Equations that are in x, or in y, or in x and y can be graphed. **The graph of such an equation is the set of all points $P(x,y)$ for which the coordinates x and y form a solution of the equation.**

In case of linear (of degree one) equations, the graph is a straight line. There are several forms of a line's equation. Two of them are as follows (there are more).

$$\begin{aligned} y &= mx+b && \text{slope-intercept form} \\ Ax+By &= C && \text{general form} \end{aligned}$$

Method 1 - Graphing by Finding Points

1. Graph the line $y=-2x+3$

Solution: We will find points on this line and connect the dots. Since the graph is a straight line, theoretically it doesn't matter which of its many points we will find. To safeguard against computational errors and to guarantee precision, at least four or five points should be plotted.

Here is how we can find a point.

Step 1. Let us freely choose any value for x. We will go with $x=4$. We will look for a point on this line with $x-$coordinate 4.

Step 2. To find the $y-$coordinate of this point, we will use the equation of the line that establishes a connection between the $x-$ and $y-$coordinates of the points.

$$\begin{aligned} y &= \ ? \ \text{ if } \ x=4 \\ x &= 4 \text{ and } y=-2x+3 \implies y=-2(4)+3=-8+3=-5 \end{aligned}$$

If $x=4$, then $y=-5$. Thus we found the point $(4,-5)$ that is on this line. We repeat the process with other values for x to find other points on the line.

Let $x=0$. We will compute the value of y.

$$\begin{aligned} y &= \ ? \ \text{ if } \ x=0 \\ x &= 0 \text{ and } y=-2x+3 \implies y=-2(0)+3=0+3=3 \implies (0,3) \end{aligned}$$

If $x=0$, then $y=3$. Thus we found the point $(0,3)$ that is on this line.

Let $x=-2$. We will compute the value of y.

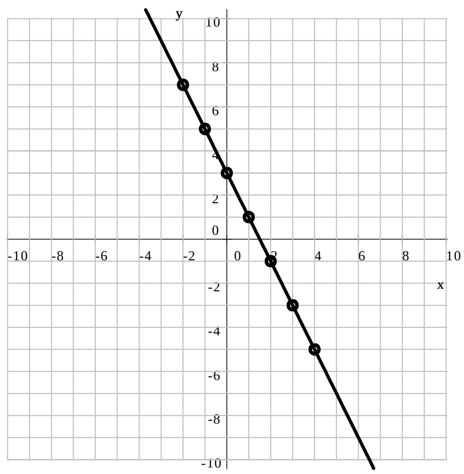
$$\begin{aligned} y &= \ ? \ \text{ if } \ x=-2 \\ x &= -2 \text{ and } y=-2x+3 \implies y=-2(-2)+3=4+3=7 \implies (-2,7) \end{aligned}$$

If $x=-2$, then $y=7$. Thus we found the point $(-2,7)$ that is on this line.

We continue to find additional points in this manner. We organize the results in a table:

x	y	$\Longrightarrow$	$P(x,y)$
-2	7		$(-2,7)$
-1	5		$(-1,5)$
0	3		$(0,3)$
1	1		$(1,1)$
2	-1		$(2,-1)$
3	-3		$(3,-3)$
4	-5		$(4,-5)$

We plot these points and connect the dots.



The point where the graph intersects the x−axis is called the x−intercept. The point where the graph intersects the y−axis is called the y−intercept. This line's y−intercept is $(0,3)$.

2. Graph the line $y = \dfrac{1}{2}x - 1$

Solution: We freely select any value for x. We find the y−coordinate of the point using the equation of the line.

Let $x = -4$. We will compute the value of y.

$$\begin{aligned} y &= \ ? \text{ if } x = -4 \\ x &= 4 \text{ and } y = \frac{1}{2}x - 1 \quad \Longrightarrow \quad y = \frac{1}{2}(-4) - 1 = -2 - 1 = -3 \end{aligned}$$

If $x = -4$, then $y = -3$. Thus we found the point $(-4,-3)$ on this line. We repeat the process with other values for x to find other points on the line.

