

## Sample Problems

Solve each of the following equations. Make sure to check your solutions.

- 1.)  $2x + 3 = 4x + 9$
- 2.)  $3w - 5 = 5(w + 1)$
- 3.)  $3y - 9 = -2y + 4$
- 4.)  $4 - x = 3(x - 7)$
- 5.)  $7(j - 5) + 9 = 2(-2j + 5) + 5j$
- 6.)  $3(x - 5) - 5(x - 1) = -2x + 1$
- 7.)  $\frac{3 - x}{4} - \frac{10 - 3x}{5} = x + 2$
- 8.)  $\frac{3x + 17}{2} = x - 1 + \frac{x + 19}{2}$
- 9.)  $\frac{2}{3}(x - 1) = \frac{3}{5}(x - 4) + 1$
- 10.)  $\frac{2}{3}(x - 7) = \frac{4}{5}(x + 1)$
- 11.)  $\frac{x + 2}{4} - \frac{x - 3}{5} = 20 - x$

## Practice Problems

Solve each of the following equations. Make sure to check your solutions.

- 1.)  $5x - 3 = x + 9$
- 2.)  $-x + 13 = 2x + 1$
- 3.)  $-2x + 4 = 5x - 10$
- 4.)  $5x - 7 = 6x + 8$
- 5.)  $8x - 1 = 3x + 19$
- 6.)  $-7x - 1 = 3x - 21$
- 7.)  $3(x - 4) + 5(x + 8) = 2(x - 1)$
- 8.)  $3(x - 4) = 2(x + 5)$
- 9.)  $4(5x + 1) = 6x + 4$
- 10.)  $3(2x - 7) - 2(5x + 2) = -5x - 30$
- 11.)  $a - 3 = 5(a - 1) - 2$
- 12.)  $3y - 2 = -2y + 18$
- 13.)  $2(b + 1) - 5(b - 3) = 2(b - 7) + 1$
- 14.)  $3(2x - 1) - 5(2 - x) = 4(x - 1) + 5$
- 15.)  $5(x - 1) - 3(x + 1) = 3x - 8$
- 16.)  $5(x - 1) - 3(-x + 1) = -3 + 8x$
- 17.)  $-2x - (3x - 1) = 2(5 - 3x)$
- 18.)  $3(x - 4) - 4(x - 3) = 3(x - 2) + 2(3 - x)$
- 19.)  $\frac{3x - 1}{5} - \frac{7 - x}{3} = 2x + 6$
- 20.)  $8(x - 3) - 3(5 - 2x) = x$
- 21.)  $\frac{3x - 1}{4} + \frac{8 - 4x}{3} = -3 - x$
- 22.)  $\frac{3x - 2}{5} + \frac{x + 4}{3} = \frac{14(x + 1)}{15}$
- 23.)  $\frac{3}{8}x + 1\frac{4}{5} = \frac{1}{4}x + 1\frac{3}{10}$
- 24.)  $\frac{2x + 1}{3} - \frac{3 - x}{2} = x - 2$
- 25.)  $\frac{2}{3}x - 1 = -\frac{2}{3}\left(x + \frac{1}{2}\right)$

## Sample Problems - Answers

- 1.)  $-3$    2.)  $-5$    3.)  $\frac{13}{5}$    4.)  $\frac{25}{4}$    5.)  $6$    6.) no solution   7.)  $-5$   
8.) identity, all numbers are solution   9.)  $-11$    10.)  $-41$    11.)  $18$

## Practice Problems - Answers

- 1.)  $3$    2.)  $4$    3.)  $2$    4.)  $-15$    5.)  $4$    6.)  $2$    7.)  $-5$    8.)  $22$    9.)  $0$   
10.)  $-5$    11.)  $1$    12.)  $4$    13.)  $6$    14.)  $2$    15.)  $0$    16.) contradiction, there is no solution  
17.)  $9$    18.)  $0$    19.)  $-8$    20.)  $3$    21.)  $-13$    22.) identity, all numbers are solution  
23.)  $-4$    24.)  $-5$    25.)  $\frac{1}{2}$

