Equations are a fundamental concept and tool in mathematics.

**Definition**: An **equation** is a statement in which two expressions (algebraic or numeric) are connected with an equal sign.

For example, $3x^2 - x = 4x + 28$ is an equation. So is $x^2 + 5y = -y^2 + x + 2$.

**Definition**: A **solution** of an equation is a number (or an ordered set of numbers) that, when substituted into the variable(s) in the equation, makes the statement of equality true.

**Example 1**: a) Verify that $-2$ is not a solution of the equation $3x^2 - x = 4x + 28$.

b) Verify that 4 is a solution of the equation $3x^2 - x = 4x + 28$.

c) Verify that the pair of numbers $x = 3$ and $y = -4$ is a solution of the equation $x^2 + 5y = -y^2 + x + 2$.

d) Verify that the pair of numbers $x = -4$ and $y = 3$ is not a solution of the equation $x^2 + 5y = -y^2 + x + 2$.

Solution: a) Consider the equation $3x^2 - x = 4x + 28$ with $x = -2$.
If $x = -2$, the left-hand side of the equation is $\text{LHS} = 3x^2 - x = 3(-2)^2 - (-2) = 3 \cdot 4 + 2 = 12 + 2 = 14$ and the right-hand side is RHS$= 4x + 28 = 4(-2) + 28 = -8 + 28 = 20$
Since the two sides are not equal, $14 \neq 20$, the number $-2$ is not a solution of this equation.

b) Consider the equation $3x^2 - x = 4x + 28$ with $x = 4$.
If $x = 4$, the left-hand side of the equation is $\text{LHS} = 3x^2 - x = 3 \cdot 4^2 - 4 = 3 \cdot 16 - 4 = 48 - 4 = 44$ and the right-hand side is $\text{RHS} = 4x + 28 = 4 \cdot 4 + 28 = 16 + 28 = 44$
Since the two sides are equal, $x = 4$ is a solution of this equation.

c) Consider the equation $x^2 + 5y = -y^2 + x + 2$ with $x = 3$ and $y = -4$.
If $x = 3$ and $y = -4$, the left-hand side of the equation is $\text{LHS} = x^2 + 5y = 3^2 + 5(-4) = 9 - 20 = -11$ and the right-hand side is $\text{RHS} = -y^2 + x + 2 = -(-4)^2 + 3 + 2 = -16 + 3 + 2 = -11$
Since the two sides are equal, $x = 3$ and $y = -4$ is a solution of this equation.

d) Consider the equation $x^2 + 5y = -y^2 + x + 2$ with $x = -4$ and $y = 3$.
If $x = -4$ and $y = 3$, the left-hand side of the equation is $\text{LHS} = x^2 + 5y = (-4)^2 + 5 \cdot 3 = 16 + 15 = 41$ and the right-hand side is $\text{RHS} = -y^2 + x + 2 = -3^2 + (-4) + 2 = -9 - 4 + 2 = -11$
Since $41 \neq -11$, the two sides are not equal, and so $x = -4$ and $y = 3$ is not a solution of this equation.

If the equation is in more than one variable, like in parts c) and d) before, it is important to identify which number is to be substituted into which variable. After all, $x = 3$ and $y = -4$ (or, as an *ordered pair*, $(3, -4)$) is a solution of the equation $x^2 + 5y = -y^2 + x + 2$, but $x = -4$ and $y = 3$, (or, as an *ordered pair*, $(-4, 3)$) is not.

**Definition**: To **solve an equation** is to find *all* solutions of it. The set of all solutions is also called the solution set.

Caution! Finding one solution for an equation is not the same as solving it. For example, we found that 4 is a solution of $3x^2 - x = 4x + 28$. As it turns out, 4 is not the only solution. We leave to the reader to verify that $-\dfrac{7}{3}$ is also a solution of the equation. We will have to deploy systematic methods to find all solutions. The methods we will use usually depends on the type of equation. We will start with the simplest equations, linear equations.

