Sample Problems

1. Simplify each of the following.

a) $\left(2x^5\right)\left(x^4\right)$

b) $(2x)^5\left(x^4\right)$

c) $\left(2x^5\right)^4$

d) $(-xy)^2\left(-xy^2\right)^3$

e) $-2a^3\left(-2a^4\right)^2$

f) $2a^3\left(-2ab^2\right)^3 ab^2$

g) $\dfrac{(-2x)^2 y^3}{2x^3y^2}$

h) $\dfrac{(2ab)^3\left(-3a^2b\right)^2}{-b\left(6ab^2\right)^2}$

i) $\left(\dfrac{-2ab}{3b^3}\right)^3\left(\dfrac{6ab^4}{4a^3b}\right)^2$

2. Write each of the following expressions in terms of a fixed number or a single exponential expression.

a) $\dfrac{3^{2x+1}}{9^{x-1}}$ b) $\dfrac{\left(8^{b-2}\right)\left(2^{b+1}\right)}{4^{2b-3}}$ c) $5^{2x-1}\cdot 25^{3-x}$

3. Let us denote 3^{100} by M. Express each of the following in terms of M.

a) 3^{101} b) $3^{100}-2\cdot 3^{101}+3^{102}$ c) 3^{99} d) 9^{100}

Practice Problems

1. Simplify each of the following.

a) -3^2

b) $(-3)^2$

c) $\left(-\dfrac{2}{3}\right)^3$

d) $\left(-\dfrac{1}{2}\right)^4$

e) $\left(2a^2b\right)^3$

f) $\left((2a)^2 b\right)^3$

g) $(2a)^2 b^3$

h) $\dfrac{m^4m^5}{m^3}$

i) $\left(3p^2q^5\right)\left(2pq^3\right)$

j) $\dfrac{\left(a^2\right)^6 a^3}{\left(-a^3\right)^2}$

k) $\dfrac{\left(-5s^3t\right)^4\left(s^2t\right)^3}{\left(10st^3\right)^2}$

l) $\left(-2xy^3\right)^2 xy^5x^2$

m) $\dfrac{\left(3ab^2\right)^2\left(-2a^3b\right)^4}{(-2ab)^3}$

n) $\dfrac{\left(-2x^2y^3\right)^4 xy^3\left(2x^2y\right)^2}{(2x)^2 y^9\left(2x^2y\right)^4}$

o) $\left(\dfrac{2a^3b}{-3ab^2}\right)^2\left(\dfrac{3ab}{6b^2}\right)^3$

2. Write each of the following expressions in terms of a fixed number or a single exponential expression.

a) $\dfrac{2^{2x-1}}{4^{x-2}}$ b) $\dfrac{100^{x+1}}{2^{2x+1}\cdot 5^{x-1}}$ c) $\dfrac{9^{x-1}\cdot 4^{x+2}}{6^{2x+1}}$ d) $\dfrac{4^{x-1}\cdot 5^{x+2}}{10^{x-1}}$

3. Let P denote 5^{2015}. Express each of the following in terms of P.

a) 5^{2016} b) 5^{2017} c) 5^{2014} d) 25^{2015} e) $5^{2015}-3\cdot 5^{2016}+5^{2017}$

Sample Problems - Answers

1. a) $2x^9$ b) $32x^9$ c) $16x^{20}$ d) $-x^5y^8$ e) $-8a^{11}$ f) $-16a^7b^8$ g) $\dfrac{2y}{x}$ h) $-2a^5$ i) $-\dfrac{2}{3a}$

2. a) 27 b) 2 c) 3125 3. a) $3M$ b) $4M$ c) $\dfrac{M}{3}$ d) M^2

Practice Problems - Answers

1. a) -9 b) 9 c) $-\dfrac{8}{27}$ d) $\dfrac{1}{16}$ e) $8a^6b^3$ f) $64a^6b^3$ g) $4a^2b^3$ h) m^6

i) $6p^3q^8$ j) a^9 k) $\dfrac{25s^{16}t}{4}$ l) $4x^5y^{11}$ m) $-18a^{11}b^5$ n) x^3y^4 o) $\dfrac{a^7}{18b^5}$

2. a) 8 b) $250 \cdot 5^x$ c) $\dfrac{8}{27}$ d) $\dfrac{125}{2} \cdot 2^x$

3. a) $5P$ b) $25P$ c) $\dfrac{P}{5}$ d) P^2 e) $11P$

Sample Problems - Solutions

Let us recall the rules of exponents.

$$\begin{array}{rrcl}
1) & a^n \cdot a^m & = & a^{n+m} \\
2) & \dfrac{a^n}{a^m} & = & a^{n-m} \\
3) & \left(a^n\right)^m & = & a^{nm} \\
4) & (ab)^n & = & a^n b^n \\
5) & \left(\dfrac{a}{b}\right)^n & = & \dfrac{a^n}{b^n}
\end{array}$$

