Part 1 - The Definition

Exponential notation expresses repeated multiplication.

> **Definition**: We define 2^7 to denote the factor 2 multiplied by itself repeatedly 7 times, such as
>
> $$\underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{\text{7 factors}} = 2^7$$
>
> The new operation defined is called **exponentiation.** The factor (in this case 2) is called the **base**. The number written above the base, in smaller font (in this case, 7) is called the **exponent**.

Since the definition does not elegantly fit the case when the exponent is one, we also define 5^1 to be 5. One factor, so technically, no multiplication.

When we enlarge our mathematical notation by the inclusion of exponential expressions, a few things might become problematic. For example, is there a difference between -3^2 and $(-3)^2$?

Recall that a negative sign in front of anything can be interpreted as '*the opposite of*', which is the same as mutliplication by -1. We can interpret -3 as $-1 \cdot 3$, and so we can re-interpret the original question from comparing -3^2 and $(-3)^2$ to a question comparing $-1 \cdot 3^2$ and $(-1 \cdot 3)^2$. The rest is really just an order of operations problem.

Recall that in our order of operations agreement, $\boxed{\begin{array}{c}\text{P}\\ \text{E}\\ \text{M D}\\ \text{A S}\end{array}}$, exponentiation superseeds multiplication. So, when presented by multiplication and exponentiation, we first execute the exponentiation and then the multiplication. And so, if there is no parentheses, we have

$$\begin{array}{ll} -1 \cdot 3^2 & \text{perform exponentiation} \\ -1 \cdot 9 & \text{perform multiplication} \\ -9 & \end{array}$$

And if we have a parentheses, that serves to overwrite the usual order of operations:

$$\begin{array}{ll} (-1 \cdot 3)^2 & \text{whatever is in the parentheses first} \\ (-3)^2 & \text{square the number } -3 \\ 9 & \end{array}$$

The difference between -3^2 and $(-3)^2$ is truly an order of operations thing: we are talking about taking the opposite and squaring, but in different orders.

-3^2 is square 3 and then take the result's opposite —- or the opposite of the square of 3

$(-3)^2$ is take the opposite of 3 and then square —— or the square of the opposite of 3

In algebra, it is important to correctly read notation. Confusing -3^2 and $(-3)^2$ is an error that commonly occurs and messes up computations.

> Caution! In the expression $(-3)^2$, the base of the exponentiation is -3. In the expression -3^2, the base of the exponentiation is 3.

Example 1. Simplify each of the given expressions.

a) -2^4 b) $(-2)^4$ c) -1^3 d) $(-1)^3$ e) $-(-2)^2$ f) $-\left(-2^2\right)$

Solution: a) The base of the exponentiation is 2.

$$-2^4 = -1 \cdot 2^4 = -1 \cdot (2 \cdot 2 \cdot 2 \cdot 2) = -1 \cdot 16 = -16$$

-2^4 can be read as the opposite of 2^4.

b) The base of the exponentiation is -2.

$$(-2)^4 = (-1 \cdot 2)^4 = (-1 \cdot 2)(-1 \cdot 2)(-1 \cdot 2)(-1 \cdot 2) = (-2)(-2)(-2)(-2) = 16$$

$(-2)^4$ can be read as the fourth power of -2.

c) The base of the exponentiation is 1.

$$-1^3 = -1 \cdot 1^3 = -1 \cdot 1 \cdot 1 \cdot 1 = -1$$

d) The base of the exponentiation is -1.

$$(-1)^3 = (-1)(-1)(-1) = -1$$

e) The base of the exponentiation is -2.

$$-(-2)^2 = -((-2)(-2)) = -(4) = -4$$

f) Careful! The base of the exponentiation is 2. This is NOT squaring -2. This is squaring 2 and then taking the opposite of the result twice.

$$-\left(-2^2\right) = -(-1 \cdot 2 \cdot 2) = -(-4) = 4$$

Discussion: Explain why in the expression $-(-5)^2$, the two negatives do not cancel out to a positive.

Part 2 - Rules of Exponents

When mathematicians agreed to define 3^5 as $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$, that was a free choice. They could have gone with other definitions. Once this definition exists, however, certain properties are automatically true, and we have no other option but to recognize them as true. The following statements are straightforward consequences of the definition - and the mathematics we already have.