Linear equations are a fundamental concept and tool in mathematics. To solve a linear equation, we isolate the unknown by applying the same operation(s) to both sides.

**Example 2**: Solve each of the given equations. Make sure to check your solutions.

a) $x - 8 = 10$ b) $3y = -12$ c) $\dfrac{y}{3} = -2$ d) $m + 10 = -5$

Equations like these are called **one-step equations** because they can be solved in only one step. We need to isolate the unknown on one side. In order to do that, we perform the inverse operation. (The inverse operation of additon is subtraction and vica versa. The inverse operation of multiplication is division and vica versa.)

Solution: a) In order to isolate the unknown, we add 8 to both sides.

$$\begin{aligned} x - 8 &= 10 \qquad \text{add } 8 \\ x &= 18 \end{aligned}$$

So the only solution of this equation is 18. We can also say that the solution set is $\{18\}$. We should check; if $x = 18$, the left-hand side is

$$\text{LHS} = x - 8 = 18 - 8 = 10 = \text{RHS}$$

So our solution, $\boxed{x = 18}$ is correct.

b) In order to isolate the unknown, we divide both sides by 3.

$$\begin{aligned} 3y &= -12 \qquad \text{divide by } 3 \\ y &= -4 \end{aligned}$$

So the only solution of this equation is $-4$. We check; if $y = -4$, then

$$\text{LHS} = 3y = 3(-4) = -12 = \text{RHS}$$

So our solution, $\boxed{y = -4}$ is correct.

c) In order to isolate the unknown, we multiply both sides by 3.

$$\begin{aligned} \frac{y}{3} &= -2 \qquad \text{multiply by } 3 \\ y &= -6 \end{aligned}$$

So the only solution of this equation is $-6$. We check; if $y = -6$, then

$$\text{LHS} = \frac{-6}{3} = -2 = \text{RHS}$$

So our solution, $\boxed{x = -6}$ is correct.

d) In order to isolate the unknown, we we subtract 10 from both sides.

$$\begin{aligned} m + 10 &= -5 \qquad \text{subtract } 10 \\ m &= -15 \end{aligned}$$

So the only solution of this equation is $-15$. We check; if $m = -15$, then

$$\text{LHS} = m + 10 = -15 + 10 = -5 = \text{RHS}$$

So our solution, $\boxed{m = -15}$ is correct.

Note: If the reader is interested in applications of one-step equations, basic percent problems and basic motion problems can be easily handled by setting up and solving one-step equations.

**Discussion**: Solve each of the following equations. How are these unusual?

a) $5x = 5$ b) $5x = 0$ c) $x - 4 = -4$ d) $\dfrac{x}{3} = 0$

**Example 3**: Solve each of the given equations. Make sure to check your solutions.

a) $10 = 3x - 11$ b) $\dfrac{t-7}{2} = -8$ c) $\dfrac{x}{-3} + 4 = 15$

Equations like these are called **two-step equations**. We need to isolate the unknown on one side. In order to do that, we perform the inverse operations, in the reverse order it was done to the unknown.

Solution: a) This equation looks unusual in the sense that two-step equations often contain the unknown on the left-hand side. We are always allowed to swap two sides of an equation. If $A = B$, then clearly, also $B = A$. We will do this first. Notice that this is an optional step.

$$\begin{aligned} 10 &= 3x - 11 && \text{swap the two sides} \\ 3x - 11 &= 10 \end{aligned}$$

We now look at the side that contains $x$ and ask: *What happened to the unknown*? The answer is: *Multiplication by* 3 *and subtraction of* 11. We need to apply the inverse operations, in a reverse order. In this case, this means that we will add 11 to both sides and then divide both sides by 3.

$$\begin{aligned} 3x - 11 &= 10 && \text{add } 11 \\ 3x &= 21 && \text{divide by } 3 \\ x &= 7 \end{aligned}$$