Let $x = -3$. We will compute the value of y.

$$\begin{aligned} y &= \ ? \text{ if } x = -3 \\ x &= -3 \text{ and } y = \frac{1}{2}x - 1 \quad \Longrightarrow \quad y = \frac{1}{2}(-3) - 1 = -\frac{3}{2} - 1 = -\frac{5}{2} \end{aligned}$$

If $x = -3$, then $y = -\dfrac{5}{2}$. Thus we found the point $\left(-3, -\dfrac{5}{2}\right)$ on this line. Although the point we found is correct, its y−coordinate is not an integer. This makes the plotting of this point more difficult and also inaccurate.

Whenever possible, we should rely on graphing lattice. points. **A lattice point is a point of whose both coordinates are integers.**

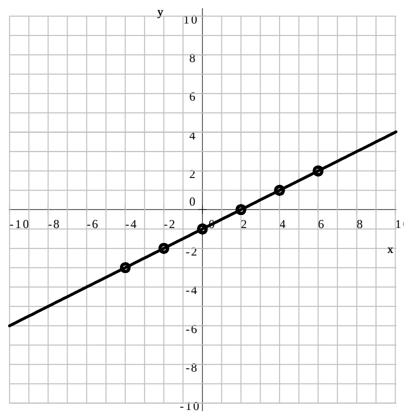
Let $x = -2$. We will compute the value of y.

$$\begin{aligned} y &= \text{ ? if } x = -2 \\ x &= -2 \text{ and } y = \frac{1}{2}x - 1 \implies y = \frac{1}{2}(-2) - 1 = -1 - 1 = -2 \end{aligned}$$

If $x = -2$, then $y = -2$. Thus we found the point $(-2, -2)$ on this line.

We continue to find additional points in this manner. Notice that we get lattice points if we use even numbers for x. We organize the results in a table, and then plot these points and connect the dots.

x	y	$\implies$	$P(x, y)$
-4	-3		$(-4, -3)$
-3	$-\frac{5}{2}$		$\left(-3, -\frac{5}{2}\right)$
-2	-2		$(-2, -2)$
-1	$-\frac{3}{2}$		$\left(-1, -\frac{3}{2}\right)$
0	-1		$(0, -1)$
1	$-\frac{1}{2}$		$\left(1, -\frac{1}{2}\right)$
2	0		$(2, 0)$
4	1		$(4, 1)$
6	2		$(6, 2)$



The point where the graph intersects the $x-$axis is called the $x-$intercept. The point where the graph intersects the $y-$axis is called the $y-$intercept. This line's $x-$intercept is $(2, 0)$ and $y-$intercept is $(0, -1)$.

3. Graph the line $2x + 3y = -12$

Solution: We freely select any value for x. We find the $y-$coordinate of the point using the equation of the line. We substitute the value for x and solve the equation for y.
Let $x = 0$. We will compute the value of y.

$$\begin{aligned} y &= \text{ ? if } x = 0 \\ x &= 0 \text{ and } 2x + 3y = -12 \implies 2(0) + 3y = -12 \end{aligned}$$

$$\begin{aligned} 2(0) + 3y &= -12 && \text{solve for } y \\ 3y &= -12 && \text{divide by 3} \\ y &= -4 \end{aligned}$$

If $x = 0$, then $y = -4$. Thus we found the point $(0, -4)$ on this line. We repeat the process with other values for x to find other points on the line.

