Simplify each of the following. Assume that all variables represent positive numbers. Present your answer without negative exponents.

1. 3^{-2}
2. $\dfrac{1}{2^{-3}}$
3. m^{-4}
4. $\dfrac{1}{x^{-5}}$
5. $a^8 \cdot a^{-1}$
6. $p^3\left(p^{-7}\right)p^8$
7. $\dfrac{x^{-4}}{x^{-9}}$
8. $\dfrac{50a^{12}}{10a^{-3}}$
9. $\dfrac{t^{-3}}{t^4}$
10. x^0
11. $-x^0$
12. $(-x)^0$
13. $\left(b^{-5}\right)\left(b^2\right)\left(b^{-1}\right)$
14. $\dfrac{1}{\left(b^{-5}\right)\left(b^2\right)\left(b^{-1}\right)}$
15. $\dfrac{m^{-2}}{m^{-5}}$
16. $\dfrac{x^3y^{-5}}{z^{-4}}$
17. $\dfrac{18q^3}{6q^{-3}}$
18. $\left(\dfrac{2}{3}\right)^{-3}$
19. $2y^{-3}$
20. $(2y)^{-3}$
21. $\left(-\dfrac{3}{5}\right)^{-2}$
22. $\dfrac{a^3b^{-5}}{a^{-2}b^3}$
23. $\left(3m^3\right)^{-2}$
24. $\left(-2ab^{-3}\right)^{-3}$
25. $\dfrac{\left(k^3\right)^{-3}}{\left(k^{-5}\right)^2}$
26. $\left(\dfrac{2a^{-3}b^5}{-3a^3b^{-2}}\right)^{-2}\left(a^3b^{-5}\right)^{-3}$
27. $\left(-2a^{-3}\right)\left(-2a^{-2}b\right)^{-4}$
28. $\dfrac{\left(-3p^3q^5\right)^2}{\left(2q^0p^3\right)^{-1}}$
29. $\left(\dfrac{2a^{-2}b^3}{-2^2\left(a^{-1}b\right)^{-3}}\right)^{-2}$
30. $\left(-\dfrac{x^3y^0x^{-5}}{y^{-3}}\right)^{-2}$
31. $\left(-\dfrac{x^3y^7x^{-5}}{y^{-3}}\right)^{0}$
32. $\dfrac{x^{-1}+y^{-1}}{x^{-2}-y^{-2}}$
33. $\dfrac{\left(-2a^{-2}\right)^{-2}b^3a^0\left(-aba^{-2}b^{-2}\right)^{-3}}{2a^2\left(-2a^{-2}b\right)^{-2}ab^0}$
34. $\left(\dfrac{-a^2\left(b^{-1}a\right)^{-5}}{b^7\left(-ab^2\right)^{-3}}\right)^{-2}$
35. $\dfrac{\left(x^{-2}\right)^{-2}y^3x^0\left(-2yx^0y^{-2}x^{-2}\right)^0}{yx^5\left(y^{-2}x\right)^{-3}\left(2x^{-1}yx^3\right)^{-1}}$

Answers

1. $\frac{1}{9}$
2. 8
3. $\frac{1}{m^4}$
4. x^5
5. a^7
6. p^4
7. x^5
8. $5a^{15}$
9. $\frac{1}{t^7}$
10. 1
11. -1
12. 1
13. $\frac{1}{b^4}$
14. b^4
15. m^3
16. $\frac{x^3 z^4}{y^5}$
17. $3q^6$
18. $\frac{27}{8}$
19. $\frac{2}{y^3}$
20. $\frac{1}{8y^3}$
21. $\frac{25}{9}$
22. $\frac{a^5}{b^8}$
23. $\frac{1}{9m^6}$
24. $-\frac{b^9}{8a^3}$
25. k
26. $\frac{9}{4}a^3 b$
27. $-\frac{a^5}{8b^4}$
28. $18p^9 q^{10}$
29. $\frac{4a^{10}}{b^{12}}$
30. $\frac{x^4}{y^6}$
31. 1
32. $\frac{xy}{y-x}$
33. $-\frac{b^8}{2}$
34. $\frac{1}{b^8}$
35. $\frac{2x^4}{y^3}$

Solutions

Simplify each of the following. Assume that all variables represent positive numbers. Present your answer without negative exponents.

1. 3^{-2}

   Solution: We just apply the rule $a^{-n} = \dfrac{1}{a^n}$.

