

Sample Problems

Solve each of the following equations. Make sure to check your solutions.

1. $2x + 3 = 4x + 9$

5. $5 - x = 3x - 21$

9. $2w + 1 = 2w - 9$

2. $3w - 5 = 5w + 15$

6. $3a - 31 = -2a + 9$

10. $\frac{1}{2}x - 1 = \frac{2}{3}x + 4$

3. $7x - 2 = 5x - 2$

7. $-2b - 6 = -9b + 8$

11. $-\frac{2}{5}x + \frac{1}{3} = \frac{4}{15}x$

4. $3y - 9 = -2y + 6$

8. $3m - 1 = 3m - 1$

12. $7x - 3 = -x + 2$

Practice Problems

Solve each of the following equations. Make sure to check your solutions.

1. $5x - 8 = -4x + 1$

7. $5b - 6 = -9b - 6$

12. $4x - 1 = 4x + 1$

2. $-2a + 5 = 7a - 13$

8. $-5y + 3 = -5y + 3$

13. $3a + 7 = 3a + 7$

3. $-7x = x - 24$

9. $2x + 1 = -2x - 5$

14. $\frac{2}{3}x - 1 = \frac{2}{3}x + 1$

4. $-3m - 9 = 2m + 16$

10. $-\frac{2}{3}x - 5 = -\frac{1}{2}x + 1$

15. $\frac{2}{3}x - 1 = \frac{2}{3}x - 1$

5. $1 - 3x = 3x - 23$

11. $4x - 1 = x - 4$

16. $\frac{2}{3}x + 1 = \frac{1}{5}x + 1$

6. $4p + 7 = 4p - 7$

Sample Problems - Answers

- 1.) -3 2.) -10 3.) 0 4.) 3 5.) $\frac{13}{2}$ 6.) 8 7.) 2 8.) identity, all numbers are solution
- 9.) there is no solution 10.) -30 11.) $\frac{1}{2}$ 12.) $\frac{5}{8}$

Practice Problems - Answers

- 1.) 1 2.) 2 3.) 3 4.) -5 5.) 4 6.) There is no solution. 7.) 0
- 8.) identity, all numbers are solution 9.) $-\frac{3}{2}$ 10.) -36 11.) -1 12.) There is no solution.
- 13.) identity, all numbers are solution 14.) There is no solution. 15.) identity, all numbers are solution 16.) 0

Sample Problems - Solutions

1. $2x + 3 = 4x + 9$

Solution:

$$\begin{array}{ll} 2x + 3 = 4x + 9 & \text{subtract } 2x \text{ from both sides} \\ 3 = 2x + 9 & \text{subtract } 9 \text{ from both sides} \\ -6 = 2x & \text{divide both sides by } 2 \\ -3 = x & \end{array}$$

We check: if $x = -3$, then

$$\begin{array}{l} \text{LHS} = 2(-3) + 3 = -6 + 3 = -3 \\ \text{RHS} = 4(-3) + 9 = -12 + 9 = -3 \end{array}$$

Thus our solution, $x = -3$ is correct.

2. $3w - 5 = 5w + 15$

Solution:

$$\begin{array}{ll} 3w - 5 = 5w + 15 & \text{subtract } 3w \text{ from both sides} \\ -5 = 2w + 15 & \text{subtract } 15 \text{ from both sides} \\ -20 = 2w & \text{divide both sides by } 2 \\ -10 = w & \end{array}$$

We check: if $w = -10$, then

$$\begin{array}{l} \text{LHS} = 3(-10) - 5 = -30 - 5 = -35 \\ \text{RHS} = 5(-10) + 15 = -50 + 15 = -35 \end{array}$$

Thus our solution, $x = -10$ is correct.

3. $7x - 2 = 5x - 2$

Solution:

$$\begin{array}{ll}
 7x - 2 = 5x - 2 & \text{subtract } 5x \text{ from both sides} \\
 2x - 2 = -2 & \text{add } 2 \text{ to both sides} \\
 2x = 0 & \text{divide both sides by } 2 \\
 x = 0 &
 \end{array}$$

We check: if $x = 0$, then

$$\begin{array}{l}
 \text{LHS} = 7(0) - 2 = 0 - 2 = -2 \\
 \text{RHS} = 5(0) - 2 = 0 - 2 = -2
 \end{array}$$

Thus our solution, $x = 0$ is correct.

