Part 1 - The 'usual' Average

Sometimes we are dealing with too much information. When we would like to describe a set of numbers using just a single number, the average is a very good start.

> **Definition:** Given a set of numbers, the **average** (also called **arithmetic mean**) of the numbers is the sum of all numbers divided by the number of numbers in the data set.

Example 1. Amy's class had 5 quizzes so far. Her scores (where the highest score was 10) were $4, 9, 7, 2,$ and 8. Compute the average score of her quizzes.

Solution: We add all these numbers and, because we addded five numbers, we divide the sum by 5.

$$\text{av} = \frac{4+9+7+2+8}{5} = \frac{30}{5} = 6$$

What is the meaning of the average? In this case, we have five different scores that add up to 30. The average would be a single number, that, when replaced all five numbers, results in the same sum.

$$4 + 9 + 7 + 2 + 8 = 30 \text{ and } 6 + 6 + 6 + 6 + 6 = 30$$

In a way, average flattens out numbers with respect to addition.

Example 2. Amy's class had 5 quizzes so far. Her scores (where the highest score was 10) were $4, 9, 7, 2,$ and 8. There will be one additional quiz.

    a) What score does Amy need to earn on the sixth quiz so that her quiz average would be 6.5?

    b) What score does Amy need to earn on the sixth quiz so that her quiz average would be 7?

Solution: a) Let us denote the desired score on the sixth quiz by x. We state the average over the six quiz scores. This time we are computing the average of six numbers. We assume the average is 6.5 and solve for x.

$$\frac{4+9+7+2+8+x}{6} = 6.5$$
$$\frac{30+x}{6} = 6.5 \qquad \text{multiply by } 6$$
$$30+x = 39 \qquad \text{subtract } 30$$
$$x = 9$$

Therefore, she needs to earn $\boxed{9 \text{ points}}$ on the last quiz.

    b) The computation is very similar, but this time we use an average of 7.

$$\frac{4+9+7+2+8+x}{6} = 7$$
$$\frac{30+x}{6} = 7 \qquad \text{multiply by } 6$$
$$30+x = 42 \qquad \text{subtract } 30$$
$$x = 12$$

If the maximum score on quizzes is 10 points, our solution 12 indicates that her average $\boxed{\text{cannot be changed to 7 by a single sixth score}}$.

Part 2 - Weighted Average

We use weighted average when some numbers are more important than others in the data set. In this case, we can compute the weighted average if the weight of each number is also given.

Example 3. Thomas is taking a class. At the end of the semester, his results were as shown below. Compute his weighted average for this class.

Assingment type	Exam 1	Exam 2	Quizzes	Homework
Weight	20%	30%	30%	20%
Score	78	92	61	95

Solution: The more important a score is, the higher weight is. To compute this weighted average, imagine that instead of four scores, we have 100. Out of this 100, 20 had score of 78, 30 had score 92, 30 had score of 61, and 20 had score 95. We compute the arithmetic mean (or average) of these 100 numbers.

$$\text{average} = \frac{20 \cdot 78 + 30 \cdot 92 + 30 \cdot 61 + 20 \cdot 95}{100} = \frac{8050}{100} = 80.5$$

So Thomas's weighted average is $\boxed{80.5}$.

Example 4. Latisha is taking a the same class. After the first exam, her results were as shown below. She did not yet take Exam 2.

Assingment type	Exam 1	Exam 2	Quizzes	Homework
Weight	20%	30%	30%	20%
Score	91		85	86

  a) Compute her weighted average for this class at this point.

  b) Can she bring up her average to 90%? If so, what score does she need to earn on Exam 2?

Solution: a) We will count the score for exam 1 as 20 scores, the quiz scores as 30 scores, and the homework score as 20 scores. At the end, we divide by the number of the exams, which is 70.

$$\text{av} = \frac{20 \cdot 91 + 30 \cdot 85 + 20 \cdot 86}{70} = \frac{6090}{70} = 87$$

So her average at this point is $\boxed{87 \text{ points}}$.

  b) Let us denote the score earned on Exam 2 by x, and write an equation for the average. This time we are averaging 100 scores, so we divide by 100.

$$\frac{20 \cdot 91 + 30 \cdot 85 + 20 \cdot 86 + 30 \cdot x}{100} = 90$$

$$\frac{6090 + 30x}{100} = 90 \qquad \text{multiply by 100}$$

$$6090 + 30x = 9000 \qquad \text{subtract 6090}$$

$$30x = 2910 \qquad \text{divide by 30}$$

$$x = 97$$

She can bring up her average to 90%, but she needs to earn $\boxed{97 \text{ points}}$ or more on Exam 2.

Example 5. Sean is taking an English class. After the first two exams, his results were as shown below. He did not yet take Exam 3.

Assingment type	Exam 1	Exam 2	Exam 3	Quizzes	Homework
Weight	15%	20%	30%	25%	10%
Score	72	73		77	96

a) Compute his weighted average for this class at this point.

b) Can he bring up his average to 80%? If so, what score does he need to earn on Exam 3?

c) Can he bring up his average to 90%? If so, what score does he need to earn on Exam 3?