So the only solution of this equation is 7. We check: if $x = 7$, then

$$\text{LHS} = 3x - 11 = 3 \cdot 7 - 11 = 21 - 11 = 10 = \text{RHS}$$

So our solution, $\boxed{x = 7}$ is correct.

b) As we look at the left-hand side and ask: *What happened to the unknown*? The answer is: on the left-hand side, there was a subtraction of 7 and then a division by 2. To reverse that, we will multiply both sides by 2 and then add 7 to both sides.

$$\begin{aligned} \frac{t-7}{2} &= -8 && \text{multiply by } 2 \\ t - 7 &= -16 && \text{add } 7 \\ t &= -9 \end{aligned}$$

So the only solution of this equation is $-9$. We check: if $t = -9$, then

$$\text{LHS} = \frac{t-7}{2} = \frac{-9-7}{2} = \frac{-16}{2} = -8 = \text{RHS}$$

So our solution, $\boxed{t = -9}$ is correct.

c) What happened to the unknown? On the left-hand side, there was a division by $-3$ and then an addition of $4$. To reverse that, we will subtract $4$ from both sides by and then multiply both sides by $-3$.

$$\begin{aligned} \frac{x}{-3} + 4 &= 15 && \text{subtract } 4 \\ \frac{x}{-3} &= 11 && \text{multiply by } -3 \\ x &= -33 \end{aligned}$$

So the only solution of this equation is $-33$. We check: if $x = -33$, then

$$\text{LHS} = \frac{x}{-3} + 4 = \frac{-33}{-3} + 4 = 11 + 4 = 15 = \text{RHS}$$

So our solution, $\boxed{x = -33}$ is correct.

**Example 4**: Solve each of the given equations. Make sure to check your solutions.

a) $2x - 8 = 5x + 10$ b) $7a - 12 = -a + 20$ c) $-4x + 2 = -x + 17$

Solution: Notice that in each equation, the unknown appears on both sides. This will be the first thing we will address.

a)

$$\begin{aligned} 2x - 8 &= 5x + 10 && \text{subtract } 2x \\ -8 &= 3x + 10 && \text{subtract } 10 \\ -18 &= 3x && \text{divide by } 3 \\ -6 &= x \end{aligned}$$

So the only solution of this equation is $-6$. We check; if $x = -6$,

$$\text{LHS} = 2(-6) - 8 = -12 - 8 = -20 \text{ and } \text{RHS} = 5(-6) + 10 = -30 + 10 = -20 \implies \text{LHS} = \text{RHS}$$

So our solution, $\boxed{x = -6}$ is correct.

b)

$$\begin{aligned} 7a - 12 &= -a + 20 && \text{add } a \\ 8a - 12 &= 20 && \text{add } 12 \\ 8a &= 32 && \text{divide by } 8 \\ a &= 4 \end{aligned}$$

So the only solution of this equation is $4$. We check; if $a = 4$,

$$\text{LHS} = 7 \cdot 4 - 12 = 28 - 12 = 16 \text{ and } \text{RHS} = -4 + 20 = 16 \implies \text{LHS} = \text{RHS}$$

So our solution, $\boxed{a = 4}$ is correct.

c)

$$\begin{aligned} -4x + 2 &= -x + 17 && \text{add } 4x \\ 2 &= 3x + 17 && \text{subtract } 17 \\ -15 &= 3x && \text{divide by } 3 \\ -5 &= x \end{aligned}$$

We check; if $x = -5$, then

$$\text{LHS} = -4(-5) + 2 = 20 + 2 = 22 \text{ and } \text{RHS} = -(-5) + 17 = 5 + 17 = 22 \implies \text{LHS} = \text{RHS}$$

So our solution, $\boxed{x = -5}$ is correct.

Linear equations might be more complicated. Most often we will be dealing with the distributive law. Also, these equations can be classified based on their solution sets. Consider each of the following.

