Sample Problems

1. Simplify each of the following.

   a) $\dfrac{2a-5}{5-2a}$ b) $\dfrac{x^3-x}{x+1}$ c) $\dfrac{2x+1}{4x^2-1}$ d) $\dfrac{x^2-4x+3}{x^2+2x-15}$ e) $\dfrac{(x+5)-2}{5(x+2)-(x-2)}$

2. Perform the indicated operations and simplify.

   a) $\dfrac{c}{5a}\cdot\dfrac{15a^2b}{3b^2c}$ c) $\dfrac{x^2-3x}{x^2-8x+15}\cdot\dfrac{x^2-16x+15}{x^2-x}$ e) $\dfrac{x^2-10x+25}{x^2-10x+24}\left(\dfrac{x^2-2x-8}{x^2-6x+5}\div\dfrac{x-5}{x-1}\right)$

   b) $\dfrac{5x-30}{x^2-36}\cdot\dfrac{3x+18}{5}$ d) $\dfrac{x^2-9}{x^2-4x-21}\div\dfrac{4x-12}{3x-21}$

Practice Problems

1. Simplify each of the following.

   a) $\dfrac{2b-5}{10-4b}$ c) $\dfrac{4t^2-9}{4t-6}$ e) $\dfrac{3m+m^2-10}{11m+m^2+30}$ g) $\dfrac{-7x-11-3(x-2)}{5x-11+3(x+5)}$

   b) $\dfrac{x^2-1}{x+1}$ d) $\dfrac{p^2-p}{p^2-1}$ f) $\dfrac{x^2-x-2}{x^2-5x+6}$

2. Perform the indicated operations and simplify.

   a) $\dfrac{x}{5yz}\cdot\dfrac{10x^2y^3z}{4xy^2}$ c) $\dfrac{x^2-5x}{x^2-2x-15}\cdot\dfrac{x^2-9}{x^2-3x}$ e) $\dfrac{5y-35}{y^2-2y-35}\cdot\dfrac{3y+15}{5y-5}$

   b) $\dfrac{a^2-8a+16}{a}\cdot\dfrac{a^3}{4-a}$ d) $\dfrac{x^2-4x-21}{x^2-49}\div\dfrac{8x+x^2+15}{2x+x^2-35}$ f) $\dfrac{2x^2-98}{x^2-6x-7}\div\dfrac{21+3x}{6x^2-6}$

Sample Problems – Answers

1. a) -1 b) $x^2 - x$ c) $\dfrac{1}{2x-1}$ d) $\dfrac{x-1}{x+5}$ e) $\dfrac{1}{4}$

2. a) $\dfrac{a}{b}$ b) 3 c) $\dfrac{x-15}{x-5}$ d) $\dfrac{3}{4}$ e) $\dfrac{x+2}{x-6}$

Practice Problems – Answers

1. a) $-\dfrac{1}{2}$ b) $x-1$ c) $\dfrac{2t+3}{2}$ d) $\dfrac{p}{p+1}$ e) $\dfrac{m-2}{m+6}$ f) $\dfrac{x+6}{x-8}$ g) $-\dfrac{5}{4}$

2. a) $\dfrac{x^2}{2}$ b) $-a^2(a-4)$ c) 1 d) $\dfrac{x-5}{x+5}$ e) $\dfrac{3}{y-1}$ f) $4x-4$

Sample Problems – Solutions

1. Simplify each of the following.

   a) $\dfrac{2a-5}{5-2a} =$ -1

   Solution: We need to notice that the numerator and denominator are opposites of each other. Indeed, the opposite of $2a-5$ is $5-2a$ since

   $$-1\,(2a-5) = -2a+5 = 5-2a$$

   Thus

   $$\frac{2a-5}{5-2a} = \frac{2a-5}{-1\,(2a-5)} = \frac{1}{-1} = -1$$

   b) $\dfrac{x^3-x}{x+1} =$ x^2-x

   Solution: In general, we factor both numerator and denominator and then simplify. In this case we only factor the numerator, since the denominator is too small to factor. After we factor out the greatest common factor (or GCF) which is x, the expression factors via the difference of squares theorem.

   $$x^3-x = x\left(x^2-1\right) = x\,(x+1)\,(x-1)$$

   Then we simplify the fraction by canceling out the same factor from numerator and denominator.

   $$\frac{x^3-x}{x+1} = \frac{x\,(x+1)\,(x-1)}{x+1} = x\,(x-1) \quad \text{or} \quad x^2-x$$

   c) $\dfrac{2x+1}{4x^2-1} =$ $\dfrac{1}{2x-1}$

   Solution: We factor the denominator via the difference of squares theorem, and then cancel.

   $$\frac{2x+1}{4x^2-1} = \frac{2x+1}{(2x+1)\,(2x-1)} = \frac{1}{2x-1}$$

   d) $\dfrac{x^2-4x+3}{x^2+2x-15} =$ $\dfrac{x-1}{x+5}$

   Solution: We factor both numerator and denominator and then simplify. We can easily factor both of these polynomials by trial and error.

