Sample Problems

1. Solve each of the following system of linear equations.

   a) $\begin{cases} 2x - y = 16 \\ 3x + 5y = 11 \end{cases}$ b) $\begin{cases} x + 3y = 11 \\ 12y = -4x + 7 \end{cases}$ c) $\begin{cases} 4y = 6x + 10 \\ 3x - 2y = -5 \end{cases}$

2. There is an animal farm where chickens and cows live. All together, there are 85 heads and 238 legs. How many chickens and how many cows are there on the farm?

3. We have a jar of coins, all pennies and dimes. All together, we have 372 coins, and the total value of all coins in the jar is \$20.91. How many pennies are there in the jar?

4. We invested \$7000 into two bank accounts. One account earns 14% per year, the other account earns 9% per year. How much did we invest into each account if after the first year, the combined interest from the two accounts is \$840?

5. How many gallons of each of a 4% and an 11% salt solutions should be mixed to obtain 35 gallons of a 7% solution?

Practice Problems

1. Solve each of the following system of linear equations.

   a) $\begin{cases} 3x + y = -4 \\ x - 3y = -8 \end{cases}$ d) $\begin{cases} \frac{1}{2}x + \frac{1}{4}y = 5 \\ \frac{1}{2}y - \frac{1}{3}x = -6 \end{cases}$ g) $\begin{cases} 2x + 3y = 3 \\ 5x - 2y = 4 \end{cases}$

   b) $\begin{cases} 5(p-1) - 2(q-1) = 22 \\ p - q = 8 \end{cases}$ e) $\begin{cases} 2x - y = 1 \\ 2(y-3) = 6(x-1) \end{cases}$ h) $\begin{cases} 3x - 2y = -8 \\ -2x + 3y = 12 \end{cases}$

   c) $\begin{cases} a + 3b = 10 \\ b = \dfrac{-a - 10}{3} \end{cases}$ f) $\begin{cases} 2a + 3b = 4 \\ 4a = -6b + 8 \end{cases}$ i) $\begin{cases} 2r - 0.5s = -1.7 \\ 1.5r + s = 0.65 \end{cases}$

2. Given the equations of two straight lines, find both coordinates of all intersection points.

   a) $2x - 5y = -41$ and $x + y = 4$

   b) $x + y = -5$ and $2y = -2x - 10$

   c) $y = \frac{3}{4}x - 2$ and $3x - 4y = -24$

   d) $5x - y = -35$ and $y = -\frac{3}{4}x + \frac{1}{2}$

   e) $y = -\frac{2}{3}x + 2$ and $2x + 3y = 6$

3. There is an animal farm where chickens and cows live. All together, there are 52 heads and 134 legs. How many chickens and how many cows are there on the farm?

4. We invested \$9700 into two bank accounts. One account earns 7% per year, the other account earns 12% per year. How much did we invest into each account if after the first year, the combined interest from the two accounts is \$1004?

5. We have 54 coins, all dimes and quarters, in the total value of \$10.05. How many quarters and how many dimes are there?

6. We invested \$7800 into two bank accounts. One account earns 9% per year, the other account earns 10% per year. How much did we invest into each account if after the first year we have a total of \$8549 in the accounts?
7. How many gallons of each of a 22% and a 10% salt solutions should be mixed to obtain 72 gallons of a 13% solution?
8. How many gallons of each of a 41% and a 20% sugar solutions should be mixed to obtain 147 gallons of a 26% sugar solution?

