Sample Problems

1. Consider the equation $2x^2+x+34=21x-8$. In case of each number given, determine whether it is a solution of the equation or not.

   a) $x=1$ b) $x=3$ c) $x=4$ d) $x=7$

2. Consider the equation $x^2-10x+x^3-4=4(x+5)$. In case of each number given, determine whether it is a solution of the equation or not.

   a) $x=0$ b) $x=-2$ c) $x=-3$ d) $x=2$

3. Consider the equation $3a-2b-1=(a-b)^2+4$. In case of each pair of numbers given, determine whether it is a solution of the equation or not.

   a) $a=8$ and $b=5$ b) $a=10$ and $b=7$

4. Consider the inequality $3(2y-1)+1\leq 5y-7$. In case of each number given, determine whether it is a solution of the inequality or not.

   a) $y=-10$ b) $y=3$ c) $y=-5$ d) $y=0$

5. Consider the inequality $\dfrac{2x+1}{3}+5<\dfrac{3x-1}{2}$. In case of each number given, determine whether it is a solution of the inequality or not.

   a) $x=1$ b) $x=13$ c) $x=7$ d) $x=-5$

Practice Problems

1. Consider the equation $\dfrac{2x^2-11x-21}{2x+3}=3x-(2x+7)$. In case of each number given, determine whether it is a solution of the equation or not.

   a) $x=8$ b) $x=13$ c) $x=10$

2. Consider the equation $-x^2-2x\left(3-x^2\right)=-x+2$. In case of each number given, determine whether it is a solution of the equation or not.

   a) $x=0$ b) $x=1$ c) $x=-1$ d) $x=2$ e) $x=-2$

3. Consider the equation $y=\dfrac{5x-3}{2}$. In case of each pair of numbers given, determine whether it is a solution of the equation or not.

   a) $x=1$ and $y=1$ b) $x=9$ and $y=4$ c) $x=3$ and $y=6$ d) $x=17$ and $y=41$

4. Consider the equation $(p-q)^2+\dfrac{3p-1}{6-q}=4(p+1)$. In case of each pair of numbers given, determine whether it is a solution of the equation or not.

   a) $p=8$ and $q=5$ b) $p=7$ and $q=1$

5. Consider the inequality $-x+2<-x^2+2(x+6)$. In case of each number given, determine whether it is a solution of the inequality or not.

   a) $x=-5$ b) $x=-2$ c) $x=0$ d) $x=3$ e) $x=7$

6. Consider the inequality $\dfrac{x}{3}+1\geq\dfrac{x+1}{2}-1$. In case of each number given, determine whether it is a solution of the inequality or not.

   a) $x=-9$ b) $x=-3$ c) $x=27$ d) $x=15$ e) $x=-15$

Answers - Sample Problems

1. a) $37 \neq 13$ no b) $55 = 55$ yes c) $70 \neq 76$ no d) $139 = 139$ yes
2. a) $-4 \neq 20$ no b) $12 = 12$ yes c) $8 = 8$ yes d) $-12 \neq 28$ no
3. a) $13 = 13$ yes b) $15 \neq 13$ no
4. a) $-62 \leq -57$ yes b) $16 \not\leq 8$ no c) $-32 \leq -32$ yes d) $-2 \not\leq -7$ no
5. a) $6 \not< 1$ no b) $14 < 19$ yes c) $10 < 10$ no d) $2 < -8$ no

Answers - Practice Problems

1. a) $1 = 1$ yes b) $6 = 6$ yes c) $3 = 3$ yes
2. a) $0 \neq 2$ no b) $-5 \neq 1$ no c) $3 = 3$ yes d) $0 = 0$ yes e) $-8 \neq 4$ no
3. a) $1 = 1$ yes b) $4 \neq 21$ no c) $6 = 6$ yes d) $41 = 41$ no
4. a) $32 \neq 36$ no b) $40 \neq 32$ no
5. a) $7 \not< -23$ no b) $4 \not< 4$ no c) $2 < 12$ yes d) $-1 < 9$ yes e) $-5 \not< -23$ no
6. a) $-2 \geq -5$ yes b) $0 \geq -2$ yes c) $10 \not\geq 13$ no d) $6 \not\geq 7$ no e) $-4 \geq -8$ yes