   $$\frac{x^2-4x+3}{x^2+2x-15} = \frac{(x-3)\,(x-1)}{(x+5)\,(x-3)} = \frac{x-1}{x+5}$$

   e) $\dfrac{(x+5)-2}{5\,(x+2)-(x-2)} =$ $\dfrac{1}{4}$

   Solution: We simplify both numerator and denominator, then if possible, factor these and then simplify the fraction by cancellation.

   $$\frac{(x+5)-2}{5\,(x+2)-(x-2)} = \frac{x+5-2}{5x+10-x+2} = \frac{x+3}{4x+12} = \frac{x+3}{4\,(x+3)} = \frac{1}{4}$$

2. Perform the indicated operations and simplify.

   a) $\dfrac{c}{5a}\cdot\dfrac{15a^2b}{3b^2c} =$ $\dfrac{a}{b}$

   Solution: We perform the multiplication among fractions (top by top, bottom by bottom) and then simplify by canceling factors appearing in both the numerator and denominator.

   $$\frac{c}{5a}\cdot\frac{15a^2b}{3b^2c} = \frac{15a^2bc}{15ab^2c} = \frac{a}{b}$$

b) $\dfrac{5x-30}{x^2-36}\cdot\dfrac{3x+18}{5} = 3$

Solution: we will factor whatever we can and then simplify by canceling factors appearing in both the numerator and denominator.

$$\frac{5x-30}{x^2-36}\cdot\frac{3x+18}{5} = \frac{5(x-6)}{(x+6)(x-6)}\cdot\frac{3(x+6)}{5} = 3$$

c) $\dfrac{x^2-3x}{x^2-8x+15}\cdot\dfrac{x^2-16x+15}{x^2-x} = \dfrac{x-15}{x-5}$

Solution: We factor whatever we can and then simplify by canceling factors appearing in both the numerator and denominator. We can factor all of these polynomials by completing the square or by factoring out the greatest common factor.

$$\frac{x^2-3x}{x^2-8x+15}\cdot\frac{x^2-16x+15}{x^2-x} = \frac{x(x-3)}{(x-3)(x-5)}\cdot\frac{(x-1)(x-15)}{x(x-1)} = \frac{x}{x-5}\cdot\frac{x-15}{x} = \frac{x-15}{x-5}$$

d) $\dfrac{x^2-9}{x^2-4x-21}\div\dfrac{4x-12}{3x-21} = \dfrac{3}{4}$

Solution: We first re-write the division as multiplication by the reciprocal.

$$\frac{x^2-9}{x^2-4x-21}\div\frac{4x-12}{3x-21} = \frac{x^2-9}{x^2-4x-21}\cdot\frac{3x-21}{4x-12}$$

We now factor the polynomials appearing in the fractions

$$\begin{aligned} x^2-9 &= (x+3)(x-3) & \qquad 4x-12 &= 4(x-3)\\ x^2-4x-21 &= (x+3)(x-7) & \qquad 3x-21 &= 3(x-7)\end{aligned}$$

We now re-write the fractions using these factored forms, and cancel out factors appearing in both numerator and denominator of the product.

$$\frac{x^2-9}{x^2-4x-21}\cdot\frac{3x-21}{4x-12} = \frac{(x+3)(x-3)}{(x+3)(x-7)}\cdot\frac{3(x-7)}{4(x-3)} = \frac{3(x+3)(x-3)(x-7)}{4(x+3)(x-3)(x-7)} = \frac{3}{4}$$

e) $\dfrac{x^2-10x+25}{x^2-10x+24}\left(\dfrac{x^2-2x-8}{x^2-6x+5}\div\dfrac{x-5}{x-1}\right) = \dfrac{x+2}{x-6}$

Solution: We first re-write the division as multiplication by the reciprocal.

$$\frac{x^2-10x+25}{x^2-10x+24}\left(\frac{x^2-2x-8}{x^2-6x+5}\div\frac{x-5}{x-1}\right) = \frac{x^2-10x+25}{x^2-10x+24}\left(\frac{x^2-2x-8}{x^2-6x+5}\cdot\frac{x-1}{x-5}\right)$$

We now factor the polynomials appearing in each fraction.

$$\begin{aligned} x^2-10x+25 &= (x-5)^2 & \qquad x^2-2x-8 &= (x+2)(x-4)\\ x^2-10x+24 &= (x-4)(x-6) & \qquad x^2-6x+5 &= (x-1)(x-5)\end{aligned}$$

We now re-write the problem, using the factored form of polynomials.

$$\frac{x^2-10x+25}{x^2-10x+24}\left(\frac{x^2-2x-8}{x^2-6x+5}\cdot\frac{x-1}{x-5}\right) = \frac{(x-5)^2}{(x-4)(x-6)}\left(\frac{(x+2)(x-4)}{(x-1)(x-5)}\cdot\frac{x-1}{x-5}\right)$$

We now perform the multiplication within the parentheses. Notice that we can cancel out $x-1$.

$$\frac{(x-5)^2}{(x-4)(x-6)}\left(\frac{(x+2)(x-4)}{(x-1)(x-5)}\cdot\frac{x-1}{x-5}\right) = \frac{(x-5)^2}{(x-4)(x-6)}\cdot\frac{(x+2)(x-4)}{(x-5)^2}$$

We can now cancel out $(x-5)^2$ and $x-4$, and the final answer is $\dfrac{x+2}{x-6}$.