4. $3y - 9 = -2y + 6$

Solution:

$$\begin{array}{ll}
 3y - 9 = -2y + 6 & \text{add } 2y \text{ to both sides} \\
 5y - 9 = 6 & \text{add } 9 \text{ to both sides} \\
 5y = 15 & \text{divide both sides by } 5 \\
 y = 3 &
 \end{array}$$

We check: if $y = 3$, then

$$\begin{array}{l}
 \text{LHS} = 3(3) - 9 = 9 - 9 = 0 \\
 \text{RHS} = -2(3) + 6 = -6 + 6 = 0
 \end{array}$$

Thus our solution, $y = 3$ is correct.

5. $5 - x = 3x - 21$

Solution:

$$\begin{array}{ll}
 5 - x = 3x - 21 & \text{add } x \text{ to both sides} \\
 5 = 4x - 21 & \text{add } 21 \text{ to both sides} \\
 26 = 4x & \text{divide both sides by } 4 \\
 \frac{26}{4} = x & \text{reduce result to lowest terms} \\
 x = \frac{13}{2} &
 \end{array}$$

We check: if $x = \frac{13}{2}$, then

$$\begin{array}{l}
 \text{LHS} = 5 - \frac{13}{2} = \frac{10}{2} - \frac{13}{2} = \frac{10 - 13}{2} = \frac{-3}{2} = -\frac{3}{2} \\
 \text{RHS} = 3\left(\frac{13}{2}\right) - 21 = \frac{39}{2} - \frac{42}{2} = \frac{-3}{2} = -\frac{3}{2}
 \end{array}$$

Thus our solution, $x = \frac{13}{2}$ is correct.

6. $3a - 31 = -2a + 9$

Solution:

$$\begin{array}{ll}
 3a - 31 = -2a + 9 & \text{add } 2a \text{ to both sides} \\
 5a - 31 = 9 & \text{add } 31 \text{ to both sides} \\
 5a = 40 & \text{divide both sides by } 5 \\
 a = 8 &
 \end{array}$$

We check: if $a = 8$, then

$$\begin{array}{l}
 \text{LHS} = 3(8) - 31 = 24 - 31 = -7 \\
 \text{RHS} = -2(8) + 9 = -16 + 9 = -7
 \end{array}$$

Thus our solution, $a = 8$ is correct.

7. $-2b - 6 = -9b + 8$

Solution:

$$\begin{array}{ll}
 -2b - 6 = -9b + 8 & \text{add } 9b \text{ to both sides} \\
 7b - 6 = 8 & \text{add } 6 \text{ to both sides} \\
 7b = 14 & \text{divide both sides by } 7 \\
 b = 2 &
 \end{array}$$

We check: if $b = 2$, then

$$\begin{array}{l}
 \text{LHS} = -2(2) - 6 = -4 - 6 = -10 \\
 \text{RHS} = -9(2) + 8 = -18 + 8 = -10
 \end{array}$$

Thus our solution, $b = 2$ is correct.

8. $3m - 1 = 3m - 1$

Solution: This equation looks different from all the others because the two sides are identical. Logically, the two sides will be equal no matter what number we substitute into the equation. Computation will confirm this idea:

$$\begin{array}{ll}
 3m - 1 = 3m - 1 & \text{add } 3m \text{ to both sides} \\
 -1 = -1 & \text{add } 1 \text{ to both sides} \\
 0 = 0 &
 \end{array}$$

The statement $0 = 0$ is true no matter what the value of m is. Such a statement is called an **unconditionally true statement** or **identity**. **All numbers are solutions** of this equation.