Consider the expression $2^3 \cdot 2^4$. If we re-write the expression using the definition of exponents, we quickly get that

$$2^3 \cdot 2^4 \overset{\text{def}}{=} (2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2) \underset{\text{associative}}{\overset{\text{mult is}}{=}} 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \overset{\text{def}}{=} 2^7$$

The computation above illustrates why the following theorem is true.

Theorem 1. If a is any number and m and n are any two positive integers, then

$$a^n \cdot a^m = a^{n+m}$$

We can see that this rule follows from the definition of exponents and from the fact that multiplication is associative.

Consider now the expression $\frac{2^5}{2^3}$. If we re-write the expression using the definition of exponents, we quickly get that

$$\frac{2^5}{2^3} \overset{\text{def}}{=} \frac{2\cdot 2\cdot 2\cdot 2\cdot 2}{2\cdot 2\cdot 2} \overset{\text{cancellation}}{=} \frac{\cancel{2}\cdot \cancel{2}\cdot \cancel{2}\cdot 2\cdot 2}{\cancel{2}\cdot \cancel{2}\cdot \cancel{2}} \overset{\text{def}}{=} \frac{2^2}{1} = 2^2$$

> **Theorem 2.** If a is any number and m and n are any two positive integers, then
> $$\frac{a^n}{a^m} = a^{n-m}$$

As our computation shows, this property is a consequence of the definition of exponentials and our rules of cancellation.

Consider now $\left(2^3\right)^5$. If we re-write the expression using the definition of exponents, we quickly get that

$$\left(2^3\right)^5 \overset{\text{def}}{=} \left(2^3\right)\cdot\left(2^3\right)\cdot\left(2^3\right)\cdot\left(2^3\right)\cdot\left(2^3\right) \overset{\text{def}}{=} (2\cdot 2\cdot 2)\cdot(2\cdot 2\cdot 2)\cdot(2\cdot 2\cdot 2)\cdot(2\cdot 2\cdot 2)\cdot(2\cdot 2\cdot 2)$$
$$\underset{\text{associative}}{\overset{\text{mult is}}{=}} 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2 \overset{\text{def}}{=} 2^{15}$$

Clearly, we have five groups of three two-factors.

> **Theorem 3.** If a is any number and m and n are any two positive integers, then
> $$\left(a^n\right)^m = a^{nm}$$

As our computation shows, this property is a consequence of the definition of exponentials and the fact that multiplication is associative.

Consider now $(2\cdot 3)^4$. If we re-write the expression using the definition of exponents, we quickly get that

$$(2\cdot 3)^4 \overset{\text{def}}{=} (2\cdot 3)\cdot(2\cdot 3)\cdot(2\cdot 3)\cdot(2\cdot 3) \underset{\text{associative}}{\overset{\text{mult is}}{=}} 2\cdot 3\cdot 2\cdot 3\cdot 2\cdot 3\cdot 2\cdot 3$$
$$\underset{\text{commutative}}{\overset{\text{mult is}}{=}} 2\cdot 2\cdot 2\cdot 2\cdot 3\cdot 3\cdot 3\cdot 3 \underset{\text{associative}}{\overset{\text{mult is}}{=}} (2\cdot 2\cdot 2\cdot 2)\cdot(3\cdot 3\cdot 3\cdot 3) \overset{\text{def}}{=} 2^4\cdot 3^4$$

> **Theorem 4.** If a and b are any numbers and n is any positive integer, then
> $$(ab)^n = a^n\cdot b^n$$

As our computation shows, this property is a consequence of the definition of exponentials and the fact that multiplication is commutative and associative.

Caution! Exponentiation denotes repeated multiplication, so it is a fundamentally multiplicative concept. Exponents will exhibit nice behvior with respect to multiplication and division, but NOT with respect to addition and subtraction. Similar-looking statements fail to be true if addition or subtraction is involved. For example, $(2\cdot 5)^2 = 2^2\cdot 5^2$, but $(2+5)^2 \neq 2^2+5^2$.