Sample Problems - Solutions

1. Consider the equation $2x^2 + x + 34 = 21x - 8$. In case of each number given, determine whether it is a solution of the equation or not.

   a) $x = 1$
   Solution: We need to substitute 1 for x into both sides of the equation and evaluate those algebraic expressions to see whether the left-hand side equals to the right-hand side. The left-hand side:

   $$\text{LHS} = 2x^2 + x + 34 = 2(1)^2 + (1) + 34 = 2 \cdot 1 + 1 + 34 = 2 + 1 + 34 = 3 + 34 = 37$$

   The right-hand side:

   $$\text{RHS} = 21x - 8 = 21(1) - 8 = 21 - 8 = 13$$

   When $x = 1$, the two expressions are not equal. Thus 1 is not a solution of the equation.

   b) $x = 3$
   Solution: We need to substitute 3 for x into both the left-hand side and right-hand side of the equation and evaluate those algebraic expressions to see whether the left-hand side equals to the right-hand side.
   The left-hand side:

   $$\text{LHS} = 2x^2 + x + 34 = 2(3)^2 + (3) + 34 = 2 \cdot 9 + 3 + 34 = 18 + 3 + 34 = 21 + 34 = 55$$

   The right-hand side:

   $$\text{RHS} = 21x - 8 = 21(3) - 8 = 63 - 8 = 55$$

   When $x = 3$, the two expressions are equal. Thus 3 is a solution of the equation.

c) $x = 4$
Solution: We need to substitute 4 for x into both sides of the equation and evaluate those algebraic expressions to see whether the left-hand side equals to the right-hand side.
The left-hand side:

$$\text{LHS} = 2x^2 + x + 34 = 2(4)^2 + (4) + 34 = 2 \cdot 16 + 4 + 34 = 32 + 4 + 34 = 36 + 34 = 70$$

The right-hand side:

$$\text{RHS} = 21x - 8 = 21(4) - 8 = 84 - 8 = 76$$

Since the two expressions are not equal when $x = 4$, 4 is not a solution of the equation.

d) $x = 7$
Solution: We need to substitute 7 for x into both sides of the equation and evaluate those algebraic expressions to see whether the left-hand side equals to the right-hand side.
The left-hand side:

$$\text{LHS} = 2x^2 + x + 34 = 2(7)^2 + (7) + 34 = 2 \cdot 49 + 7 + 34 = 98 + 7 + 34 = 105 + 34 = 139$$

The right-hand side:

$$\text{RHS} = 21x - 8 = 21(7) - 8 = 147 - 8 = 139$$

Since the two expressions are equal when $x = 7$, 7 is a solution of the equation.

2. Consider the equation $x^2 - 10x + x^3 - 4 = 4(x + 5)$. In each case, determine whether the number given is a solution of the equation or not.

a) $x = 0$
Solution: We simply evaluate both sides of the equation when $x = 0$.

$$\text{LHS} = (0)^2 - 10(0) + (0)^3 - 4 = 0 - 10 \cdot 0 + 0 - 4 = 0 - 0 + 0 - 4 = -4$$

$$\text{RHS} = 4((0) + 5) = 4(0 + 5) = 4 \cdot 5 = 20$$

Since $-4 \neq 20$, $x = 0$ is not a solution of this equation.

b) $x = -2$
Solution: We simply evaluate both sides of the equation when $x = -2$.

$$\begin{aligned} \text{LHS} &= (-2)^2 - 10(-2) + (-2)^3 - 4 = 4 - 10(-2) + (-8) - 4 = 4 - (-20) + (-8) - 4 \\ &= 24 + (-8) - 4 = 16 - 4 = 12 \end{aligned}$$

$$\text{RHS} = 4((-2) + 5) = 4 \cdot 3 = 12$$

Since $12 = 12$, $x = -2$ is a solution of this equation.

c) $x = -3$
Solution: We simply evaluate both sides of the equation when $x = -3$.

$$\begin{aligned} \text{LHS} &= (-3)^2 - 10(-3) + (-3)^3 - 4 = 9 - 10(-3) + (-27) - 4 = 9 - (-30) + (-27) - 4 \\ &= 39 + (-27) - 4 = 12 - 4 = 8 \end{aligned}$$

$$\text{RHS} = 4((-3) + 5) = 4(-3 + 5) = 4 \cdot 2 = 8$$

Since $8 = 8$, $x = -3$ is a solution of this equation.