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

2. $\dfrac{1}{2^{-3}}$

   Solution: We apply the rule $a^{-n} = \dfrac{1}{a^n}$.

$$\frac{1}{2^{-3}} = \frac{1}{\frac{1}{2^3}} = \frac{1}{\frac{1}{8}}$$

   To divide is to multiply by the reciprocal:

$$\frac{1}{\frac{1}{8}} = 1 \cdot \frac{8}{1} = 8$$

   This is true in general: $\dfrac{1}{a^{-n}} = a^n$

$$\frac{1}{a^{-n}} = \frac{1}{\frac{1}{a^n}} = 1 \cdot \frac{a^n}{1} = a^n$$

3. m^{-4}

   Solution: We apply the rule $a^{-n} = \dfrac{1}{a^n}$.

$$m^{-4} = \frac{1}{m^4}$$

4. $\dfrac{1}{x^{-5}}$

   Solution: We have already proven that $\dfrac{1}{a^{-n}} = a^n$

$$\frac{1}{x^{-5}} = x^5$$

5. $a^8 \cdot a^{-1}$

   Solution 1: We can apply the rule $a^n \cdot a^m = a^{n+m}$

$$a^8 \cdot a^{-1} = a^{8+(-1)} = a^7$$

   Solution 2: We can apply the rule $a^{-n} = \dfrac{1}{a^n}$ and then the rule $\dfrac{a^n}{a^m} = a^{n-m}$.

$$a^8 \cdot a^{-1} = a^8 \cdot \frac{1}{a^1} = \frac{a^8}{1} \cdot \frac{1}{a} = \frac{a^8}{a} = \frac{a^8}{a^1} = a^{8-1} = a^7$$

6. $p^3\left(p^{-7}\right)p^8$

Solution 1: We can apply the rule $a^n \cdot a^m = a^{n+m}$

$$p^3\left(p^{-7}\right)p^8 = p^{3+(-7)+8} = p^4$$

Solution 2: We can apply the rules $a^{-n} = \dfrac{1}{a^n}$ and $a^n \cdot a^m = a^{n+m}$ and $\dfrac{a^n}{a^m} = a^{n-m}$.

$$p^3\left(p^{-7}\right)p^8 = p^3 \cdot \frac{1}{p^7} \cdot p^8 = \frac{p^3}{1} \cdot \frac{1}{p^7} \cdot \frac{p^8}{1} = \frac{p^3 \cdot p^8}{p^7} = \frac{p^{3+8}}{p^7} = \frac{p^{11}}{p^7} = p^{11-7} = p^4$$

7. $\dfrac{x^{-4}}{x^{-9}}$

Solution 1: We can apply the rule $\dfrac{a^n}{a^m} = a^{n-m}$.

$$\frac{x^{-4}}{x^{-9}} = x^{-4-(-9)} = x^{-4+9} = x^5$$

Solution 2: We can apply the rules $a^{-n} = \dfrac{1}{a^n}$ and $\dfrac{a^n}{a^m} = a^{n-m}$.

$$\frac{x^{-4}}{x^{-9}} = \frac{x^9}{x^4} = x^{9-4} = x^5$$

8. $\dfrac{50a^{12}}{10a^{-3}}$

Solution 1: We can apply the rule $\dfrac{a^n}{a^m} = a^{n-m}$.

$$\frac{50a^{12}}{10a^{-3}} = 5a^{12-(-3)} = 5a^{12+3} = 5a^{15}$$

Solution 2: We can apply the rules $a^{-n} = \dfrac{1}{a^n}$ and $\dfrac{a^n}{a^m} = a^{n-m}$.

$$\frac{50a^{12}}{10a^{-3}} = \frac{50a^{12}a^3}{10} = 5a^{12+3} = 5a^{15}$$

9. $\dfrac{t^{-3}}{t^4}$

Solution 1: We can apply the rules $\dfrac{a^n}{a^m} = a^{n-m}$ and then $a^{-n} = \dfrac{1}{a^n}$.

$$\frac{t^{-3}}{t^4} = t^{-3-4} = t^{-7} = \frac{1}{t^7}$$

Solution 2: We can apply the rule $a^{-n} = \dfrac{1}{a^n}$ and then $a^n \cdot a^m = a^{n+m}$.

$$\frac{t^{-3}}{t^4} = \frac{1}{t^4 \cdot t^3} = \frac{1}{t^7}$$

10. x^0

Solution: There is a separate rule stating that as long as x is not zero, then $x^0 = 1$. So the answer is 1.