1. Simplify each of the following.

a) $\left(2x^5\right)\left(x^4\right)$
Solution: $\left(2x^5\right)\left(x^4\right) = 2x^5x^4 = 2x^{5+4} = 2x^9$ by rule 1.

b) $(2x)^5\left(x^4\right)$
Solution:

$$\begin{array}{rcll}
(2x)^5\left(x^4\right) & = & 2^5x^5x^4 & \text{by rule 4} \\
& = & 32x^{5+4} & \text{by rule 1} \\
& = & 32x^9 &
\end{array}$$

c) $\left(2x^5\right)^4$
Solution:

$$\begin{array}{rcll}
\left(2x^5\right)^4 & = & 2^4\left(x^5\right)^4 & \text{by rule 4} \\
& = & 16x^{20} & \text{by rule 3}
\end{array}$$

d) $(-xy)^2\left(-xy^2\right)^3$
Solution:

$$\begin{array}{rcll}
(-xy)^2\left(-xy^2\right)^3 & = & (-1xy)^2\left(-1xy^2\right)^3 & \text{the 1's will help with signs} \\
& = & (-1)^2x^2y^2(-1)^3x^3\left(y^2\right)^3 & \text{by rule 4} \\
& = & 1\cdot x^2y^2(-1)x^3y^6 & \text{by rule 3} \\
& = & 1(-1)x^2x^3y^2y^6 & \text{multiplication is commutative} \\
& = & -1x^{2+3}y^{2+6} & \text{by rule 1} \\
& = & -x^5y^8 &
\end{array}$$

e) $-2a^3\left(-2a^4\right)^2$
Solution:

$$\begin{array}{rcll}
-2a^3\left(-2a^4\right)^2 & = & -2a^3\left(-2a^4\right)^2 & \text{rule 4} \\
& = & -2a^3(-2)^2\left(a^4\right)^2 & \text{rule 3} \\
& = & -2a^3(4)a^8 & \text{multiplication is commutative} \\
& = & -2(4)a^3a^8 & \text{rule 1} \\
& = & -8a^{3+8} = -8a^{11} &
\end{array}$$

f) $2a^3\left(-2ab^2\right)^3ab^2$
Solution:

$$\begin{aligned}
2a^3\left(-2ab^2\right)^3ab^2 &= 2a^3\left(-2ab^2\right)^3ab^2 && \text{rule 4}\\
&= 2a^3\left(-2\right)^3a^3\left(b^2\right)^3ab^2 && \text{rule 3}\\
&= 2a^3\left(-8\right)a^3b^6ab^2 && \text{multiplication is commutative}\\
&= 2\left(-8\right)a^3a^3ab^6b^2 && \text{rule 1}\\
&= -16a^{3+3+1}b^{6+2} = -16a^7b^8
\end{aligned}$$

g) $\dfrac{\left(-2x\right)^2y^3}{2x^3y^2}$
Solution:

$$\begin{aligned}
\frac{\left(-2x\right)^2y^3}{2x^3y^2} &= \frac{\left(-2\right)^2x^2y^3}{2x^3y^2} && \text{rule 4}\\
&= \frac{4x^2y^3}{2x^3y^2} = \frac{2x^2y^3}{x^3y^2} && \text{cancel out } x^2y^2\\
&= \frac{2y}{x}
\end{aligned}$$

h) $\dfrac{\left(2ab\right)^3\left(-3a^2b\right)^2}{-b\left(6ab^2\right)^2}$
Solution:

$$\begin{aligned}
\frac{\left(2ab\right)^3\left(-3a^2b\right)^2}{-b\left(6ab^2\right)^2} &= \frac{\left(2ab\right)^3\left(-3a^2b\right)^2}{-1b\left(6ab^2\right)^2} && \text{the 1 will help with signs}\\
&= \frac{2^3a^3b^3\left(-3\right)^2\left(a^2\right)^2b^2}{-1\cdot b\cdot 6^2\cdot a^2\left(b^2\right)^2} && \text{by rule 4}\\
&= \frac{8a^3b^3\cdot 9\cdot a^4b^2}{-1\cdot b\cdot 36\cdot a^2b^4} && \text{by rule 3}\\
&= \frac{8\cdot 9\cdot a^3a^4b^3b^2}{-1\cdot 36\cdot a^2\cdot b\cdot b^4} && \text{multiplication is commutative}\\
&= \frac{72a^7b^5}{-36a^2b^5} && \text{by rule 1}\\
&= \frac{-2a^7b^5}{a^2b^5} && \text{simplify numbers: } \frac{72}{-36} = \frac{-72}{36} = \frac{-2}{1}\\
&= \frac{-2a^7}{a^2} && \text{cancel out } a^2 \text{ (or use rule 2)}\\
&= -2a^5
\end{aligned}$$

i) $\left(\dfrac{-2ab}{3b^3}\right)^3\left(\dfrac{6ab^4}{4a^3b}\right)^2$
Solution: We first simplify each expression within the parentheses.