Caution! Another common mistake is to confuse $(ab)^n$ with ab^n. (It is an order of operations thing.) The base of exponentiation is ab in $(ab)^n$ but only b in ab^n.

$$ab^n = a\cdot\underbrace{b\cdot b\cdot \ldots\cdot b}_{n\text{ times}} \quad\text{and}\quad (ab)^n = \underbrace{(ab)\cdot(ab)\cdot\ldots\cdot(ab)}_{n\text{ times}}$$

Consider now the expression $\left(\frac{3}{4}\right)^5$. If we re-write the expression using the definition of exponents, we quickly get that

$$\left(\frac{3}{4}\right)^5 \overset{\text{def}}{=} \left(\frac{3}{4}\right)\cdot\left(\frac{3}{4}\right)\cdot\left(\frac{3}{4}\right)\cdot\left(\frac{3}{4}\right)\cdot\left(\frac{3}{4}\right) \underset{\text{fractions}}{\overset{\text{rules of multipying}}{=}} \frac{3\cdot 3\cdot 3\cdot 3\cdot 3}{4\cdot 4\cdot 4\cdot 4\cdot 4} \overset{\text{def}}{=} \frac{3^5}{4^5}$$

Theorem 5. If a and b are any numbers and n is any positive integer, then

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

As our computation shows, this property is a consequence of the definition of exponentials and the rule of how we multiply fractions.

Caution! Exponentiation denotes repeated multiplication, so it is a fundamentally multiplicative concept. Exponents will exhibit nice behavior with respect to multiplication and division, but NOT with respect to addition and subtraction. Similar-looking statements fail to be true if addition or subtraction is involved. For example, $\left(\frac{2}{5}\right)^2 = \frac{2^2}{5^2}$, but $(5-2)^2 \neq 5^2 - 2^2$.

To summarize what just happened: once we defined exponentiation as repeated multiplication, certain properties immediately followed from the definition. These properties are as follows.

Theorem: If a, b are any numbers and m, n are any positive integers, then

$$\begin{aligned} a^n \cdot a^m &= a^{n+m} \\ \frac{a^n}{a^m} &= a^{n-m} \\ (a^n)^m &= a^{nm} \\ (ab)^n &= a^n b^n \\ \left(\frac{a}{b}\right)^n &= \frac{a^n}{b^n} \end{aligned}$$

Sample Problems

1. Simplify each of the following.

a) $\left(2x^5\right)\left(x^4\right)$

b) $(2x)^5\left(x^4\right)$

c) $\left(2x^5\right)^4$

d) $(-xy)^2\left(-xy^2\right)^3$

e) $-2a^3\left(-2a^4\right)^2$

f) $2a^3\left(-2ab^2\right)^3ab^2$

g) $\dfrac{(-2x)^2y^3}{2x^3y^2}$

h) $\dfrac{(2ab)^3\left(-3a^2b\right)^2}{-b\left(6ab^2\right)^2}$

i) $\left(\dfrac{-2ab}{3b^3}\right)^3\left(\dfrac{6ab^4}{4a^3b}\right)^2$

2. Write each of the following expressions in terms of a fixed number or a single exponential expression.

a) $\dfrac{3^{2x+1}}{9^{x-1}}$ b) $\dfrac{\left(8^{b-2}\right)\left(2^{b+1}\right)}{4^{2b-3}}$ c) $5^{2x-1}\cdot 25^{3-x}$

3. Let us denote 3^{100} by M. Express each of the following in terms of M.

a) 3^{101} b) $3^{100}-2\cdot 3^{101}+3^{102}$ c) 3^{99} d) 9^{100}

Practice Problems

1. Simplify each of the following.

a) -3^2

b) $(-3)^2$

c) $\left(-\dfrac{2}{3}\right)^3$

d) $\left(-\dfrac{1}{2}\right)^4$

e) $\left(2a^2b\right)^3$

f) $\left((2a)^2b\right)^3$

g) $(2a)^2b^3$

h) $\dfrac{m^4m^5}{m^3}$

i) $\left(3p^2q^5\right)\left(2pq^3\right)$

j) $\dfrac{\left(a^2\right)^6a^3}{\left(-a^3\right)^2}$

k) $\dfrac{\left(-5s^3t\right)^4\left(s^2t\right)^3}{\left(10st^3\right)^2}$

l) $\left(-2xy^3\right)^2xy^5x^2$

m) $\dfrac{\left(3ab^2\right)^2\left(-2a^3b\right)^4}{(-2ab)^3}$

n) $\dfrac{\left(-2x^2y^3\right)^4xy^3\left(2x^2y\right)^2}{(2x)^2y^9\left(2x^2y\right)^4}$