Sample Problems – Answers

1. a) $x = 7, y = -2$ b) there is no solution
   c) x can be any number, and then $y = \dfrac{3x+5}{2}$
2. 51 chickens and 34 cows
3. 181 pennies
4. \$4200 at 14% and \$2800 at 9%
5. 20 gallons of 4% solution with 15 gallons of 11% solution

Practice Problems – Answers

1. a) $x = -2, y = 2$ b) $p = 3, q = -5$ c) there is no solution d) $x = 12, y = -4$
   e) $x = -1, y = -3$
   f) there are infintely many solutions; a can be any number and then $b = \dfrac{4-2a}{3}$
   g) $x = \dfrac{18}{19}, y = \dfrac{7}{19}$ h) $x = 0, y = 4$ i) $r = -0.5, s = 1.4$
2. a) $(-3, 7)$ b) all points on $y = -x - 5$ are common; the two lines given are identical
   c) no common points; the two lines given are parallel d) $(-6, 5)$
   e) all points on $y = -\dfrac{2}{3}x + 2$ are common; the two lines given are identical.
3. 37 chickens, 15 cows
4. \$3200 at 7% and \$6500 at 12%
5. 23 dimes and 31 quarters
6. \$3100 at 9% and \$4700 at 10%
7. 18 gallons of 22% and 54 gallons of 10% solution
8. 42 gallons of 41% solution and 105 gallons of 20% solution

Sample Problems – Solutions

1. a) $\begin{cases} 2x - y = 16 \\ 3x + 5y = 11 \end{cases}$

Solution: We will first solve for y in terms of x in the first equation.

$$\begin{aligned} 2x - y &= 16 && \text{add } y \\ 2x &= y + 16 && \text{subtract } 16 \\ 2x - 16 &= y \end{aligned}$$

We substitute $y = 2x - 16$ into the second equation and solve for x.

$$\begin{aligned} 3x + 5y &= 11 \\ 3x + 5(2x - 16) &= 11 \\ 3x + 10x - 80 &= 11 \\ 13x - 80 &= 11 && \text{add } 80 \\ 13x &= 91 && \text{divide by } 13 \\ x &= 7 \end{aligned}$$

Now that we know the value of x, we can easily compute y since $y = 2x - 16$

$$y = 2x - 16 = 2 \cdot 7 - 16 = -2$$

Thus the solution is $x = 7,\ \ y = -2$. We check: the pair should be a solution for both equations.

$$\begin{aligned} 2 \cdot 7 - (-2) &= 14 + 2 = 16 \ \checkmark \\ 3 \cdot 7 + 5(-2) &= 21 - 10 = 11 \ \checkmark \end{aligned}$$

Thus our solution is correct.

b) $\begin{cases} x + 3y = 11 \\ 12y = -4x + 7 \end{cases}$

Solution: We will first solve for x in terms of y in the first equation.

$$\begin{aligned} x + 3y &= 11 && \text{subtract } 3y \\ x &= -3y + 11 && \text{divide by } 3 \end{aligned}$$

We substitute $x = -3y + 11$ into the second equation and solve for y.

$$\begin{aligned} 12y &= -4x + 7 \\ 12y &= -4(-3y + 11) + 7 && \text{distribute} \\ 12y &= 12y - 44 + 7 \\ 12y &= 12y - 37 && \text{subtract } 12y \\ 0 &= -37 \end{aligned}$$

Since there is no value for y that could make the statement $0 = -37$ true, there is no solution for this system. A linear system like this is called an **inconsistent system**.

c) $\begin{cases} 4y = 6x + 10 \\ 3x - 2y = -5 \end{cases}$

Solution: We will first solve for y in terms of x in the second equation.

$$\begin{aligned} 3x - 2y &= -5 && \text{add } 2y \\ 3x &= 2y - 5 && \text{add } 5 \\ 3x + 5 &= 2y && \text{divide by } 2 \\ \frac{3x+5}{2} &= y \end{aligned}$$

We substitute $y = \dfrac{3x+5}{2}$ into the first equation and solve for x.

$$\begin{aligned} 4y &= 6x + 10 \\ 4\left(\frac{3x+5}{2}\right) &= 6x + 10 \\ \frac{4(3x+5)}{2} &= 6x + 10 && \text{simplify} \\ 2(3x+5) &= 6x + 10 && \text{distribute} \\ 6x + 10 &= 6x + 10 && \text{subtract } 6x \\ 10 &= 10 \end{aligned}$$

$10 = 10$ is in fact true for all values of x. What happens here, one equation establishes a connection between x and y, namely, $y = \dfrac{3x+5}{2}$. The other equation does not contain any new information, it is just a disguised re-statement of the same connection. A system like this is called a **dependent system**. This system has infinitely many solutions. x can take any value, and then y must be $y = \dfrac{3x+5}{2}$. Thus, there are infinitely many solutions.

