Work word problems are quite different from the word problems we have seen so far. As we will soon see, the labeling of unknowns and the subsequent equation is unusual at first.

Example 1. It takes Randy four hours to paint a room. It takes Cecile six hours to paint the same room. How long does it take for them to paint the room together?

Solution: Let us denote the job of painting the room by 1. If Randy paints the room in four hours, then he paints $\frac{1}{4}$ of the room in an hour, $\frac{2}{4}$ of the room in two hours, $\frac{3}{4}$ of the room in three hours. In four hours, he paints $\frac{4}{4}$ of the room, which is what we denoted by 1.

Similarly, Cecile paints $\frac{1}{6}$ of the room in an hour, $\frac{2}{6}$ in two hours, $\frac{3}{6}$ in three hours, etc.

Let x denote the time it takes for the two of them to paint the room, measured in hours. In x hours, Randy paints $\frac{x}{4}$ of the room and Ceclie paints $\frac{x}{6}$ of the room. Together, they paint the entire room, denoted by 1.

$$\frac{x}{4}+\frac{x}{6}=1 \qquad \text{bring to common denominator}$$

$$\frac{3x+2x}{12}=1$$

$$\frac{5x}{12}=1 \qquad \text{multiply by } 12$$

$$5x=12 \qquad \text{divide by } 5$$

$$x=\frac{12}{5}=2.4$$

So it takes them together 2.4 hours to paint the room. If we convert the 0.4 hours to minutes, we get

$$0.4\,\text{h}=\frac{0.4\cancel{\text{h}}}{1}\cdot\frac{60\,\text{min}}{1\cancel{\text{h}}}=\frac{24\,\text{min}}{1}=24\,\text{min}$$

Our answer is that they will need 2.4 hours or $\boxed{\text{2 hour and 24 minutes}}$ to paint the room together.

Example 2. A swimming pool can be filled by three water pipes, A, B and C. Pipe A alone can fill the pool in 8 hours. Pipe B alone can fill the pool in 10 hours, and pipe C can fill it in 12 hours. How long does it take to fill the swimming pool using all three pipes?

Solution: Although the situation sounds different, mathematically this problem is similar to the previous example. If pipe A alone can fill the pool in 8 hours, then it fills up $\frac{1}{8}$ of the pool in an hour, $\frac{2}{8}$ in two hours, and $\frac{x}{8}$ in x hours. Pipe B fills up the pool in 10 hours, so it fills up $\frac{1}{10}$ in one hour and $\frac{x}{10}$ in x hours. Similarly, pipe C fills up $\frac{1}{12}$ in an hour and $\frac{x}{12}$ in x hours. We set up the equation as before.

$$\frac{x}{8}+\frac{x}{10}+\frac{x}{12}=1 \qquad \text{factor out } x$$

$$x\left(\frac{1}{8}+\frac{1}{10}+\frac{1}{12}\right)=1$$

$$x\cdot\frac{15+12+10}{120}=1$$

$$x\cdot\frac{37}{120}=1 \qquad \text{divide by } \frac{37}{120}$$

$$x=1\div\frac{37}{120}=1\cdot\frac{120}{37}=\frac{120}{37}$$

Therefore, it will take $\frac{120}{37}\approx 3.2432$ hours to fill the pool using all three pipes. For a more precise answer, we can convert the answer to minutes.

$$\frac{120}{37}\,\text{h} = \frac{120\cancel{h}}{37} \cdot \frac{60\,\text{min}}{1\cancel{h}} = \frac{7200\,\text{min}}{37} \approx 194.6\,\text{min}$$

It will take 194.6 minutes or $\boxed{\text{3 hours and 14.6}}$ minutes for the three pipes to fill the pool together.

Example 3. Ann can paint her room in 10 hours. If Ann and Britney work together, it takes them 6 hours to paint the room. How long would it take for Britney to paint the room alone?

Solution: Let x denote the time it takes for Britney to paint the room. The equation will express that together it will take 6 hours to paint the room.

$6\left(\frac{1}{10}+\frac{1}{x}\right)=1$	distribute 6
$\frac{6}{10}+\frac{6}{x}=1$	subtract $\frac{6}{10}=\frac{3}{5}$
$\frac{6}{x}=1-\frac{3}{5}$	

$\frac{6}{x}=\frac{2}{5}$	take reciprocal of both sides
$\frac{x}{6}=\frac{5}{2}$	multiply by 6
$x=\frac{30}{2}=15$	

Thus it will take $\boxed{\text{15 hours}}$ for Britney to paint the room alone.

Example 4. A swimming pool can be filled by 3 water pipes, A, B and C. Pipe C alone can fill the pool in 8 hours. If pipes B and C are used together, it can be filled in 6 hours. If pipes A and B are used together, it takes 10 hours. How long does it take to fill the pool if all three pipes are used?

Solution: Let x denote the time it takes for pipe C to fill the pool alone. Then pipe C pumps $\frac{1}{x}$ of the pool in one hour. Pipes B and C fill up the pool in 6 hours.

$6\left(\frac{1}{8}+\frac{1}{x}\right)=1$	distribute 6
$\frac{6}{8}+\frac{6}{x}=1$	subtract $\frac{6}{8}=\frac{3}{4}$
$\frac{3}{4}+\frac{6}{x}=1$	

$\frac{6}{x}=1-\frac{3}{4}$	
$\frac{6}{x}=\frac{1}{4}$	take reciprocal of both sides
$\frac{x}{6}=4$	multiply by 6
$x=24$	

Therefore, it takes 24 hours for pipe B to fill up the pool. Let y denote the time it takes for pipe A to fill up the pool. Pipes A and B take 10 hours to fill up the pool.