**Example 5**: Solve each of the given equations. Make sure to check your solutions.

a) $3x - 2(4 - x) = 3(3x - 1) - (x - 7)$ b) $4(y - 2) - 6(3y - 5) = 5 - 2(7y + 1)$ c) $2 - (3 - 5x) = 2(2x - 1) + 1$

Solution: a) We first eliminate the parentheses by applying the distributive law.

$$\begin{aligned} 3x - 2(4 - x) &= 3(3x - 1) - (x - 7) && \text{eliminate parentheses} && \text{Caution! } -2(-x) = 2x \\ 3x - 8 + 2x &= 9x - 3 - x + 7 && \text{combine like terms} && \text{and } -(-7) = 7 \\ 5x - 8 &= 8x + 4 && \text{subtract } 5x \\ -8 &= 3x + 4 && \text{subtract } 4 \\ -12 &= 3x && \text{divide by } 3 \\ -4 &= x \end{aligned}$$

We check: if $x = -4$, then

$$\begin{aligned} \text{LHS} &= 3(-4) - 2(4 - (-4)) = 3(-4) - 2 \cdot 8 = -12 - 16 = -28 \quad \text{and} \\ \text{RHS} &= 3(3(-4) - 1) - (-4 - 7) = 3(-12 - 1) - (-11) = 3(-13) + 11 = -39 + 11 = -28 \quad \Longrightarrow \quad \text{LHS} = \text{RHS} \end{aligned}$$

So our solution, $\boxed{x = -4}$ is correct.

b) We first eliminate the parentheses by applying the distributive law.

$$\begin{aligned} 4(y-2) - 6(3y-5) &= 5 - 2(7y+1) && \text{eliminate parentheses} \\ 4y - 8 - 18y + 30 &= 5 - 14y - 2 && \text{combine like terms} \\ -14y + 22 &= -14y + 3 && \text{add } 14y \\ 22 &= 3 \end{aligned}$$

Something different happened here. When we tried to eliminate the unknown from one side, it disappeared from both sides. We are left with the statement $22 = 3$. No matter what the value of the unknown is, this statement can not be made true. Indeed, our last line is an **unconditionally false statement**. This means that there is no number that could make this statement true, and so this equation **has no solution**. An equation like this is called a **contradiction**.

c) We first eliminate the parentheses by applying the distributive law.

$$\begin{aligned} 2 - (3 - 5x) &= 2(2x-1) + 1 && \text{eliminate parentheses} \\ 2 - 3 + 5x &= 4x - 2 + 1 && \text{combine like terms} \\ 5x - 1 &= 5x - 1 && \text{subtract } 5x \\ -1 &= -1 \end{aligned}$$

When we tried to eliminate the unknown from one side, it disappeared again from both sides. We are left with the statement $-1 = -1$. No matter what the value of the unknown is, this statement is always true. Indeed, our last line is an **unconditionally true statement**. This means that every number makes make this statement true, and so the solution set of this equation is the set of all numbers. An equation like this is called an **identity**.

We often use identities in mathematics, although it seems at first that we would not need equations whose solution set is every number. Consider the following equation: $a + b = b + a$. This equation is an identity, because every pair of numbers is a solution. We use this identity to express a property of *addition*: that the sum of two numbers does not depend on the order of the two numbers.

Based on their solution sets, these equations can be classified as belonging to one of the following three groups.

1. If the last line is of the form $x = 5$, the equation is called **conditional**. (This is because the truth value of the statement depends on the value of $x$. True if $x$ is 5, false otherwise.) A conditional equation has exactly one solution.
2. If the last line is of the form $1 = 1$, the equation is unconditionally true. Such an equation is called an **identity** and all numbers are solutions of it.
3. If the last line is of the form $3 = 14$, the equation is unconditionally false. Such an equation is called a **contradiction** its solution set is the empty set.

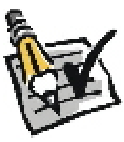
**Discussion**: Classify each of the following equations as conditional, identity, or contradiction.

a) $3x + 1 = 3x - 1$ b) $2x - 4 = 7x - 4$ c) $x - 4 = 4 - x$ d) $x - 1 = -1 + x$

## Sample Problems

Solve each of the following equations. Make sure to check your solutions.