11. $-x^0$

Solution: This is the opposite of x^0 and so the answer is -1.

$$-x^0 = -1 \cdot x^0 = -1 \cdot 1 = -1$$

12. $(-x)^0$

Solution: This is again 1 because any non-zero riased to the power zero is 1.

13. $\left(b^{-5}\right)\left(b^2\right)\left(b^{-1}\right)$

Solution 1: We can apply the rules $a^n \cdot a^m = a^{n+m}$ and then $a^{-n} = \dfrac{1}{a^n}$.

$$\left(b^{-5}\right)\left(b^2\right)\left(b^{-1}\right) = b^{-5+2+(-1)} = b^{-4} = \frac{1}{b^4}$$

Solution 2: We can apply the rule $a^{-n} = \dfrac{1}{a^n}$ and then just cancel.

$$\left(b^{-5}\right)\left(b^2\right)\left(b^{-1}\right) = \frac{1}{b^5} \cdot b^2 \cdot \frac{1}{b^1} = \frac{1}{b^5} \cdot \frac{b^2}{1} \cdot \frac{1}{b^1} = \frac{b^2}{b^6} = \frac{\cancel{b} \cdot \cancel{b}}{\cancel{b} \cdot \cancel{b} \cdot b \cdot b \cdot b \cdot b} = \frac{1}{b^4}$$

14. $\dfrac{1}{\left(b^{-5}\right)\left(b^2\right)\left(b^{-1}\right)}$

Solution 1: We can apply the rules $a^n \cdot a^m = a^{n+m}$ and then $a^{-n} = \dfrac{1}{a^n}$.

$$\frac{1}{\left(b^{-5}\right)\left(b^2\right)\left(b^{-1}\right)} = \frac{1}{b^{-5+2+(-1)}} = \frac{1}{b^{-4}} = \frac{1}{\dfrac{1}{b^4}} = 1 \cdot \frac{b^4}{1} = b^4$$

Solution 2: We can apply the rule $a^{-n} = \dfrac{1}{a^n}$ and then $\dfrac{a^n}{a^m} = a^{n-m}$.

$$\frac{1}{\left(b^{-5}\right)\left(b^2\right)\left(b^{-1}\right)} = \frac{b^5 \cdot b^1}{b^2} = \frac{b^6}{b^2} = b^{6-2} = b^4$$

15. $\dfrac{m^{-2}}{m^{-5}}$

Solution 1: We can apply the rules $\dfrac{a^n}{a^m} = a^{n-m}$ and then $a^{-n} = \dfrac{1}{a^n}$.

$$\frac{m^{-2}}{m^{-5}} = m^{-2-(-5)} = m^{-2+5} = m^3$$

Solution 2: We can apply the rule $a^{-n} = \dfrac{1}{a^n}$ and then $\dfrac{a^n}{a^m} = a^{n-m}$.

$$\frac{m^{-2}}{m^{-5}} = \frac{m^5}{m^2} = m^{5-2} = m^3$$

16. $\dfrac{x^3y^{-5}}{z^{-4}}$

Solution: Each variable occurs only once and so this problem is just about bringing it to the form required. We can apply the rule $a^{-n} = \dfrac{1}{a^n}$. We hve alread shown that $\dfrac{1}{a^{-n}} = a^n$.

$$\frac{x^3y^{-5}}{z^{-4}} = \frac{x^3z^4}{y^5}$$

17. $\dfrac{18q^3}{6q^{-3}}$

Solution 1: We can apply the rule $\dfrac{a^n}{a^m} = a^{n-m}$.

$$\frac{18q^3}{6q^{-3}} = \frac{\cancel{6}\cdot 3q^{3-(-3)}}{\cancel{6}\cdot 1} = \frac{3q^{3+3}}{1} = 3q^6$$

Solution 2: We can apply the rules $a^{-n} = \dfrac{1}{a^n}$ and then $a^n \cdot a^m = a^{n+m}$.

$$\frac{18q^3}{6q^{-3}} = \frac{\cancel{6}\cdot 3q^3q^3}{\cancel{6}\cdot 1} = 3q^6$$

18. $\left(\dfrac{2}{3}\right)^{-3}$

Solution: We can apply the rule $a^{-n} = \dfrac{1}{a^n}$.

$$\left(\frac{2}{3}\right)^{-3} = \frac{1}{\left(\dfrac{2}{3}\right)^3} = \frac{1}{\dfrac{2}{3}\cdot\dfrac{2}{3}\cdot\dfrac{2}{3}} = \frac{1}{\dfrac{8}{27}} = 1\cdot\frac{27}{8} = \frac{27}{8}$$

Note that we basically proved here that $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$.