## Sample Problems - Solutions

1.  $2x + 3 = 4x + 9$

Solution:

$$\begin{aligned} 2x + 3 &= 4x + 9 && \text{subtract } 2x \text{ from both sides} \\ 3 &= 2x + 9 && \text{subtract } 9 \text{ from both sides} \\ -6 &= 2x && \text{divide both sides by } 2 \\ -3 &= x \end{aligned}$$

We check: if  $x = -3$ , then

$$\begin{aligned} \text{RHS} &= 2(-3) + 3 = -6 + 3 = -3 \\ \text{LHS} &= 4(-3) + 9 = -12 + 9 = -3 \end{aligned}$$

Thus our solution,  $x = -3$  is correct.

2.  $3w - 5 = 5(w + 1)$

Solution: we first apply the law of distributivity to simplify the right-hand side.

$$\begin{aligned} 3w - 5 &= 5(w + 1) \\ 3w - 5 &= 5w + 5 && \text{subtract } 3w \text{ from both sides} \\ -5 &= 2w + 5 && \text{subtract } 5 \text{ from both sides} \\ -10 &= 2w && \text{divide both sides by } 2 \\ -5 &= w \end{aligned}$$

We check. If  $w = -5$ , then

$$\begin{aligned} \text{LHS} &= 3(-5) - 5 = -15 - 5 = -20 \\ \text{RHS} &= 5((-5) + 1) = 5(-4) = -20 \end{aligned}$$

Thus our solution,  $w = -5$  is correct.

3.  $3y - 9 = -2y + 4$

Solution:

$$\begin{aligned} 3y - 9 &= -2y + 4 && \text{add } 2y \text{ to both sides} \\ 5y - 9 &= 4 && \text{add } 9 \text{ to both sides} \\ 5y &= 13 && \text{divide both sides by } 5 \\ y &= \frac{13}{5} \end{aligned}$$

We check. If  $x = \frac{13}{5}$ , then

$$\begin{aligned} \text{LHS} &= 3\left(\frac{13}{5}\right) - 9 = \frac{3}{1} \cdot \frac{13}{5} - 9 = \frac{39}{5} - \frac{9}{1} = \frac{39}{5} - \frac{45}{5} = \frac{-6}{5} = -\frac{6}{5} \\ \text{RHS} &= -2\left(\frac{13}{5}\right) + 4 = \frac{-2}{1} \cdot \frac{13}{5} + \frac{4}{1} = \frac{-26}{5} + \frac{20}{5} = \frac{-6}{5} = -\frac{6}{5} \end{aligned}$$

Thus  $x = \frac{13}{5}$  is the correct solution.

4.  $4 - x = 3(x - 7)$

Solution: We first apply the law of distributivity to simplify the right-hand side.

$$\begin{aligned}
 4 - x &= 3(x - 7) && \text{distribute } 3 \\
 4 - x &= 3x - 21 && \text{add } x \text{ to both sides} \\
 4 &= 4x - 21 && \text{add } 21 \text{ to both sides} \\
 25 &= 4x && \text{divide both sides by } 4 \\
 \frac{25}{4} &= x
 \end{aligned}$$

We check. If  $x = \frac{25}{4}$ , then

$$\begin{aligned}
 \text{LHS} &= 4 - x = 4 - \frac{25}{4} = \frac{4}{1} - \frac{25}{4} = \frac{16}{4} - \frac{25}{4} = \frac{16 - 25}{4} = \frac{-9}{4} = -\frac{9}{4} \\
 \text{RHS} &= 3(x - 7) = 3\left(\frac{25}{4} - 7\right) = 3\left(\frac{25}{4} - \frac{7}{1}\right) = 3\left(\frac{25}{4} - \frac{28}{4}\right) = 3\left(\frac{25 - 28}{4}\right) \\
 &= 3\left(\frac{-3}{4}\right) = \frac{3}{1} \cdot \frac{-3}{4} = \frac{-9}{4} = -\frac{9}{4}
 \end{aligned}$$

Thus our solution,  $x = \frac{25}{4}$  is correct.