$$\begin{aligned}
\left(\frac{-2ab}{3b^3}\right)^3\left(\frac{6ab^4}{4a^3b}\right)^2 &= \left(\frac{-2a}{3b^2}\right)^3\left(\frac{3b^3}{2a^2}\right)^2 && \text{rule 5}\\
&= \frac{(-2a)^3}{(3b^2)^3}\cdot\frac{(3b^3)^2}{(2a^2)^2} && \text{rule 4}\\
&= \frac{(-2)^3(a)^3}{3^3(b^2)^3}\cdot\frac{3^2(b^3)^2}{2^2(a^2)^2} && \text{rule 3}\\
&= \frac{-8a^3}{27b^6}\cdot\frac{9b^6}{4a^4} && \text{multiplication of fractions}\\
&= \frac{-8a^3(9b^6)}{27b^6(4a^4)} = \frac{-8\cdot 9a^3b^6}{4\cdot 27a^4b^6} && \text{cancel}\\
&= \frac{-2a^3b^6}{3a^4b^6} = \frac{-2}{3a}
\end{aligned}$$

2. Write each of the following expressions in terms of a fixed number or a single exponential expression.

a) $\dfrac{3^{2x+1}}{9^{x-1}}$
Solution 1:

$$\frac{3^{2x+1}}{9^{x-1}} = \frac{3^{2x+1}}{(3^2)^{x-1}} \overset{\text{Rule 3}}{=} \frac{3^{2x+1}}{3^{2(x-1)}} = \frac{3^{2x+1}}{3^{2x-2}} \overset{\text{Rules 1,2}}{=} 3^{2x+1-(2x-2)} = 3^{2x+1-2x+2} = 3^3 = 27$$

Solution 2:

$$\frac{3^{2x+1}}{9^{x-1}} \overset{\text{Rules 1,2}}{=} \frac{3^{2x}\cdot 3^1}{\dfrac{9^x}{9^1}} = \frac{3^{2x}\cdot 3}{9^x\cdot\dfrac{1}{9}} = \frac{3^{2x}\cdot 3}{9^x}\cdot\frac{9}{1} = \frac{27\cdot 3^{2x}}{9^x} \overset{\text{Rule 3}}{=} \frac{27\cdot(3^2)^x}{9^x} = \frac{27\cdot 9^x}{9^x} = 27$$

b) $\dfrac{(8^{b-2})(2^{b+1})}{4^{2b-3}}$

$$= \frac{(8^{b-2})(2^{b+1})}{4^{2b-3}} \overset{\text{Rules 1,2}}{=} \frac{\dfrac{8^b}{8^2}\cdot 2^b\cdot 2^1}{\dfrac{4^{2b}}{4^3}} \overset{\text{Rule 3}}{=} \frac{\dfrac{8^b\cdot 2^b\cdot 2}{64}}{\dfrac{(4^2)^b}{64}} \qquad \text{to divide is to multiply by reciprocal}$$

$$= \frac{8^b\cdot 2^b\cdot 2}{64}\cdot\frac{64}{(4^2)^b} \overset{\text{Rule 4}}{=} \frac{(8\cdot 2)^b\cdot 2}{16^b} = \frac{16^b\cdot 2}{16^b} = 2$$

c) $5^{2x-1}\cdot 25^{3-x}$

$$5^{2x-1}\cdot 25^{3-x} = 5^{2x-1}\cdot(5^2)^{3-x} \overset{\text{Rule 3}}{=} 5^{2x-1}\cdot 5^{2(3-x)} = 5^{2x-1}\cdot 5^{6-2x} = \overset{\text{Rule 1}}{=} 5^{2x-1+6-2x} = 5^5 = 3125$$

3. Let us denote 3^{100} by M. Express each of the following in terms of M.

a) 3^{101}

Solution: Using rule 1, we write $3^{101} = 3^{100+1} = 3^{100} \cdot 3^1 = M \cdot 3 = 3M$

b) $3^{100} - 2 \cdot 3^{101} + 3^{102}$

Solution: Using rule 1, we re-write 3^{101} and 3^{102}

$$\begin{aligned} 3^{101} &= 3^{100+1} = 3^{100} \cdot 3^1 = M \cdot 3 = 3M \\ 3^{102} &= 3^{100+2} = 3^{100} \cdot 3^2 = M \cdot 9 = 9M \end{aligned}$$

Then our expression becomes

$$3^{100} - 2 \cdot 3^{101} + 3^{102} = M - 2 \cdot (3M) + 9M = M - 6M + 9M = -5M + 9M = 4M$$

c) 3^{99}

Solution: Using rule 2, we write $3^{99} = 3^{100-1} = \dfrac{3^{100}}{3^1} = \dfrac{M}{3}$

d) 9^{100}

Solution: This time we will use rule 3 in a novel way: $(a^n)^m = (a^m)^n$

$$9^{100} = \left(3^2\right)^{100} = \left(3^{100}\right)^2 = M^2$$

We can also solve this problem using rule 4

$$9^{100} = (3 \cdot 3)^{100} = 3^{100} \cdot 3^{100} = M \cdot M = M^2$$