19. $2y^{-3}$

Solution: We can apply the rule $a^{-n} = \dfrac{1}{a^n}$. It is important to note that the base of exponentiation is y and not $2y$.

$$2y^{-3} = 2\cdot\frac{1}{y^3} = \frac{2}{1}\cdot\frac{1}{y^3} = \frac{2}{y^3}$$

20. $(2y)^{-3}$

Solution: We can apply the rule $a^{-n} = \dfrac{1}{a^n}$. This time the base of exponentiation is $2y$. So we will apply the rule $(ab)^n = a^nb^n$.

$$(2y)^{-3} = \frac{1}{(2y)^3} = \frac{1}{2^3y^3} = \frac{1}{8y^3}$$

21. $\left(-\frac{3}{5}\right)^{-2}$

Solution 1: We can apply the rule $a^{-n} = \frac{1}{a^n}$.

$$\left(-\frac{3}{5}\right)^{-2} = \frac{1}{\left(-\frac{3}{5}\right)^2} = \frac{1}{\left(-\frac{3}{5}\right)\left(-\frac{3}{5}\right)} = \frac{1}{\frac{-3}{5}\cdot\frac{-3}{5}} = \frac{1}{\frac{9}{25}} = 1\cdot\frac{25}{9} = \frac{25}{9}$$

Solution 2: We proved previously that $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$. Using that,

$$\left(-\frac{3}{5}\right)^{-2} = \left(-\frac{5}{3}\right)^2 = \left(-\frac{5}{3}\right)\left(-\frac{5}{3}\right) = \frac{25}{9}$$

22. $\frac{a^3b^{-5}}{a^{-2}b^3}$

Solution 1: We can apply the rule $\frac{a^n}{a^m} = a^{n-m}$ and then $a^{-n} = \frac{1}{a^n}$.

$$\frac{a^3b^{-5}}{a^{-2}b^3} = a^{3-(-2)}b^{-5-3} = a^{3+2}b^{-5-3} = a^5b^{-8} = a^5\cdot\frac{1}{b^8} = \frac{a^5}{1}\cdot\frac{1}{b^8} = \frac{a^5}{b^8}$$

Solution 2: We can apply the rules $a^{-n} = \frac{1}{a^n}$ and $a^n\cdot a^m = a^{n+m}$.

$$\frac{a^3b^{-5}}{a^{-2}b^3} = \frac{a^3a^2}{b^3b^5} = \frac{a^5}{b^8}$$

23. $\left(3m^3\right)^{-2}$

Solution: We can apply the rule $a^{-n} = \frac{1}{a^n}$ and then $(ab)^n = a^nb^n$ and also $(a^n)^m = a^{nm}$.

$$\left(3m^3\right)^{-2} = \frac{1}{\left(3m^3\right)^2} = \frac{1}{3^2\left(m^3\right)^2} = \frac{1}{9m^{3\cdot 2}} = \frac{1}{9m^6}$$

24. $\left(-2ab^{-3}\right)^{-3}$

Solution: We can apply the rule $(ab)^n = a^nb^n$ and then $(a^n)^m = a^{nm}$.

$$\left(-2ab^{-3}\right)^{-3} = (-2)^{-3}a^{-3}\left(b^{-3}\right)^{-3} = (-2)^{-3}a^{-3}b^{-3(-3)} = (-2)^{-3}a^{-3}b^9$$

We now apply $a^{-n} = \frac{1}{a^n}$.

$$(-2)^{-3}a^{-3}b^9 = \frac{1}{(-2)^3}\cdot\frac{1}{a^3}\cdot b^9 = \frac{1}{-8}\cdot\frac{1}{a^3}\cdot\frac{b^9}{1} = \frac{b^9}{-8a^3} = -\frac{b^9}{8a^3}$$

25. $\frac{\left(k^3\right)^{-3}}{\left(k^{-5}\right)^2}$

Solution: We can apply the rule $(a^n)^m = a^{nm}$ and then $\frac{a^n}{a^m} = a^{n-m}$.