9. $2w + 1 = 2w - 9$

Solution:

$$\begin{array}{ll}
 2w + 1 = 2w - 9 & \text{subtract } 2w \text{ from both sides} \\
 1 = -9 &
 \end{array}$$

The statement $1 = -9$ is false no matter what the value of w is. Such a statement is called an **unconditionally false statement**, or **contradiction**. **This equation has no solution.**

$$10. \frac{1}{2}x - 1 = \frac{2}{3}x + 4$$

Solution: Structurally, this equation is no different from the previous equations. However, because the coefficients of x are fractions, each step will take a bit more work.

$$\begin{aligned} \frac{1}{2}x - 1 &= \frac{2}{3}x + 4 && \text{subtract } \frac{1}{2}x \text{ from both sides} \\ -1 &= \frac{1}{6}x + 4 && \text{subtract 4 from both sides} \\ -5 &= \frac{1}{6}x && \text{divide both sides by } \frac{1}{6} \\ -30 &= x \end{aligned}$$

Here are the computations for each step. To subtract $\frac{1}{2}x$ from the right-hand side:

$$\frac{2}{3}x - \frac{1}{2}x = \left(\frac{2}{3} - \frac{1}{2}\right)x = \left(\frac{4}{6} - \frac{3}{6}\right)x = \frac{4-3}{6}x = \frac{1}{6}x$$

To divide both sides by $\frac{1}{6}$ is to multiply by its reciprocal:

$$-5 \div \frac{1}{6} = \frac{-5}{1} \div \frac{1}{6} = \frac{-5}{1} \cdot \frac{6}{1} = \frac{-30}{1} = -30$$

We check: if $x = -30$, then

$$\begin{aligned} \text{LHS} &= \frac{1}{2}(-30) - 1 = -15 - 1 = -16 \\ \text{RHS} &= \frac{2}{3}(-30) + 4 = \frac{2}{3} \cdot \frac{-30}{1} + 4 = \frac{-60}{3} + 4 = -20 + 4 = -16 \end{aligned}$$

Thus our solution, $x = -30$ is correct.

$$11. -\frac{2}{5}x + \frac{1}{3} = \frac{4}{15}x$$

Solution:

$$\begin{aligned} -\frac{2}{5}x + \frac{1}{3} &= \frac{4}{15}x && \text{add } \frac{2}{5}x \text{ to both sides} \\ \frac{1}{3} &= \frac{2}{3}x && \text{divide both sides by } \frac{2}{3} \\ \frac{1}{2} &= x \end{aligned}$$

The computation for each step are as follows. To add $\frac{2}{5}x$ to the right-hand side:

$$\frac{4}{15}x + \frac{2}{5}x = \left(\frac{4}{15} + \frac{2}{5}\right)x = \left(\frac{4}{15} + \frac{6}{15}\right)x = \frac{10}{15}x = \frac{2}{3}x$$

To divide by $\frac{2}{3}$ is to multiply by its reciprocal:

$$\frac{1}{3} \div \frac{2}{3} = \frac{1}{3} \cdot \frac{3}{2} = \frac{1}{2}$$

We check: if $x = \frac{1}{2}$, then

$$\begin{aligned}\text{LHS} &= -\frac{2}{5} \left(\frac{1}{2} \right) + \frac{1}{3} = -\frac{1}{5} + \frac{1}{3} = \frac{-3}{15} + \frac{5}{15} = \frac{-3+5}{15} = \frac{2}{15} \\ \text{RHS} &= \frac{4}{15} \left(\frac{1}{2} \right) = \frac{4}{30} = \frac{2}{15}\end{aligned}$$

Thus our solution, $x = \frac{1}{2}$ is correct.

12. $7x - 3 = -x + 2$

Solution: As we will see, this equation is interesting in the sense that solving it will require less work than checking the solution.

$$\begin{aligned}7x - 3 &= -x + 2 && \text{add } x \text{ to both sides} \\ 8x - 3 &= 2 && \text{add } 3 \text{ to both sides} \\ 8x &= 5 && \text{divide both sides by } 8 \\ x &= \frac{5}{8} && \text{reduce result to lowest terms}\end{aligned}$$

We check: if $x = \frac{5}{8}$, then

$$\begin{aligned}\text{LHS} &= 7 \left(\frac{5}{8} \right) - 3 = \frac{7}{1} \cdot \frac{5}{8} - 3 = \frac{35}{8} - \frac{24}{8} = \frac{35-24}{8} = \frac{11}{8} \\ \text{RHS} &= - \left(\frac{5}{8} \right) + 2 = \frac{-5}{8} + \frac{16}{8} = \frac{11}{8}\end{aligned}$$

Thus our solution, $x = \frac{5}{8}$ is correct.