5.  $7(j - 5) + 9 = 2(-2j + 5) + 5j$

Solution:

$$\begin{aligned}
 7(j - 5) + 9 &= 2(-2j + 5) + 5j && \text{distribute on both sides} \\
 7j - 35 + 9 &= -4j + 10 + 5j && \text{combine like terms} \\
 7j - 26 &= j + 10 && \text{subtract } j \\
 6j - 26 &= 10 && \text{add } 26 \\
 6j &= 36 && \text{divide by } 6 \\
 j &= 6
 \end{aligned}$$

We check: if  $j = 6$ , then

$$\begin{aligned}
 \text{LHS} &= 7(6 - 5) + 9 = 7 \cdot 1 + 9 = 7 + 9 = 16 \\
 \text{RHS} &= 2(-2 \cdot 6 + 5) + 5 \cdot 6 = 2(-12 + 5) + 30 = 2(-7) + 30 = -14 + 30 = 16
 \end{aligned}$$

Thus our solution is correct.

6.  $3(x - 5) - 5(x - 1) = -2x + 1$

Solution:

$$\begin{aligned}
 3(x - 5) - 5(x - 1) &= -2x + 1 && \text{multiply out parentheses} \\
 3x - 15 - 5x + 5 &= -2x + 1 && \text{combine like terms} \\
 -2x - 10 &= -2x + 1 && \text{add } 2x \\
 -10 &= 1
 \end{aligned}$$

Since  $x$  disappeared from the equation and we are left with an unconditionally false statement, there is no solution for this equation. This type of an equation is called a **contradiction**.

$$7. \frac{3-x}{4} - \frac{10-3x}{5} = x+2$$

Solution:

$$\begin{aligned} \frac{3-x}{4} - \frac{10-3x}{5} &= x+2 && \text{make everything a fraction} \\ \frac{3-x}{4} - \frac{10-3x}{5} &= \frac{x+2}{1} && \text{common denominator} \\ \frac{5(3-x)}{20} - \frac{4(10-3x)}{20} &= \frac{20(x+2)}{20} && \text{multiply by 20} \\ 5(3-x) - 4(10-3x) &= 20(x+2) && \text{distribute} \\ 15 - 5x - 40 + 12x &= 20x + 40 && \text{combine like terms} \\ 7x - 25 &= 20x + 40 && \text{subtract } 7x \\ -25 &= 13x + 40 && \text{subtract } 40 \\ -65 &= 13x && \text{divide by 13} \\ -5 &= x \end{aligned}$$

We check:

$$\begin{aligned} \text{LHS} &= \frac{3 - (-5)}{4} - \frac{10 - 3(-5)}{5} = \frac{8}{4} - \frac{25}{5} = 2 - 5 = -3 \\ \text{RHS} &= -5 + 2 = -3 \end{aligned}$$

Thus our solution,  $-5$  is correct.

$$8. \frac{3x+17}{2} = x-1 + \frac{x+19}{2}$$

Solution:

$$\begin{aligned} \frac{3x+17}{2} &= x-1 + \frac{x+19}{2} && \text{express everything as a fraction} \\ \frac{3x+17}{2} &= \frac{x-1}{1} + \frac{x+19}{2} && \text{bring everything to the common denominator} \\ \frac{3x+17}{2} &= \frac{2(x-1)}{2} + \frac{x+19}{2} && \text{add fractions on right hand side} \\ \frac{3x+17}{2} &= \frac{2(x-1) + x + 19}{2} && \text{multiply out parentheses} \\ \frac{3x+17}{2} &= \frac{2x - 2 + x + 19}{2} && \text{combine like terms} \\ \frac{3x+17}{2} &= \frac{3x+17}{2} && \text{multiply by 2} \\ 3x+17 &= 3x+17 \end{aligned}$$