1. $2x - 5 = 17$
2. $\dfrac{a-10}{5} = -3$
3. $\dfrac{t}{4} - 10 = -4$
4. $\dfrac{t-5}{12} = 4$
5. $2x - 7 = -3$
6. $\dfrac{x+8}{3} = -2$
7. $\dfrac{x}{3} + 8 = -2$
8. $-2x + 3 = 3$
9. $3(x+7) = 36$
10. $2x + 5 = 4x + 11$
11. $3w - 5 = 5(w+1)$
12. $7(j-5) + 9 = 2(-2j+5) + 5j$
13. $3(x-5) - 5(x-1) = -2x + 1$
14. $3x - 8 = 3(x-2) - 2$



## Practice Problems

Solve each of the following equations. Make sure to check your solutions.

1. $2x - 3 = -11$
2. $-2x - 3 = 7$
3. $5x - 3 = 17$
4. $\dfrac{x-3}{7} = -2$
5. $\dfrac{x}{7} - 3 = -1$
6. $-4x - 3 = 13$
7. $\dfrac{a+1}{4} = -9$
8. $5x - 6 = -6$
9. $\dfrac{x}{7} - 1 = -3$
10. $-x + 5 = -7$
11. $\dfrac{2x-1}{7} = -3$
12. $5(x-2) = -20$
13. $5x - 3 = x + 9$
14. $-x + 13 = 2x + 1$
15. $-2x + 4 = 5x - 10$
16. $5x - 7 = 6x + 8$
17. $8x - 1 = 3x + 19$
18. $-7x - 1 = 3x - 21$
19. $3(x-4) + 5(x+8) = 2(x-1)$
20. $3(x-4) = 2(x+5)$
21. $4(5x+1) = 6x + 4$
22. $3(2x-7) - 2(5x+2) = -5x - 30$
23. $a - 3 = 5(a-1) - 2$
24. $3y - 2 = -2y + 18$
25. $2(b+1) - 5(b-3) = 2(b-7) + 1$
26. $3(2x-1) - 5(2-x) = 4(x-1) + 5$
27. $5(x-1) - 3(x+1) = 3x - 8$
28. $5(x-1) - 3(-x+1) = -3 + 8x$
29. $-2x - (3x-1) = 2(5-3x)$
30. $3(x-4) - 4(x-3) = 3(x-2) + 2(3-x)$
31. $8(x-3) - 3(5-2x) = x$

# Answers

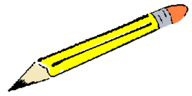
Sample Problems

1. $11$  2. $-5$  3. $24$  4. $53$  5. $2$  6. $-14$  7. $-30$  8. $0$  9. $5$  10. $-3$  11. $-5$  12. $6$  13. no solution  14. all numbers are solution

Practice Problems

1. $-4$  2. $-5$  3. $4$  4. $-11$  5. $14$  6. $-4$  7. $-37$  8. $0$  9. $-14$  10. $12$  11. $-10$  12. $-2$  13. $3$  14. $4$  15. $2$  16. $-15$  17. $4$  18. $2$  19. $-5$  20. $22$  21. $0$  22. $-5$  23. $1$  24. $4$  25. $6$  26. $2$  27. $0$  28. contradiction, there is no solution  29. $9$  30. $0$  31. $3$

# Sample Problems  Solutions

1. $2x - 5 = 17$

   Solution:

$$\begin{aligned} 2x - 5 &= 17 && \text{add 5 to both sides} \\ 2x &= 22 && \text{divide by 2} \\ x &= 11 \end{aligned}$$

   We check: if $x = 11$, then

$$\text{RHS} = 2(11) - 5 = 22 - 5 = 17 = \text{LHS}$$

   Thus our solution, $\boxed{x = 11}$ is correct.