2. There is an animal farm where chickens and cows live. All together, there are 85 heads and 238 legs. How many chickens and how many cows are there on the farm?
Solution: We will denote the number of chickens by x and the number of cows by y. The first equation will express the number of heads, the second equation will express the number of legs.

$$\begin{aligned} x + y &= 85 \\ 2x + 4y &= 238 \end{aligned}$$

To simplify our system, we divide the second equation by 2.

$$\begin{aligned} x + y &= 85 \\ x + 2y &= 119 \end{aligned}$$

We solve for y in terms of x in the first equation: $y = 85 - x$ We substitute this into the second equation:

$$\begin{aligned} x + 2y &= 119 \\ x + 2(85 - x) &= 119 && \text{distribute } 2 \\ x + 170 - 2x &= 119 && \text{combine like terms} \\ -x + 170 &= 119 && \text{subtract } 170 \\ -x &= -51 && \text{multiply by } -1 \\ x &= 51 \end{aligned}$$

Now that we know the value of x, we compute y.

$$y = 85 - x = 85 - 51 = 34$$

Thus we have 51 chickens and 34 cows. We check: the number of heads is $51 + 34 = 85$, and the number of legs is $2\,(51) + 4\,(34) = 102 + 136 = 238$. So our solution is correct.

3. We have a jar of coins, all pennies and dimes. All together, we have 372 coins, and the total value of all coins in the jar is \$20.91. How many pennies are there in the jar?

   Solution: Let us denote the number of pennies by x and the number of dimes by y. The number of coins will give us one equation:

   $$x + y = 372$$

   The second equation will express the total value of the coins. We can express it in dollars: each penny is worth 0.01 dollars and each dime is worth 0.1 dollars. So the equation is

   $$0.01x + 0.1y = 20.91$$

   We would want to get rid of the decimals by multiplying both sides of this equation by 100 and then get

   $$x + 10y = 2091$$

   Our second choice is to express the total value of the coins in pennies. That will immediately give us the equation $x + 10y = 2091$. So we need to solve the system

   $$\begin{cases} x + y = 372 \\ x + 10y = 2091 \end{cases}$$

   We will solve this using substitution. From the first equation, $y = 372 - x$. Then the second equation becomes

   $$\begin{aligned} x + 10\,(372 - x) &= 2091 && \text{distribute } 10 \\ x + 3720 - 10x &= 2091 && \text{combine like terms} \\ -9x + 3720 &= 2091 && \text{subtract } 3720 \\ -9x &= -1629 && \text{divide by } -9 \\ x &= 181 \end{aligned}$$

   So we have 181 pennies. To check, we should also figure out the number of dimes.
   It is $y = 372 - x = 372 - 181 = 191$. So, if we have 181 pennies and 191 dimes. Now we check against the conditions stated in the problem:

   $$\begin{aligned} x + y &= 181 + 191 = 372 && \text{total number of coins is } 372 \\ 0.01x + 0.1y &= 0.01\,(181) + 0.1\,(191) = 1.81 + 19.1 = 20.91 && \text{total value of coins is \$20.91} \end{aligned}$$

   and so our solution is correct.