Let $x = 2$. We will compute the value of y.

$$\begin{aligned} y &= \text{ ? if } x = 2 \\ x &= 2 \text{ and } 2x + 3y = -12 \implies 2(2) + 3y = -12 \end{aligned}$$

$$\begin{aligned} 2(2)+3y &= -12 && \text{solve for } y \\ 3y+4 &= -12 && \text{subtract } 4 \\ 3y &= -16 && \text{divide by } 3 \\ y &= -\frac{16}{3} \end{aligned}$$

If $x = 2$, then $y = -\dfrac{16}{3}$. Thus we found the point $\left(2, -\dfrac{16}{3}\right)$ on this line. Although the point we found is correct, its $y-$coordinate is not an integer. This makes the plotting of this point more difficult and also inaccurate. Whenever possible, we should rely on graphing lattice points. **A lattice point is a point of whose both coordinates are integers.**

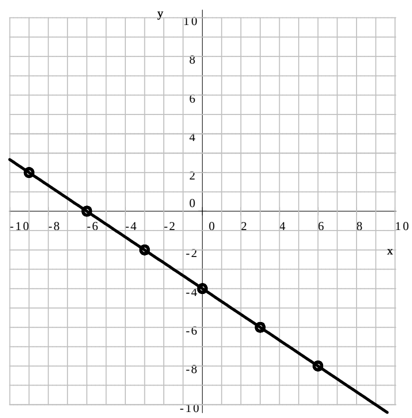
Let $x = 3$. We will compute the value of y.

$$\begin{aligned} y &= \ ? \text{ if } x = 3 \\ x &= \text{ and } 2x+3y=-12 \implies 2(3)+3y=-12 \end{aligned}$$

$$\begin{aligned} 2(3)+3y &= -12 && \text{solve for } y \\ 3y+6 &= -12 && \text{subtract } 6 \\ 3y &= -18 && \text{divide by } 3 \\ y &= -6 \end{aligned}$$

If $x = 3$, then $y = -6$. Thus we found the point $(3, -6)$ on this line.
We continue to find additional points in this manner. Notice that we get lattice points if we use numbers for x that are divisible by 3. We organize the results in a table. We plot the lattice points and connect the dots.

x	y	$\Longrightarrow$	$P(x,y)$
-6	0		$(-6,0)$
-3	-2		$(-3,-2)$
0	-4		$(0,-4)$
3	-6		$(3,-6)$
-9	2		$(-9,2)$
-12	4		$(-12,4)$



The point where the graph intersects the $x-$axis is called the $x-$intercept. The point where the graph intersects the $y-$axis is called the $y-$intercept. This line's $x-$intercept is $(-6,0)$ and $y-$intercept is $(0,-4)$.

4. Graph the line $y = -2$

Solution: We freely select any value for x. We find the $y-$coordinate of the point using the equation of the line. We substitute the value for x and solve the equation for y.

$$\begin{aligned} y &= \ ? \text{ if } x = 0 \\ x &= 0 \text{ and } y = -2 \implies P(0, -2) \end{aligned}$$

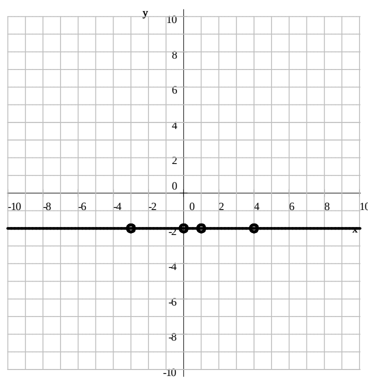
If $x = 0$, then $y = -2$. Thus we found the point $(0, -2)$ on this line. We repeat the process with other values for x to find other points on the line.

$$\begin{aligned} y &= \ ? \text{ if } x = -3 \\ x &= -3 \text{ and } y = -2 \implies P(-3, -2) \end{aligned}$$

If $x = -3$, then $y = -2$. Thus we found the point $(-3, -2)$ on this line.

We continue to find additional points in this manner. It is clear that no matter what the value of x is, y will always be -2. We organize the results in a table. We plot the lattice points and connect the dots.

x	y	$\implies$	$P(x, y)$
-3	-2		$(-3, -2)$
0	-2		$(0, -2)$
1	-2		$(1, -2)$
4	-2		$(4, -2)$



This line's $y-$intercept is $(0, -2)$ and it does not have an $x-$intercept.