Solution: a) We compute the average using the data known so far. We will need to divide by 70.

$$av = \frac{15 \cdot 72 + 20 \cdot 73 + 25 \cdot 77 + 10 \cdot 96}{70} = \frac{5425}{70} = 77.5$$

So his current weighted average is $\boxed{77.5 \text{ points}}$.

b) Let us denote the score earned on Exam 3 by x, and write an equation for the average. This time we are averaging 100 scores, so we divide by 100.

$$\frac{15 \cdot 72 + 20 \cdot 73 + 25 \cdot 77 + 10 \cdot 96 + 30 \cdot x}{100} = 80$$

$$\frac{5425 + 30x}{100} = 80 \qquad \text{multiply by 100}$$

$$5425 + 30x = 8000 \qquad \text{subtract 5425}$$

$$30x = 2575 \qquad \text{divide by 30}$$

$$x = 85.83333$$

He can bring up his average to 80%, and he needs to earn $\boxed{86 \text{ points}}$ or more on Exam 3.

b) Let us denote the score earned on Exam 3 by x, and write an equation expressing the average. This time we are averaging 100 scores, so we divide by 100.

$$\frac{15 \cdot 72 + 20 \cdot 73 + 25 \cdot 77 + 10 \cdot 96 + 30 \cdot x}{100} = 90$$

$$\frac{5425 + 30x}{100} = 90 \qquad \text{multiply by 100}$$

$$5425 + 30x = 9000 \qquad \text{subtract 5425}$$

$$30x = 3575 \qquad \text{divide by 30}$$

$$x = 119.1666$$

This number tells us that $\boxed{\text{it is not possible}}$ to bring up the average to 90%.

Example 6. Samantha is taking a Chemistry class. Her grades are as shown. She did not yet take Exams 3 or 4.

Assingment type	Exam 1	Exam 2	Exam 3	Exam 4	Quizzes	Homework
Weight	10%	15%	20%	25%	20%	10%
Score	59	73			81	92

    a) Compute her weighted average for this class at this point.

    b) Is it possible for her to get an A in this class? (In most cases, a grade of A means at least 90% course average.)

    c) How much does she need to ear to get a B in this class? (In most cases, a grade of B means at least 80% course average.)

Solution: a) We compute the average using the data known so far. We will need to divide by 55.

$$av = \frac{10 \cdot 59 + 15 \cdot 73 + 20 \cdot 81 + 10 \cdot 92}{55} = \frac{4225}{55} = 77.727$$

So her current average is about 77.73%.

    b) Let us treat Exam 3 and Exam 4 as a single exam with a weight of 45%.

$$\frac{10 \cdot 59 + 15 \cdot 73 + 20 \cdot 81 + 10 \cdot 92 + 45 \cdot x}{100} = 90$$

$$\frac{4225 + 45x}{100} = 90 \qquad \text{multiply by 100}$$

$$4225 + 45x = 9000 \qquad \text{subtract 4225}$$

$$45x = 4775 \qquad \text{divide by 45}$$

$$x = 106.11$$

This number indicates that given the current scores, $\boxed{\text{it is not possible}}$ to finish this class with an A.

    c) Let us treat Exam 3 and Exam 4 as a single exam with a weight of 45%.

$$\frac{10 \cdot 59 + 15 \cdot 73 + 20 \cdot 81 + 10 \cdot 92 + 45 \cdot x}{100} = 80$$

$$\frac{4225 + 45x}{100} = 80 \qquad \text{multiply by 100}$$

$$4225 + 45x = 8000 \qquad \text{subtract 4225}$$

$$45x = 3775 \qquad \text{divide by 45}$$

$$x = 83.866$$

She can get a B but she needs to earn at least $\boxed{84 \text{ points}}$ on both Exam 3 and Exam 4.

Practice Problems

1. Milan is taking a Biology class. After the first two exams, his results were as shown. He did not yet take Exam 3.

Assingment type	Exam 1	Exam 2	Exam 3	Quizzes	Homework
Weight	15%	20%	25%	25%	15%
Score	63	71		68	91

    a) Compute his weighted average for this class at this point.

    b) Can he bring up his average to 80%? If so, what score does he need to earn on Exam 3?

2. Wanda is having a difficult semester. Her grades in her Physics class are as shown. She did not yet take Exam 4.

Assingment type	Exam 1	Exam 2	Exam 3	Exam 4	Quizzes	Homework
Weight	10%	15%	20%	20%	25%	10%
Score	43	61	72		73	83

    a) Compute her weighted average for this class at this point.

    b) What score does she need to earn on Exam 4 for a course average of 70%?

    c) What score does she need to earn on Exam 4 for a course average of 80%?

Answers - Practice Problems

1. a) 72.4% b) It is not possible (we get that he needs to earn 102.8% on Exam 3)

2. a) 68% b) 78% or higher   c) It is not possible (a score of 128% needed)

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