$$\frac{\left(k^3\right)^{-3}}{\left(k^{-5}\right)^2} = \frac{k^{3(-3)}}{k^{-5\cdot 2}} = \frac{k^{-9}}{k^{-10}} = k^{-9-(-10)} = k^{-9+10} = k^1 = k$$

26. $\left(\dfrac{2a^{-3}b^{5}}{-3a^{3}b^{-2}}\right)^{-2}\left(a^{3}b^{-5}\right)^{-3}$

Solution:

$$\begin{aligned}
E &= \left(\frac{2a^{-3}b^{5}}{-3a^{3}b^{-2}}\right)^{-2}\left(a^{3}b^{-5}\right)^{-3} = \left(\frac{2a^{-3-3}b^{5-(-2)}}{-3}\right)^{-2}\left(a^{3}b^{-5}\right)^{-3} && \text{apply } \frac{a^n}{a^m}=a^{n-m}\\
&= \left(\frac{2a^{-6}b^{5+2}}{-3}\right)^{-2}\left(a^{3}b^{-5}\right)^{-3}\\
&= \left(\frac{2a^{-6}b^{7}}{-3}\right)^{-2}\left(a^{3}b^{-5}\right)^{-3} && \text{apply } \left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^{n}\\
&= \left(\frac{-3}{2a^{-6}b^{7}}\right)^{2}\left(a^{3}b^{-5}\right)^{-3} && \text{apply } \left(\frac{a}{b}\right)^{n}=\frac{a^n}{b^n}\\
&= \frac{(-3)^2}{\left(2a^{-6}b^{7}\right)^2}\left(a^{3}b^{-5}\right)^{-3} && \text{apply } (ab)^n=a^nb^n \text{ and } a^{-n}=\frac{1}{a^n}\\
&= \frac{9}{2^2\left(a^{-6}\right)^2\left(b^{7}\right)^2}\cdot\frac{1}{\left(a^{3}b^{-5}\right)^{3}} && \text{apply } (a^n)^m=a^{nm} \text{ and } (ab)^n=a^nb^n\\
&= \frac{9}{4a^{-12}b^{14}}\cdot\frac{1}{\left(a^{3}\right)^{3}\left(b^{-5}\right)^{3}} && \text{apply } a^{-n}=\frac{1}{a^n} \text{ and } (ab)^n=a^nb^n\\
&= \frac{9a^{12}}{4b^{14}}\cdot\frac{1}{a^{3\cdot 3}b^{(-5)3}}\\
&= \frac{9a^{12}}{4b^{14}}\cdot\frac{1}{a^{9}b^{-15}} && \text{apply } a^{-n}=\frac{1}{a^n}\\
&= \frac{9a^{12}}{4b^{14}}\cdot\frac{b^{15}}{a^{9}} = \frac{9a^{12}b^{15}}{4b^{14}a^{9}} && \text{apply } \frac{a^n}{a^m}=a^{n-m}\\
&= \frac{9a^{12-9}b^{15-14}}{4} = \frac{9a^{3}b^{1}}{4} = \frac{9}{4}a^{3}b
\end{aligned}$$

27. $\left(-2a^{-3}\right)\left(-2a^{-2}b\right)^{-4}$

Solution:

$$\begin{aligned}
E &= \left(-2a^{-3}\right)\left(-2a^{-2}b\right)^{-4} && \text{apply } a^{-n}=\frac{1}{a^n}\\
&= \left(-2\cdot\frac{1}{a^3}\right)\frac{1}{\left(-2a^{-2}b\right)^{4}} && \text{apply } (ab)^n=a^nb^n\\
&= \left(\frac{-2}{1}\cdot\frac{1}{a^3}\right)\frac{1}{(-2)^4\left(a^{-2}\right)^4b^4} && \text{apply } (a^n)^m=a^{nm}\\
&= \frac{-2}{a^3}\cdot\frac{1}{16a^{-8}b^4} && \text{apply } a^{-n}=\frac{1}{a^n}\\
&= \frac{-2}{a^3}\cdot\frac{a^8}{16b^4}\\
&= \frac{-2a^8}{a^3\cdot 16b^4} = \frac{-2a^8}{16a^3b^4} && \text{apply } \frac{a^n}{a^m}=a^{n-m}\\
&= \frac{-1\cdot\cancel{2}a^{8-3}}{8\cdot\cancel{2}b^4} = \frac{-a^5}{8b^4}
\end{aligned}$$