2. $\dfrac{a - 10}{5} = -3$

   Solution:

$$\begin{aligned} \frac{a - 10}{5} &= -3 && \text{multiply both sides by 5} \\ a - 10 &= -15 && \text{add } 10 \text{ to both sides} \\ a &= -5 \end{aligned}$$

   We check: if $a = -5$, then

$$\text{LHS} = \frac{-5 - 10}{5} = \frac{-15}{5} = -3 = \text{RHS}$$

   Thus our solution, $\boxed{a = -5}$ is correct.

3. $\dfrac{t}{4} - 10 = -4$

   Solution:

$$\begin{aligned} \frac{t}{4} - 10 &= -4 && \text{add } 10 \text{ to both sides} \\ \frac{t}{4} &= 6 && \text{multiply both sides by 4} \\ t &= 24 \end{aligned}$$

   We check: if $t = 24$, then

$$\text{RHS} = \frac{t}{4} - 10 = \frac{24}{4} - 10 = 6 - 10 = -4 = \text{LHS}$$

   Thus our solution, $\boxed{t = 24}$ is correct.

4. $\dfrac{t - 5}{12} = 4$

   Solution:

$$\begin{aligned} \frac{t - 5}{12} &= 4 && \text{multiply both sides by 12} \\ t - 5 &= 48 && \text{add 5 to both sides} \\ t &= 53 \end{aligned}$$

   We check: if $t = 53$, then

$$\text{RHS} = \frac{53 - 5}{12} = \frac{48}{12} = 4 = \text{LHS}$$

   Thus our solution, $\boxed{t = 53}$ is correct.

5. $2x - 7 = -3$

Solution: We apply all operations to both sides.

$$\begin{array}{rcll} 2x - 7 & = & -3 & \text{add } 7 \\ 2x & = & 4 & \text{divide by } 2 \\ x & = & 2 & \end{array}$$

We check: if $x = 2$, then

$$\text{LHS} = 2(2) - 7 = 4 - 7 = -3 = \text{RHS}$$

Thus our solution, $\boxed{x = 2}$ is correct.

6. $\dfrac{x+8}{3} = -2$

Solution: We apply all operations to both sides.

$$\begin{array}{rcll} \dfrac{x+8}{3} & = & -2 & \text{multiply by } 3 \\ x + 8 & = & -6 & \text{subtract } 8 \\ x & = & -14 & \end{array}$$

We check:

$$\text{LHS} = \frac{-14+8}{3} = \frac{-6}{3} = -2 = \text{RHS}$$

Thus our solution, $\boxed{x = -14}$ is correct.

7. $\dfrac{x}{3} + 8 = -2$

Solution: We apply all operations to both sides.

$$\begin{array}{rcll} \dfrac{x}{3} + 8 & = & -2 & \text{subtract } 8 \\ \dfrac{x}{3} & = & -10 & \text{multiply by } 3 \\ x & = & -30 & \end{array}$$

We check:

$$\text{LHS} = \frac{-30}{3} + 8 = -10 + 8 = -2 = \text{RHS}$$

Thus our solution, $\boxed{x = -30}$ is correct.

8. $-2x + 3 = 3$

Solution: We apply all operations to both sides.

$$\begin{array}{rcll} -2x + 3 & = & 3 & \text{subtract } 3 \\ -2x & = & 0 & \text{divide by } -2 \\ x & = & 0 & \end{array}$$

We check: if $x = 0$, then

$$\text{LHS} = -2 \cdot 0 + 3 = 0 + 3 = 3 = \text{RHS}$$

Thus our solution, $\boxed{x = 0}$ is correct.

9. $3(x+7)=36$

   Solution: We apply all operation to both sides,

$$\begin{aligned} 3(x+7) &= 36 && \text{divide by } 3 \\ x+7 &= 12 && \text{subtract } 7 \\ x &= 5 \end{aligned}$$

   We check: if $x=5$, then

$$\text{LHS} = 3(5+7) = 3 \cdot 12 = 36 = \text{RHS}$$

   Thus our solution, $\boxed{x=5}$ is correct.

