What does factoring mean and why do we do it?

Definition: To factor something means to re-write it as a product.

We factor things for several reasons. For example, reducing a fraction to lowest terms involves factoring both numerator and denominator and then cancelling out all common factors. Another, very important reason for factoring is the Zero Product Rule. It is our only method to solve equations of degree 2, 3, 4, and so on.

Theorem: (The Zero Product Rule) Suppose that we multiply some numbers and the result is zero.
Then:
1.) One of the factors must be zero, and
2.) the values of all other factors are irrelevant.

Example 1. Solve the equation $(x-3)(x-7)=0$.

Solution: If we were to expand the left-hand side, we would get a quadratic expression, and so this equation is quadratic. We will solve this equation by applying the Zero Product Rule.

We are multiplying only two factors, $x-3$ and $x-7$, and the result is zero. The only way this is possible if one of the two factors is zero.

Either $x-3$ is zero (and then we can comfortably ignore the other factor, $x-7$) and solve the linear equation $x-3=0$ for x. Or, the other factor, $x-7$ is zero (and now we don't need to worry about $x-3$). Again, we solve the linear equation for x.

$$(x-3)(x-7)=0$$

$$\begin{aligned} \text{Either } x-3 &= 0 \quad \text{or} \quad x-7=0 \\ x &= 3 \quad \text{or} \quad x=7 \end{aligned}$$

Thus this equation has two solutions, $\boxed{x_1=3 \text{ and } x_2=7}$.

Example 2. Solve the equation $(x-1)(3x+1)(2x-5)=0$

Solution: We multiplied three quantities, and the result was zero. There are only three ways that can happen:

$$\text{Either} \quad x-1=0 \quad \text{or} \quad 3x+1=0 \quad \text{or} \quad 2x-5=0$$

We solve each of the linear equations for x and obtain:

$$\begin{array}{ccccc} x=1 & \text{or} & 3x+1=0 & \text{or} & 2x-5=0 \\ & & 3x=-1 & & 2x=5 \\ x=1 & & x=-\dfrac{1}{3} & & x=\dfrac{5}{2} \end{array}$$

So this equation has three solutions: $\boxed{x_1=1,\ x_2=-\dfrac{1}{3}, \text{ and } x_3=\dfrac{5}{2}}$.

We will leave checking to the reader. It is clear that when we substitute each solution into the original equation, a different factor will be zero, making the product zero.

Example 3. Solve the equation $5(x-2)(x+8)=0$.

Solution: When we apply the Zero Product Rule for the three factors, the first factor, 5 will never be zero, no matter what the value of x. So, even though we have three factors, there are only two solutions of this equation, $x=2$ and $x=-8$.

Example 4. Solve the equation $x^2(x+1)(x-3)=0$.

Solution: We can re-write the product on the left-hand side without exponents:

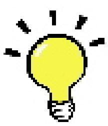
$$x \cdot x \cdot (x+1)(x-3)=0$$

When we apply the Zero Product Rule, the four factors will give us four solutions: 0, 0, -1, and 3. It is clear that these four factors will produce only three solutions since the first and second solutions are identical. The solutions of this equation are -1, 0, and 3.

Practice Problems

Solve each of the following equations.

1. $(x+2)(x-5)=0$
2. $x(x-3)(x+1)=0$
3. $x(x+7)^2(x-10)^3=0$
4. $5(x+2)(x-4)=0$
5. $x(x+1)(x-1)(x+6)=0$
6. $4x^2(2x-1)(3x+7)^2(x+8)=0$
7. Write an equation (it can be in a factored form) with solutions 3 and -6.
8. Write an equation (it can be in a factored form) with solutions 0, 8 and -4.
9. Is it possible to have a seven degree equation with just one solution? Find an example.



Answers - Practice Problems

1. $-2, 5$ 2. $-1, 0, 3$ 3. $-7, 0, 10$ 4. $-2, 4$ 5. $-6, -1, 0, 1$ 7.$-8, -\dfrac{3}{7}, 0, \dfrac{1}{2}$

8. $(x-3)(x+6)=0$ 9. $x(x-8)(x+4)=0$ 10.yes, for example $x^7=0$ has only one solution: $x=0$

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