28. $\dfrac{\left(-3p^3q^5\right)^2}{\left(2q^0p^3\right)^{-1}}$

Solution:

$$\begin{aligned}
E &= \frac{\left(-3p^3q^5\right)^2}{\left(2q^0p^3\right)^{-1}} && \text{apply } \ q^0=1 \ \text{ and } \ \frac{1}{a^{-n}}=a^n \\
&= \left(-3p^3q^5\right)^2\left(2\cdot 1p^3\right)^1 \\
&= \left(-3p^3q^5\right)^2\cdot 2p^3 && \text{apply } \ (ab)^n=a^nb^n \\
&= (-3)^2\left(p^3\right)^2\left(q^5\right)^2\cdot 2p^3 && \text{apply } \ (a^n)^m=a^{nm} \\
&= 9p^{3\cdot 2}q^{5\cdot 2}\cdot 2p^3 \\
&= 18p^6q^{10}p^3 && \text{apply } \ a^n\cdot a^m=a^{n+m} \\
&= 18p^{6+3}q^{10}=18p^9q^{10}
\end{aligned}$$

29. $\left(\dfrac{2a^{-2}b^3}{-2^2\left(a^{-1}b\right)^{-3}}\right)^{-2}$

Solution:

$$\begin{aligned}
E &= \left(\frac{2a^{-2}b^3}{-2^2\left(a^{-1}b\right)^{-3}}\right)^{-2} && \text{apply } \left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n \\
&= \left(\frac{-2^2\left(a^{-1}b\right)^{-3}}{2a^{-2}b^3}\right)^2 && \text{apply } (ab)^n=a^nb^n \\
&= \left(\frac{-4\left(a^{-1}\right)^{-3}b^{-3}}{2a^{-2}b^3}\right)^2 && \text{apply } (a^n)^m=a^{nm} \\
&= \left(\frac{-4a^{-1(-3)}b^{-3}}{2a^{-2}b^3}\right)^2 \\
&= \left(\frac{-2a^3b^{-3}}{a^{-2}b^3}\right)^2 && \text{apply } \frac{a^n}{a^m}=a^{n-m} \\
&= \left(-2a^{3-(-2)}b^{-3-3}\right)^2 \\
&= \left(-2a^{3+2}b^{-3-3}\right)^2 \\
&= \left(-2a^5b^{-6}\right)^2 && \text{apply } \ (ab)^n=a^nb^n \\
&= (-2)^2\left(a^5\right)^2\left(b^{-6}\right)^2 && \text{apply } (a^n)^m=a^{nm} \\
&= 4a^{5\cdot 2}b^{-6\cdot 2}=4a^{10}b^{-12} && \text{apply } a^{-n}=\frac{1}{a^n} \\
&= 4a^{10}\cdot\frac{1}{b^{12}} \\
&= \frac{4a^{10}}{1}\cdot\frac{1}{b^{12}}=\frac{4a^{10}}{b^{12}}
\end{aligned}$$

30. $\left(-\dfrac{x^3y^0x^{-5}}{y^{-3}}\right)^{-2}$

Solution:

$$\begin{aligned}
E &= \left(-\frac{x^3y^0x^{-5}}{y^{-3}}\right)^{-2} && y^0=1 \text{ and } a^n\cdot a^m=a^{n+m}\\
&= \left(-\frac{x^{3+(-5)}}{y^{-3}}\right)^{-2} = \left(\frac{-1x^{-2}}{y^{-3}}\right)^{-2} && \text{apply } \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}\\
&= \frac{\left(-1x^{-2}\right)^{-2}}{\left(y^{-3}\right)^{-2}} && \text{apply } (ab)^n=a^nb^n\\
&= \frac{(-1)^{-2}\left(x^{-2}\right)^{-2}}{\left(y^{-3}\right)^{-2}} && \text{apply } (a^n)^m=a^{nm} \text{ and } a^{-n}=\frac{1}{a^n}\\
&= \frac{x^{-2(-2)}}{(-1)^2y^{-3(-2)}} = \frac{x^4}{1y^6} = \frac{x^4}{y^6}
\end{aligned}$$

31. $\left(-\dfrac{x^3y^7x^{-5}}{y^{-3}}\right)^{0}$

Solution: Any non-zero quantity raised to the power zero is 1. So the answer is 1.