4. We invested \$7000 into two bank accounts. One account earns 14% per year, the other account earns 9% per year. How much did we invest into each account if after the first year, the combined interest from the two accounts is \$840?
Solution: Let us denote the amount invested at 14% by x and the amount invested at 9% by y. The two equations express that

$$\begin{aligned} x + y &= 7000 && \text{the amounts add up to \$7000} \\ 0.14x + 0.09y &= 840 && \text{the interests earned add up to \$840} \end{aligned}$$

We solve the system of equation by substitution. But let us first make the second equation simpler:

$$\begin{aligned} 0.14x + 0.09y &= 840 && \text{multiply by 100} \\ 14x + 9y &= 84\,000 \end{aligned}$$

We now have

$$\begin{aligned} x + y &= 7000 \\ 14x + 9y &= 84\,000 \end{aligned}$$

We will solve for y in terms of x in the first equation: $y = 7000 - x$ and substitute that into the second equation.

$$\begin{aligned} 14x + 9y &= 84\,000 && \\ 14x + 9\,(7000 - x) &= 84\,000 && \text{distribute 9} \\ 14x + 63\,000 - 9x &= 84\,000 && \text{combine like terms} \\ 5x + 63\,000 &= 84\,000 && \text{subtract 63 000} \\ 5x &= 21\,000 && \text{divide by 5} \\ x &= 4200 \end{aligned}$$

Then $y = 7000 - x = 7000 - 4200 = 2800$. Thus we invested \$4200 at 14% and \$2800 at 9%. We check: the amounts add up to \$4200 + \$2800 = \$7000. The interest from the accounts are:

$$14\% \text{ of } 4200 \text{ is } 0.14\,(4200) = 588 \text{ and } 9\% \text{ of } 2800 \text{ is } 0.09\,(2800) = 252$$

Since $588 + 252 = 840$, our solution is correct.

5. How many gallons of each of a 4% and an 11% salt solutions should be mixed to obtain 35 gallons of a 7% solution?

   Solution: Let us denote by x the amount of 4% solution and by y the amount of 11% solution. Clearly, the two amounts should add up to 35 gallons, giving us the equation $x + y = 35$.

	Amount of Solution (gallons)	Percentage	Amount of Solvant (gallons)
Component 1	x	0.04	$0.04x$
Component 2	y	0.11	$0.11y$
Mixture	35	0.07	

   Since we have two unknown variables, we will need two equations. The first one is easy, the volume of the mixture should be 35.

$$x + y = 35$$

   We obtain the second equation by stating that the amount of solvant in the components must add up to the amount of solvant. (In other words, the last entry in the third row can be written in two different ways: the product of 35 and 7%; and the sum of $0.04x$ and 0.11)

$$0.07\,(35) = 0.04x + 0.11y$$

   And this equation can be immediately made much nicer by simply multiplying both sides by 100. Then we have:

$$\begin{aligned} 7\,(35) &= 4x + 11y \\ 4x + 11y &= 245 \end{aligned}$$

   So our system is now

$$\begin{cases} x + y = 35 \\ 4x + 11y = 245 \end{cases}$$

   We solve this system using substitution: $y = 35 - x$ from the first equation. Then the second equation becomes

$$\begin{aligned} 4x + 11\,(35 - x) &= 245 && \text{distribute 11} \\ 4x + 385 - 11x &= 245 && \text{combine like terms} \\ -7x + 385 &= 245 && \text{subtract 385} \\ -7x &= -140 && \text{divide by } -7 \\ x &= 20 \end{aligned}$$

   If $x = 20$, then the other amount, denoted by $y = 35 - x$ must be $35 - 20 = 15$.

   Thus we need to mix 20 gallons of 4% solution with 15 gallons of 11% solution.

   We check our solution: suppose we mix the two solutions specified above. We need to find how much solution and how much solvant we have, hoping that the amount of solvant indeed will be 8% of the amount of mixture.

	Amount of Solution		Amount of Solvant
Component 1	20 gallons of 4% solution	$\Longrightarrow$	$0.04\,(20) = 0.8$ gallons
Component 2	15 gallons of 11% solution	$\Longrightarrow$	$0.11\,(15) = 1.65$ gallons
	$\Downarrow$		$\Downarrow$
	35 gallons		$0.8 + 1.65 = 2.45$ gallons

   7% of 35 is $0.07\,(35) = 2.45$ Thus our solution is correct.