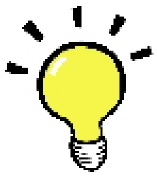
o) $\left(\dfrac{2a^3b}{-3ab^2}\right)^2\left(\dfrac{3ab}{6b^2}\right)^3$

2. Write each of the following expressions in terms of a fixed number or a single exponential expression.

a) $\dfrac{2^{2x-1}}{4^{x-2}}$ b) $\dfrac{100^{x+1}}{2^{2x+1}\cdot 5^{x-1}}$ c) $\dfrac{9^{x-1}\cdot 4^{x+2}}{6^{2x+1}}$ d) $\dfrac{4^{x-1}\cdot 5^{x+2}}{10^{x-1}}$

3. Let P denote 5^{2015}. Express each of the following in terms of P.

a) 5^{2016} b) 5^{2017} c) 5^{2014} d) 25^{2015} e) $5^{2015}-3\cdot 5^{2016}+5^{2017}$



Answers

Discussion

Two minus signs still cancel out each other, it is just that there are really three negative signs in the expression $-(-5)^2 = -(-5)(-5)$.

Sample Problems

1. a) $2x^9$ b) $32x^9$ c) $16x^{20}$ d) $-x^5y^8$ e) $-8a^{11}$ f) $-16a^7b^8$ g) $\dfrac{2y}{x}$ h) $-2a^5$ i) $-\dfrac{2}{3a}$

2. a) 27 b) 2 c) 3125

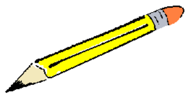
3. a) $3M$ b) $4M$ c) $\dfrac{M}{3}$ d) M^2

Practice Problems

1. a) -9 b) 9 c) $-\dfrac{8}{27}$ d) $\dfrac{1}{16}$ e) $8a^6b^3$ f) $64a^6b^3$ g) $4a^2b^3$ h) m^6 i) $6p^3q^8$ j) a^9 k) $\dfrac{25s^{16}t}{4}$ l) $4x^5y^{11}$ m) $-18a^{11}b^5$ n) x^3y^4 o) $\dfrac{a^7}{18b^5}$

2. a) 8 b) $250 \cdot 5^x$ c) $\dfrac{8}{27}$ d) $\dfrac{125}{2} \cdot 2^x$

3. a) $5P$ b) $25P$ c) $\dfrac{P}{5}$ d) P^2 e) $11P$



Sample Problems - Solutions

Let us recall the rules of exponents.

$$
\begin{array}{lrcl}
1) & a^n \cdot a^m & = & a^{n+m} \\
2) & \dfrac{a^n}{a^m} & = & a^{n-m} \\
3) & \left(a^n\right)^m & = & a^{nm} \\
4) & (ab)^n & = & a^n b^n \\
5) & \left(\dfrac{a}{b}\right)^n & = & \dfrac{a^n}{b^n}
\end{array}
$$

1. Simplify each of the following.

a) $\left(2x^5\right)\left(x^4\right)$
Solution: $\left(2x^5\right)\left(x^4\right) = 2x^5x^4 = 2x^{5+4} = 2x^9$ by rule 1.

b) $(2x)^5\left(x^4\right)$
Solution:

$$
\begin{array}{rcll}
(2x)^5\left(x^4\right) & = & 2^5x^5x^4 & \text{by rule 4} \\
 & = & 32x^{5+4} & \text{by rule 1} \\
 & = & 32x^9 &
\end{array}
$$

c) $\left(2x^5\right)^4$
Solution:

$$
\begin{array}{rcll}
\left(2x^5\right)^4 & = & 2^4\left(x^5\right)^4 & \text{by rule 4} \\
 & = & 16x^{20} & \text{by rule 3}
\end{array}
$$

d) $(-xy)^2\left(-xy^2\right)^3$
Solution:

$$
\begin{array}{rcll}
(-xy)^2\left(-xy^2\right)^3 & = & (-1xy)^2\left(-1xy^2\right)^3 & \text{the 1s will help with signs} \\
 & = & (-1)^2x^2y^2(-1)^3x^3\left(y^2\right)^3 & \text{by rule 4} \\
 & = & 1\cdot x^2y^2(-1)x^3y^6 & \text{by rule 3} \\
 & = & 1(-1)x^2x^3y^2y^6 & \text{multiplication is commutative} \\
 & = & -1\,x^{2+3}y^{2+6} & \text{by rule 1} \\
 & = & -x^5y^8 &
\end{array}
$$