3. Consider the equation $3a - 2b - 1 = (a-b)^2 + 4$. In case of each pair of numbers given, determine whether it is a solution of the equation or not.

a) $a = 8$ and $b = 5$
Solution: We need to substitute $a = 8$ and $b = 5$ into both sides of the equation and evaluate those algebraic expressions to see whether the left-hand side equals to the right-hand side. The left-hand side:

$$\begin{aligned} \text{LHS} &= 3a - 2b - 1 = 3(8) - 2(5) - 1 = 3 \cdot 8 - 2 \cdot 5 - 1 = 24 - 2 \cdot 5 - 1 = 24 - 10 - 1 \\ &= 14 - 1 = 13 \end{aligned}$$

The right-hand side:

$$\text{RHS} = (a-b)^2 + 4 = ((8) - (5))^2 + 4 = (8-5)^2 + 4 = 3^2 + 4 = 9 + 4 = 13$$

Since the two expressions are equal when $a = 8$ and $b = 5$, this pair is a solution of the equation.

b) $a = 10$ and $b = 7$
Solution: We need to substitute $a = 10$ and $b = 7$ into both sides of the equation and evaluate those algebraic expressions to see whether the left-hand side equals to the right-hand side.
The left-hand side:

$$\begin{aligned} \text{LHS} &= 3a - 2b - 1 = 3(10) - 2(7) - 1 = 3 \cdot 10 - 2 \cdot 7 - 1 = 30 - 2 \cdot 7 - 1 = 30 - 14 - 1 \\ &= 16 - 1 = 15 \end{aligned}$$

The right-hand side:

$$\text{RHS} = (a-b)^2 + 4 = ((10) - (7))^2 + 4 = (10-7)^2 + 4 = 3^2 + 4 = 9 + 4 = 13$$

Since the two expressions are not equal when $a = 8$ and $b = 5$, this pair is NOT a solution of the equation.

4. Consider the inequality $3(2y - 1) + 1 \leq 5y - 7$. In case of each number given, determine whether it is a solution of the inequality or not.

a) $y = -10$
Solution: We need to substitute $y = -10$ into both sides of the inequality and evaluate those algebraic expressions to see whether the left-hand side is indeed less than or equal to the right-hand side. The left-hand side:

$$\text{LHS} = 3(2(-10) - 1) + 1 = 3(-20 - 1) + 1 = 3(-21) + 1 = -63 + 1 = -62$$

The right-hand side:

$$\text{RHS} = 5(-10) - 7 = -50 - 7 = -57$$

So the statement $3(2y - 1) + 1 \leq 5y - 7$ becomes $-62 \leq -57$. Since this is a true statement, $y = -10$ is a solution of the inequality.

b) $y = 3$
Solution: We need to substitute $y = 3$ into both sides of the inequality and evaluate those algebraic expressions to see whether the left-hand side is indeed less than or equal to the right-hand side. The left-hand side:

$$\text{LHS} = 3(2(3) - 1) + 1 = 3(6 - 1) + 1 = 3(5) + 1 = 15 + 1 = 16$$

The right-hand side:

$$\text{RHS} = 5(3) - 7 = 15 - 7 = 8$$

So the statement $3(2y - 1) + 1 \leq 5y - 7$ becomes $16 \leq 8$. Since this is a false statement, $y = 3$ is not a solution of the inequality.