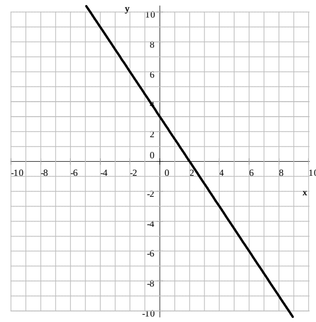
Practice Problems

Graph each of the following lines.

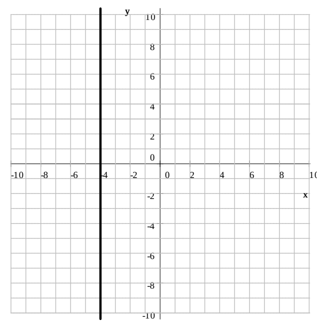
1. $3x + 2y = 6$
2. $x = -4$
3. $y = \frac{2}{5}x - 3$
4. $2x - 3y = 10$
5. $y = 1$
6. $y = 3x + 6$
7. $3x + 5y = -30$
8. $2x - y = 7$
9. $y = \frac{1}{3}x$

Practice Problems - Answers

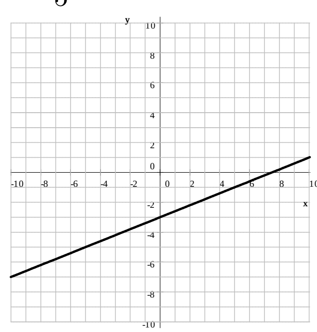
1. $3x + 2y = 6$



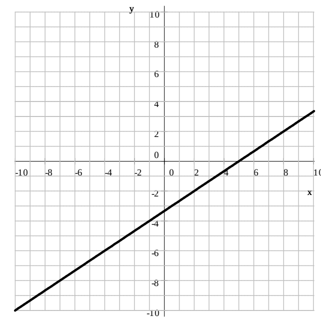
2. b) $x = -4$



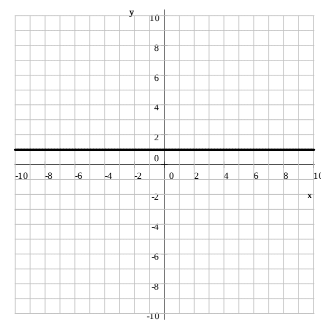
3. $y = \dfrac{2}{5}x - 3$



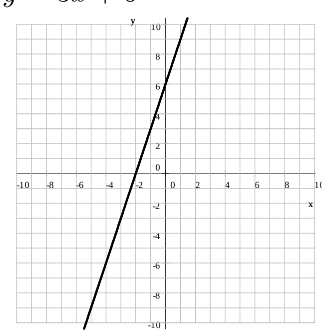
4. $2x - 3y = 10$



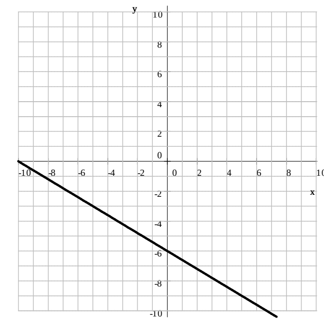
5. $y = 1$



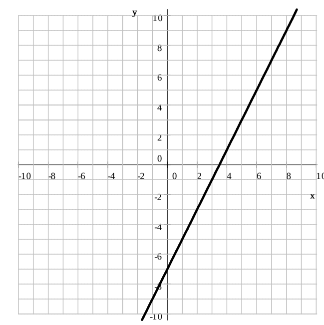
6. $y = 3x + 6$



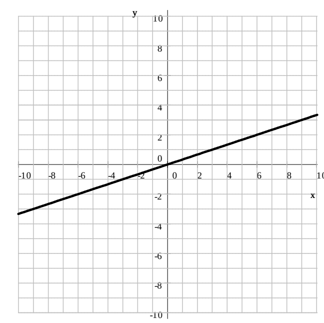
7. $3x + 5y = -30$



8. $2x - y = 7$



9. $y = \dfrac{1}{3}x$



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