e) $-2a^3\left(-2a^4\right)^2$
Solution:

$$
\begin{array}{rcll}
-2a^3\left(-2a^4\right)^2 & = & -2a^3(-2)^2\left(a^4\right)^2 & \text{rule 4} \\
 & = & -2a^3(4)a^8 & \text{rule 3} \\
 & = & -2(4)a^3a^8 & \text{multiplication is commutative} \\
 & = & -8a^{3+8} = -8a^{11} & \text{rule 1}
\end{array}
$$

f) $2a^3\left(-2ab^2\right)^3 ab^2$
Solution:

$$\begin{aligned}
2a^3\left(-2ab^2\right)^3 ab^2 &= 2a^3(-2)^3 a^3\left(b^2\right)^3 ab^2 && \text{rule 4}\\
&= 2a^3(-8)\,a^3b^6ab^2 && \text{rule 3}\\
&= 2(-8)\,a^3a^3ab^6b^2 && \text{multiplication is commutative}\\
&= -16a^{3+3+1}b^{6+2} = -16a^7b^8 && \text{rule 1}
\end{aligned}$$

g) $\dfrac{(-2x)^2 y^3}{2x^3y^2}$
Solution:

$$\begin{aligned}
\frac{(-2x)^2 y^3}{2x^3y^2} &= \frac{(-2)^2 x^2y^3}{2x^3y^2} && \text{rule 4}\\
&= \frac{4x^2y^3}{2x^3y^2} = \frac{2x^2y^3}{x^3y^2} && \text{cancel out } x^2y^2\\
&= \frac{2y}{x}
\end{aligned}$$

h) $\dfrac{(2ab)^3\left(-3a^2b\right)^2}{-b\left(6ab^2\right)^2}$
Solution:

$$\begin{aligned}
\frac{(2ab)^3\left(-3a^2b\right)^2}{-b\left(6ab^2\right)^2} &= \frac{(2ab)^3\left(-3a^2b\right)^2}{-1b\left(6ab^2\right)^2} && \text{the 1 will help with signs}\\
&= \frac{2^3a^3b^3(-3)^2\left(a^2\right)^2b^2}{-1\cdot b\cdot 6^2\cdot a^2\left(b^2\right)^2} && \text{by rule 4}\\
&= \frac{8a^3b^3\cdot 9\cdot a^4b^2}{-1\cdot b\cdot 36\cdot a^2b^4} && \text{by rule 3}\\
&= \frac{8\cdot 9\cdot a^3a^4b^3b^2}{-1\cdot 36\cdot a^2\cdot b\cdot b^4} && \text{multiplication is commutative}\\
&= \frac{72a^7b^5}{-36a^2b^5} && \text{by rule 1}\\
&= \frac{-2a^7b^5}{a^2b^5} && \text{simplify numbers: } \frac{72}{-36} = \frac{-72}{36} = \frac{-2}{1}\\
&= \frac{-2a^7}{a^2} && \text{cancel out } b^5\\
&= \frac{-2a^7}{a^2} && \text{cancel out } a^2 \text{ (or use rule 2)}\\
&= -2a^5
\end{aligned}$$

i) $\left(\dfrac{-2ab}{3b^3}\right)^3\left(\dfrac{6ab^4}{4a^3b}\right)^2$

Solution: We first simplify each expression within the parentheses.

$$\begin{aligned}
\left(\frac{-2ab}{3b^3}\right)^3\left(\frac{6ab^4}{4a^3b}\right)^2 &= \left(\frac{-2a}{3b^2}\right)^3\left(\frac{3b^3}{2a^2}\right)^2 && \text{rule 5}\\
&= \frac{(-2a)^3}{(3b^2)^3}\cdot\frac{(3b^3)^2}{(2a^2)^2} && \text{rule 4}\\
&= \frac{(-2)^3(a)^3}{3^3(b^2)^3}\cdot\frac{3^2(b^3)^2}{2^2(a^2)^2} && \text{rule 3}\\
&= \frac{-8a^3}{27b^6}\cdot\frac{9b^6}{4a^4} && \text{multiplication of fractions}\\
&= \frac{-8a^3(9b^6)}{27b^6(4a^4)} = \frac{-8\cdot 9a^3b^6}{4\cdot 27a^4b^6} && \text{cancel}\\
&= \frac{-2a^3b^6}{3a^4b^6} = \frac{-2}{3a}
\end{aligned}$$