$10\left(\frac{1}{24}+\frac{1}{y}\right)=1$	distribute 10
$\frac{10}{24}+\frac{10}{y}=1$	subtract $\frac{10}{24}=\frac{5}{12}$
$\frac{10}{y}=1-\frac{5}{12}$	

$\frac{10}{y}=\frac{7}{12}$	take reciprocal of both sides
$\frac{y}{10}=\frac{12}{7}$	multiply by 10
$y=\frac{120}{7}$	

Now we are ready to find the time it takes to fill the pool using all three pipes. Pipe A takes $\frac{120}{7} \approx 17.143$ hours to fill the pool, pipe B takes 24 hours, and pipe C takes 8 hours. Together they fill the pool in t hours.

$$t\left(\frac{1}{8}+\frac{1}{24}+\frac{1}{\frac{120}{7}}\right)=1 \qquad \text{distribute 10}$$

$$t\left(\frac{1}{8}+\frac{1}{24}+\frac{7}{120}\right)=1 \qquad \text{bring to common denominator}$$

$$t\cdot\frac{15+5+7}{120}=1$$

$$t\cdot\frac{27}{120}=1 \qquad \text{divide by } \frac{27}{120}=\frac{9}{40}$$

$$t=1\div\frac{9}{40}=1\cdot\frac{40}{9}=\frac{40}{9}$$

Therefore, it will take $\frac{40}{9} = 4.\overline{4}$ hours to fill up the pool using all three pipes. We can convert our result to minutes. $\frac{40}{9}=4\frac{4}{9}$ and converting $\frac{4}{9}$ hours to minutes, we get

$$\frac{4}{9}\,\text{h}=\frac{4\cancel{\text{h}}}{9}\cdot\frac{60\,\text{min}}{1\cancel{\text{h}}}=\frac{240\,\text{min}}{9}=26.\overline{6}\,\text{min}$$

It will take about $\boxed{5 \text{ hours and } 27 \text{ minutes}}$ to fill up the pool using all three pipes.

Example 5. A swimming pool can be filled by a water pipe in 8 hours and can be emptied by another pipe in10 hours. One night, when the pool is empty, we accidentally leave both pipes open. How long will it take for the pool to be filled with water?

Solution: This problem is similar, the unknown will end up in the denominator like before. Only, in this case, the two pips work against each other. One pipe adds water to the pool, the other removes water from the pool.

$$x\left(\frac{1}{8}-\frac{1}{10}\right)=1 \qquad \text{subtract fractions}$$

$$x\cdot\frac{5-4}{40}=1$$

$$x\cdot\frac{1}{40}=1 \qquad \text{multiply by } 40$$

$$x=40$$

So it will take $\boxed{40 \text{ hours}}$ to fill up the pool.

Example 6. Andy and Lilian together clean their house in 6 hours. Working alone, it takes Andy 5 hours longer to clean the house than for Lilian. How long does it take for Andy to clean the house?

Solution: Let x denote the time it takes for Lilian to clean the house. Then Andy needs $x+5$ hours. Together, they clean $\frac{1}{x}$ and $\frac{1}{x+5}$ of the house in each hour.

$$6\left(\frac{1}{x}+\frac{1}{x+5}\right)=1 \qquad \text{distribute } 6$$

$$\frac{6}{x}+\frac{6}{x+5}=1 \qquad \text{multiply by } x\,(x+5)$$

$$6\,(x+5)+6x=x\,(x+5)$$

$$6x+30+6x=x^2+5x$$

$$12x+30=x^2+5x$$

$$0=x^2-7x-30$$

$$0=(x-10)\,(x+3)$$

$$x=10 \text{ or } x=-3$$

The negative number can be obviously eliminated. Therefore, $x=10$. This means that Lilian cleans the house in 10 hours, and Andy in $\boxed{15 \text{ hours}}$.

Practice Problems

1. Ayesha paints her room in 12 hours. Bella can paint the same room in 15 hours. Courtney can paint the same room in 10 hours. How long would it take for the three of them to paint the room together?
2. Jimmy can clean the house in 5 hours. If Susan helps to clean the house the job can be done in 2 hours. How long should it take Susan to clean the house alone?
3. Emily can mow the lawn in 5 hours. If Sammie helps her, the lawn is mowed in 3 hours. How long would it take for Sammie to mow the lawn alone?
4. A swimming pool can be filled by a water pipe in 12 hours and can be emptied by another pipe in 20 hours. One night, when the pool is empty, we accidentally leave both pipes open. How long will it take for the pool to be filled with water?
5. A swimming pool can be filled by three water pipes, A, B and C. Pipe A alone can fill the pool in 30 hours, pipe B alone is 6 hours, and pipe C in 5 hours. How long does it take to fill the pool if all three pipes are used?
6. *A swimming pool can be filled by three water pipes, A, B and C. Pipe A and B together fill the pool in 1.2 hours. Pipes A and C fill the pool in 1.5 hours, and pipes B and C fill the pool in 2 hours. How long does it take to fill the pool if all three pipes are used?
7. *A swimming pool can be filled by three water pipes, A, B and C. Pipe A alone can fill the pool in 12 hours. Pipe C alone fills the pool four times as fast as pipe B alone. When all three pipes are used, the pool is filled in 3 hours. How long would pipe B alone fill up the pool? Pipe C?

Answers

1. 4 hrs 2. 3 hour 20 minutes 3. 7.5 hours 4. 30 hours 5. $2\frac{1}{2}$ hours

6. 1 hour 7. 5 hours and 20 hours