c) $y = -5$
Solution: We need to substitute $y = -5$ into both sides of the inequality and evaluate those algebraic expressions to see whether the left-hand side is indeed less than or equal to the right-hand side. The left-hand side:

$$\text{LHS} = 3(2(-5) - 1) + 1 = 3(-10 - 1) + 1 = 3(-11) + 1 = -33 + 1 = -32$$

The right-hand side:

$$\text{RHS} = 5(-5) - 7 = -25 - 7 = -32$$

So the statement $3(2y - 1) + 1 \leq 5y - 7$ becomes $-32 \leq -32$. Since this is a true statement, $y = -5$ is a solution of the inequality.

d) $y = 0$
Solution: We need to substitute $y = 0$ into both sides of the inequality and evaluate those algebraic expressions to see whether the left-hand side is indeed less than or equal to the right-hand side. The left-hand side:

$$\text{LHS} = 3(2(0) - 1) + 1 = 3(0 - 1) + 1 = 3(-1) + 1 = -3 + 1 = -2$$

The right-hand side:

$$\text{RHS} = 5(0) - 7 = 0 - 7 = -7$$

So the statement $3(2y - 1) + 1 \leq 5y - 7$ becomes $-2 \leq -7$. Since this is a false statement, $y = 0$ is not a solution of the inequality.

5. Consider the inequality $\dfrac{2x+1}{3} + 5 < \dfrac{3x-1}{2}$. In case of each number given, determine whether it is a solution of the inequality or not.

a) $x = 1$
Solution: We need to substitute $x = 1$ into both sides of the inequality and evaluate those algebraic expressions to see whether the left-hand side is indeed less than the right-hand side. The left-hand side:

$$\text{LHS} = \frac{2(1)+1}{3} + 5 = \frac{2+1}{3} + 5 = \frac{3}{3} + 5 = 1 + 5 = 6$$

The right-hand side:

$$\text{RHS} = \frac{3(1)-1}{2} = \frac{3-1}{2} = \frac{2}{2} = 1$$

So the statement $\dfrac{2x+1}{3} + 5 < \dfrac{3x-1}{2}$ becomes $6 < 1$. Since this is a false statement, $x = 1$ is not a solution of the inequality.

b) $x = 13$
Solution: We need to substitute $x = 13$ into both sides of the inequality and evaluate those algebraic expressions to see whether the left-hand side is indeed less than the right-hand side. The left-hand side:

$$\text{LHS} = \frac{2(13)+1}{3} + 5 = \frac{26+1}{3} + 5 = \frac{27}{3} + 5 = 9 + 5 = 14$$

The right-hand side:

$$\text{RHS} = \frac{3(13)-1}{2} = \frac{39-1}{2} = \frac{38}{2} = 19$$

So the statement $\dfrac{2x+1}{3} + 5 < \dfrac{3x-1}{2}$ becomes $14 < 19$. Since this is a true statement, $x = 13$ is a solution of the inequality.

c) $x = 7$
Solution: We need to substitute $x = 7$ into both sides of the inequality and evaluate those algebraic expressions to see whether the left-hand side is indeed less than the right-hand side. The left-hand side:

$$\text{LHS} = \frac{2(7)+1}{3} + 5 = \frac{14+1}{3} + 5 = \frac{15}{3} + 5 = 5 + 5 = 10$$

The right-hand side:

$$\text{RHS} = \frac{3(7)-1}{2} = \frac{21-1}{2} = \frac{20}{2} = 10$$

So the statement $\dfrac{2x+1}{3} + 5 < \dfrac{3x-1}{2}$ becomes $10 < 10$. Since this is a false statement, $x = 7$ is not a solution of the inequality.

d) $x = -5$
Solution: We need to substitute $x = -5$ into both sides of the inequality and evaluate those algebraic expressions to see whether the left-hand side is indeed less than the right-hand side. The left-hand side:

$$\text{LHS} = \frac{2(-5)+1}{3} + 5 = \frac{-10+1}{3} + 5 = \frac{-9}{3} + 5 = -3 + 5 = 2$$

The right-hand side:

$$\text{RHS} = \frac{3(-5)-1}{2} = \frac{-15-1}{2} = \frac{-16}{2} = -8$$

So the statement $\dfrac{2x+1}{3} + 5 < \dfrac{3x-1}{2}$ becomes $2 < -8$. Since this is a false statement, $x = -5$ is not a solution of the inequality.