Because the left hand side is now identical to the right hand side, this equation is an identity, and all real numbers are solution.

$$9. \frac{2}{3}(x-1) = \frac{3}{5}(x-4) + 1$$

Solution: We re-write the expressions as fractions.

$$\begin{aligned} \frac{2(x-1)}{3} &= \frac{3(x-4)}{5} + \frac{1}{1} && \text{common denominator is 15} \\ \frac{5 \cdot 2(x-1)}{5 \cdot 3} &= \frac{3 \cdot 3(x-4)}{3 \cdot 5} + \frac{15}{15} \\ \frac{10(x-1)}{15} &= \frac{9(x-4)}{15} + \frac{15}{15} && \text{multiply by 15} \\ 10(x-1) &= 9(x-4) + 15 && \text{distribute} \\ 10x - 10 &= 9x - 36 + 15 && \text{combine like terms} \\ 10x - 10 &= 9x - 21 && \text{subtract } 9x \\ x - 10 &= -21 && \text{add 21} \\ x &= -11 \end{aligned}$$

We check. If  $x = -11$ , then

$$\begin{aligned} \text{LHS} &= \frac{2}{3}(-11-1) = \frac{2}{3}(-12) = -8 \\ \text{RHS} &= \frac{3}{5}(-11-4) + 1 = \frac{3}{5}(-15) + 1 = -9 + 1 = -8 \end{aligned}$$

Thus our solution,  $-11$  is correct.

$$10. \frac{2}{3}(x-7) = \frac{4}{5}(x+1)$$

Solution:

$$\begin{aligned} \frac{2}{3}(x-7) &= \frac{4}{5}(x+1) \\ \frac{2}{3} \cdot \frac{x-7}{1} &= \frac{4}{5} \cdot \frac{x+1}{1} \\ \frac{2(x-7)}{3} &= \frac{4(x+1)}{5} && \text{bring fractions to common denominator} \\ \frac{5 \cdot 2(x-7)}{15} &= \frac{3 \cdot 4(x+1)}{15} && \text{multiply both sides by 15} \\ 10(x-7) &= 12(x+1) && \text{multiply out parentheses} \\ 10x - 70 &= 12x + 12 && \text{subtract } 10x \\ -70 &= 2x + 12 && \text{subtract 12} \\ -82 &= 2x && \text{divide by 2} \\ -41 &= x \end{aligned}$$

We check:

$$\begin{aligned} \text{LHS} &= \frac{2}{3}(-41-7) = \frac{2}{3}(-48) = -32 \\ \text{RHS} &= \frac{4}{5}(-41+1) = \frac{4}{5}(-40) = -32 \end{aligned}$$

Thus our solution,  $-41$  is correct.

$$11. \frac{x+2}{4} - \frac{x-3}{5} = 20 - x$$

Solution:

$$\begin{aligned} \frac{x+2}{4} - \frac{x-3}{5} &= 20 - x && \text{make everything a fraction} \\ \frac{x+2}{4} - \frac{x-3}{5} &= \frac{20-x}{1} && \text{common denominator is 20} \\ \frac{5(x+2)}{20} - \frac{4(x-3)}{20} &= \frac{20(20-x)}{20} && \text{multiply by 20} \\ 5(x+2) - 4(x-3) &= 20(20-x) && \text{distribute} \\ 5x + 10 - 4x + 12 &= 400 - 20x && \text{combine like terms} \\ x + 22 &= -20x + 400 && \text{add } 20x \\ 21x + 22 &= 400 && \text{subtract 22} \\ 21x &= 378 && \text{divide by 21} \\ x &= 18 \end{aligned}$$

We check. If  $x = 18$ , then

$$\begin{aligned} \text{LHS} &= \frac{18+2}{4} - \frac{18-3}{5} = \frac{20}{4} - \frac{15}{5} = 5 - 3 = 2 \\ \text{RHS} &= 20 - 18 = 2 \end{aligned}$$

Thus 18 is indeed the solution.