32. $\dfrac{x^{-1}+y^{-1}}{x^{-2}-y^{-2}}$

Solution: This problem is very different because there are addition and subtraction involved. Because of that, we can not simply move the expressions with negative exponents. Instead, this will be a problem involving complex fractions.

$$\begin{aligned}
E &= \frac{x^{-1}+y^{-1}}{x^{-2}-y^{-2}}\\
&= \frac{\dfrac{1}{x^1}+\dfrac{1}{y^1}}{\dfrac{1}{x^2}-\dfrac{1}{y^2}}\\
&= \frac{\dfrac{1}{x}+\dfrac{1}{y}}{\dfrac{1}{x^2}-\dfrac{1}{y^2}} && \text{bring fractions to the common denominator}\\
&= \frac{\dfrac{1\cdot y}{x\cdot y}+\dfrac{1\cdot x}{y\cdot x}}{\dfrac{1\cdot y^2}{x^2\cdot y^2}-\dfrac{1\cdot x^2}{y^2\cdot x^2}} = \frac{\dfrac{y}{xy}+\dfrac{x}{xy}}{\dfrac{y^2}{x^2y^2}-\dfrac{x^2}{x^2y^2}}\\
&= \frac{\dfrac{y+x}{xy}}{\dfrac{y^2-x^2}{x^2y^2}} && \text{to divide is to multiply by the reciprocal}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{y+x}{xy} \cdot \frac{x^2y^2}{y^2-x^2} && \text{cancel out } xy \\
&= \frac{y+x}{1} \cdot \frac{xy}{y^2-x^2} \\
&= \frac{xy\,(x+y)}{y^2-x^2} && \text{factor } y^2-x^2 \text{ via the difference of} \\
&= \frac{xy\,(x+y)}{(y-x)\,(y+x)} && \text{squares theorem, cancel out } x+y \\
&= \frac{xy}{y-x}
\end{aligned}
$$

33. $\dfrac{\left(-2a^{-2}\right)^{-2} b^3 a^0 \left(-aba^{-2}b^{-2}\right)^{-3}}{2a^2\left(-2a^{-2}b\right)^{-2} ab^0}$

Solution:

$$
\begin{aligned}
E &= \frac{\left(-2a^{-2}\right)^{-2} b^3 a^0 \left(-aba^{-2}b^{-2}\right)^{-3}}{2a^2\left(-2a^{-2}b\right)^{-2} ab^0} && a^0 = b^0 = 1 \text{ and } x^n x^m = x^{n+m} \\
&= \frac{\left(-2a^{-2}\right)^{-2} b^3 \left(-a^{1+(-2)}b^{1+(-2)}\right)^{-3}}{2a^{2+1}\left(-2a^{-2}b\right)^{-2}} \\
&= \frac{\left(-2a^{-2}\right)^{-2} b^3 \left(-1a^{-1}b^{-1}\right)^{-3}}{2a^3\left(-2a^{-2}b\right)^{-2}} && \text{apply } \ (xy)^n = x^n y^n \\
&= \frac{(-2)^{-2}\left(a^{-2}\right)^{-2} b^3 (-1)^{-3}\left(a^{-1}\right)^{-3}\left(b^{-1}\right)^{-3}}{2a^3(-2)^{-2}\left(a^{-2}\right)^{-2} b^{-2}} && \text{apply } \ (x^n)^m = x^{nm} \\
&= \frac{(-2)^{-2} a^{-2(-2)} b^3 (-1)^{-3} a^{-1(-3)} b^{-1(-3)}}{2a^3(-2)^{-2} a^{-2(-2)} b^{-2}} \\
&= \frac{(-2)^{-2} a^4 b^3 (-1)^{-3} a^3 b^3}{2a^3(-2)^{-2} a^4 b^{-2}} && \text{cancel out } a^4 \text{ and } a^3 \text{ and } \ (-2)^{-2} \\
&= \frac{b^3(-1)^{-3} b^3}{2b^{-2}} && \text{apply } x^n x^m = x^{n+m} \\
&= \frac{(-1)^{-3} b^{3+3}}{2b^{-2}} \\
&= \frac{(-1)^{-3} b^6}{2b^{-2}} && \text{apply } x^{-n} = \frac{1}{x^n} \\
&= \frac{b^6 b^2}{(-1)^3\, 2} && \text{apply } x^n x^m = x^{n+m} \\
&= \frac{b^{6+2}}{-1\cdot 2} = \frac{b^8}{-2} = -\frac{b^8}{2}
\end{aligned}
$$

34. $\left(\dfrac{-a^2\left(b^{-1}a\right)^{-5}}{b^7\left(-ab^2\right)^{-3}}\right)^{-2}$