10. $2x+5=4x+11$

    Solution:

$$\begin{aligned} 2x+5 &= 4x+11 && \text{subtract } 2x \text{ from both sides} \\ 5 &= 2x+11 && \text{subtract } 11 \text{ from both sides} \\ -6 &= 2x && \text{divide both sides by } 2 \\ -3 &= x \end{aligned}$$

    We check: if $x=-3$, then

$$\begin{aligned} \text{LHS} &= 2(-3)+5 = -6+5 = -1 \\ \text{RHS} &= 4(-3)+11 = -12+11 = -1 \end{aligned}$$

    Thus our solution, $\boxed{x=-3}$ is correct. (Note: LHS is short for the left-hand side and RHS is short for the right-hand side.)

11. $3w-5=5(w+1)$

    Solution: we first apply the law of distributivity to simplify the right-hand side.

$$\begin{aligned} 3w-5 &= 5(w+1) \\ 3w-5 &= 5w+5 && \text{subtract } 3w \text{ from both sides} \\ -5 &= 2w+5 && \text{subtract } 5 \text{ from both sides} \\ -10 &= 2w && \text{divide both sides by } 2 \\ -5 &= w \end{aligned}$$

    We check. If $w=-5$, then

$$\begin{aligned} \text{LHS} &= 3(-5)-5 = -15-5 = -20 \\ \text{RHS} &= 5((-5)+1) = 5(-4) = -20 \end{aligned}$$

    Thus our solution, $\boxed{w=-5}$ is correct.

12. $7(j-5)+9=2(-2j+5)+5j$

    Solution:

$$\begin{aligned} 7(j-5)+9 &= 2(-2j+5)+5j && \text{distribute on both sides} \\ 7j-35+9 &= -4j+10+5j && \text{combine like terms} \\ 7j-26 &= j+10 && \text{subtract } j \\ 6j-26 &= 10 && \text{add } 26 \\ 6j &= 36 && \text{divide by } 6 \\ j &= 6 \end{aligned}$$

We check: if $j = 6$, then

$$\begin{aligned} \text{LHS} &= 7(6-5)+9 = 7\cdot 1+9 = 7+9 = 16 \\ \text{RHS} &= 2(-2\cdot 6+5)+5\cdot 6 = 2(-12+5)+30 = 2(-7)+30 = -14+30 = 16 \end{aligned}$$

Thus our solution, $\boxed{j = 6}$ is correct.

13. $3(x-5) - 5(x-1) = -2x+1$
Solution:

$$\begin{aligned} 3(x-5)-5(x-1) &= -2x+1 && \text{multiply out parentheses} \\ 3x-15-5x+5 &= -2x+1 && \text{combine like terms} \\ -2x-10 &= -2x+1 && \text{add } 2x \\ -10 &= 1 \end{aligned}$$

Since $x$ disappeared from the equation and we are left with an unconditionally false statement, there is $\boxed{\text{no solution}}$ for this equation. This type of an equation is called a **contradiction**.

14. $3x - 8 = 3(x-2) - 2$

Solution: We first get rid of the parentheses in the right-hand side, then combine like terms.

$$\begin{aligned} 3x-8 &= 3(x-2)-2 && \text{distribute 3} \\ 3x-8 &= 3x-6-2 && \text{combine like terms} \\ 3x-8 &= 3x-8 && \text{subtract } 3x \\ -8 &= -8 \end{aligned}$$

Consider the previous line, $3x - 8 = 3x - 8$. What numbers would make this statement true? Since the algebraic expressions on both sides are the same, every number would work. This becomes even more apparent when $x$ simply disappears from the equation. When that happens, we get an unconditional equation. In this case,, $-8 = -8$ is true, no matter what the value of $x$ is. An equation like this is called an **identity**, and $\boxed{\text{all numbers are solution}}$

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