2. Write each of the following expressions in terms of a fixed number or a single exponential expression.

a) $\dfrac{3^{2x+1}}{9^{x-1}}$

Solution 1:

$$\frac{3^{2x+1}}{9^{x-1}} = \frac{3^{2x+1}}{(3^2)^{x-1}} \overset{\text{Rule 3}}{=} \frac{3^{2x+1}}{3^{2(x-1)}} = \frac{3^{2x+1}}{3^{2x-2}} \overset{\text{Rule 2}}{=} 3^{2x+1-(2x-2)} = 3^{2x+1-2x+2} = 3^3 = 27$$

Solution 2:

$$\frac{3^{2x+1}}{9^{x-1}} \overset{\text{Rules 1,2}}{=} \frac{3^{2x}\cdot 3^1}{\dfrac{9^x}{9^1}} = \frac{3^{2x}\cdot 3}{9^x\cdot\dfrac{1}{9}} = \frac{3^{2x}\cdot 3}{9^x}\cdot\frac{9}{1} = \frac{27\cdot 3^{2x}}{9^x} \overset{\text{Rule 3}}{=} \frac{27\cdot(3^2)^x}{9^x} = \frac{27\cdot 9^x}{9^x} = 27$$

b) $\dfrac{(8^{b-2})(2^{b+1})}{4^{2b-3}}$

$$\begin{aligned}
&= \frac{(8^{b-2})(2^{b+1})}{4^{2b-3}} \overset{\text{Rules 1,2}}{=} \frac{\dfrac{8^b}{8^2}\cdot 2^b\cdot 2^1}{\dfrac{4^{2b}}{4^3}} \overset{\text{Rule 3}}{=} \frac{\dfrac{8^b\cdot 2^b\cdot 2}{64}}{\dfrac{(4^2)^b}{64}} && \text{to divide is to multiply by reciprocal}\\
&= \frac{8^b\cdot 2^b\cdot 2}{64}\cdot\frac{64}{(4^2)^b} \overset{\text{Rule 4}}{=} \frac{(8\cdot 2)^b\cdot 2}{16^b} = \frac{16^b\cdot 2}{16^b} = 2
\end{aligned}$$

c) $5^{2x-1}\cdot 25^{3-x}$

$$5^{2x-1}\cdot 25^{3-x} = 5^{2x-1}\cdot(5^2)^{3-x} \overset{\text{Rule 3}}{=} 5^{2x-1}\cdot 5^{2(3-x)} = 5^{2x-1}\cdot 5^{6-2x} = \overset{\text{Rule 1}}{=} 5^{2x-1+6-2x} = 5^5 = 3125$$

3. Let us denote 3^{100} by M. Express each of the following in terms of M.

a) 3^{101}

Solution: Using rule 1, we write $3^{101} = 3^{100+1} = 3^{100} \cdot 3^1 = M \cdot 3 = 3M$

b) $3^{100} - 2 \cdot 3^{101} + 3^{102}$

Solution: Using rule 1, we re-write 3^{101} and 3^{102}

$$\begin{aligned} 3^{101} &= 3^{100+1} = 3^{100} \cdot 3^1 = M \cdot 3 = 3M \\ 3^{102} &= 3^{100+2} = 3^{100} \cdot 3^2 = M \cdot 9 = 9M \end{aligned}$$

Then our expression becomes

$$3^{100} - 2 \cdot 3^{101} + 3^{102} = M - 2 \cdot (3M) + 9M = M - 6M + 9M = -5M + 9M = 4M$$

c) 3^{99}

Solution: Using rule 2, we write $3^{99} = 3^{100-1} = \dfrac{3^{100}}{3^1} = \dfrac{M}{3}$

d) 9^{100}

Solution: This time we will use rule 3 in a novel way: $(a^n)^m = (a^m)^n$

$$9^{100} = \left(3^2\right)^{100} = \left(3^{100}\right)^2 = M^2$$

We can also solve this problem using rule 4

$$9^{100} = (3 \cdot 3)^{100} = 3^{100} \cdot 3^{100} = M \cdot M = M^2$$

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