Solution:

$$\begin{aligned}
E &= \left(\frac{-a^2\left(b^{-1}a\right)^{-5}}{b^7\left(-ab^2\right)^{-3}}\right)^{-2} && \text{apply } (xy)^n = x^n y^n \\
&= \left(\frac{-a^2\left(b^{-1}\right)^{-5}a^{-5}}{b^7\left(-1\right)^{-3}a^{-3}\left(b^2\right)^{-3}}\right)^{-2} && \text{apply } (x^n)^m = x^{nm} \\
&= \left(\frac{-a^2b^{-1(-5)}a^{-5}}{b^7\left(-1\right)^{-3}a^{-3}b^{2(-3)}}\right)^{-2} = \left(\frac{-a^2b^5a^{-5}}{b^7\left(-1\right)^{-3}a^{-3}b^{-6}}\right)^{-2} && \text{apply } x^n x^m = x^{n+m} \\
&= \left(\frac{-a^{2+(-5)}b^5}{\left(-1\right)^{-3}b^{7+(-6)}a^{-3}}\right)^{-2} = \left(\frac{-1\cdot a^{-3}b^5}{\left(-1\right)^{-3}b^1a^{-3}}\right)^{-2} && \text{cancel out } a^{-3} \\
&= \left(\frac{-1b^5}{\left(-1\right)^{-3}b^1}\right)^{-2} && \text{apply } a^{-n} = \frac{1}{a^n} \\
&= \left(\frac{-1\left(-1\right)^3b^5}{b^1}\right)^{-2} = \left(\frac{-1\left(-1\right)b^5}{b^1}\right)^{-2} = \left(\frac{1b^5}{b^1}\right)^{-2} && \text{apply } \frac{x^n}{x^m} = x^{n-m} \\
&= \left(b^{5-1}\right)^{-2} = \left(b^4\right)^{-2} && \text{apply } (x^n)^m = x^{nm} \\
&= b^{4(-2)} = b^{-8} = \frac{1}{b^8}
\end{aligned}$$

35. $\dfrac{\left(x^{-2}\right)^{-2}y^3x^0\left(-2yx^0y^{-2}x^{-2}\right)^0}{yx^5\left(y^{-2}x\right)^{-3}\left(2x^{-1}yx^3\right)^{-1}}$

Solution:

$$\begin{aligned}
E &= \frac{\left(x^{-2}\right)^{-2}y^3x^0\left(-2yx^0y^{-2}x^{-2}\right)^0}{yx^5\left(y^{-2}x\right)^{-3}\left(2x^{-1}yx^3\right)^{-1}} && \text{apply } a^0 = 1 \text{ and } a^n a^m = a^{n+m} \\
&= \frac{\left(x^{-2}\right)^{-2}y^3}{yx^5\left(y^{-2}x\right)^{-3}\left(2x^{-1+3}y\right)^{-1}} && \text{apply } (ab)^n = a^n b^n \\
&= \frac{\left(x^{-2}\right)^{-2}y^3}{yx^5\left(y^{-2}\right)^{-3}x^{-3}\left(2x^2y\right)^{-1}} && \text{apply } (ab)^n = a^n b^n \text{ and } a^n a^m = a^{n+m} \\
&= \frac{\left(x^{-2}\right)^{-2}y^3}{yx^{5+(-3)}\left(y^{-2}\right)^{-3}\left(2\right)^{-1}\left(x^2\right)^{-1}y^{-1}} && \text{apply } (a^n)^m = a^{nm} \\
&= \frac{x^{-2(-2)}y^3}{yx^2y^{-2(-3)}2^{-1}x^{2(-1)}y^{-1}} = \frac{x^4y^3}{2^{-1}yx^2y^6x^{-2}y^{-1}} && \text{apply } a^n a^m = a^{n+m} \\
&= \frac{x^4y^3}{2^{-1}y^{1+6+(-1)}x^{2+(-2)}} = \frac{x^4y^3}{2^{-1}y^6x^0} && x^0 = 1 \text{ and } \frac{a^n}{a^m} = a^{n-m} \\
&= \frac{x^4y^{3-6}}{2^{-1}} = \frac{x^4y^{-3}}{2^{-1}} && \text{apply } a^{-n} = \frac{1}{a^n} \\
&= \frac{2^1x^4}{y^3} = \frac{2x^4}{y^3}
\end{aligned}$$

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