted the prime factor 2. So we roll up to 3 and ask: is 75 divisible by 3? If yes, we divide 75 by 3. Since $75 \div 3 = 25$, so now $300 = 2 \cdot 2 \cdot 3 \cdot 25$ and we are looking for the prime factorization of 25. Although we know the final answer now, we continue the process. Every time we find a factor, we write it down and divide. The quotient is written down under the pair in the first column. Once that column reaches 1, the second column is the prime factorization of our number. Thus $300 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 = \boxed{2^2 \cdot 3 \cdot 5^2}$

Practice Problems

1. Find the prime factorization of 240.
2. Find the prime factorization of 2016.
3. Find the prime factorization of 1001.
4. Is 2017 a prime number?

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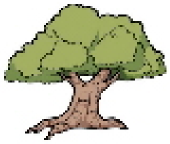
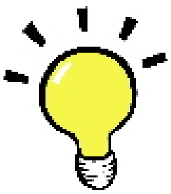


Enrichment

1. Suppose that given a number n, we need to determine whether it is a prime number or not. Until what number must we check all the prime numbers whether they are a divisor of n or not? When can we stop and say that this number must be a prime?
2. Magic: think of a three digit number. Enter a six-digit number into your calculator by repeating your three-digit number twice. For example, if you thought of the three-digit number 275, then enter 275275 into your calculator.

   Done? No matter what number you used to start, the number in your calculator is divisible by 7. Divide by 7. The number in your calculator now is still divisible by 11. Divide it by 11. The number in your calculator is still divisible by 13. Divide it by 13. What do you see? Can you explain it?



Answers

Practice Problems

1. $240 = 2^4 \cdot 3 \cdot 5$ 2. $2016 = 2^5 \cdot 3^2 \cdot 7$ 3. $1001 = 7 \cdot 11 \cdot 13$ 4. yes

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