Sample Problems

1.) Small ones weigh 3 lb, big ones weigh 4 lb. The number of small ones is 3 more than twice the number of big ones. All together, they weigh 79 lb. How many small ones are there?

2.) One side of a rectangle is 7 cm shorter than five times the other side. Find the length of the sides if the perimeter of the rectangle is 118 cm.

3.) The sum of two consecutive even integers is -170. Find these numbers.

4.) We have a jar of coins, all quarters and dimes. All together, they are worth \$17. 60 We have 13 more quarters than dimes. How many quarters, how many dimes?

5.) The sum of three consecutive odd integers is 57. Find these numbers.

6.) One side of a rectangle is 3 cm shorter than four times the other side. Find the sides if the perimeter of the rectangle is 204 cm.

7.) The opposite of a number is 18 more than twice the number. Find this number.

8.) Two times a number is 5 less than the sum of 80 and the opposite of the number. Find this number.

9.) Red pens cost \$1 each, blue ones cost \$1.50 each. We bought some pens. The number of red pens is 7 less than five times the number of blue pens. How many of each did we buy if we paid \$58?

10.) 55 people showed up on the party. There were 3 less women than men. How many men were there?

11.) Ann and Betty dine together. The total bill is \$38. Ann paid \$2 more than Betty. How much did Betty pay?

Motion Problems

12.) We traveled for nine hours. Then we increased our velocity by 10 miles per hour and traveled an additional five hours. What was our original velocity if all together we have traveled 750 miles?

13.) A bicycle leaves Chicago, heading East at 10 $\frac{\text{mi}}{\text{h}}$. Three hours later, a second bicycle leaves Chicago, heading East at 12 $\frac{\text{mi}}{\text{h}}$. How long will to take for the second bicycle to overtake the first bicycle?

14.) Town A and town B are located 55 miles apart. A jogger starts in town A and jogs toward town B. At the same time, a bicycle starts in town B and travels toward town A. The difference between the speed of the jogger and that of the bicycle is 3 miles per hour. Find the speeds if the jogger and the bicycle meet exactly 5 hours after the start.

15.) Ann headed south at 35 miles per hour. Two hours later Sue followed her, at 45 miles per hour. How long until Sue catches up with Ann?

16.) The cop was chasing the crook who had a 100 ft head start. The velocity of the cop was 15 feet per second while that of the crook was 11 feet per second. How long until the cop catches up with him?

17.) A plane leaves an airport and flies south at 300 miles per hours. Later, a second plane leaves the same airport and flies south at 450 miles per hour. If the second plane overtakes the first one in 1 hour, how much earlier did the first plane leave?

18.) Chicago, IL and Montpelier, VT are about 1000 miles apart. A car leaves Chicago to Montpelier at the same time as a train leaves Montpelier for Chicago. The train is 50 $\frac{\text{mi}}{\text{h}}$ faster than the car. Find the speed of the car if it takes 5 hours until the train and car meet.

19.) A bicycle leaves Chicago, heading East at 14 $\frac{\text{mi}}{\text{h}}$. Three hours later, a second bicycle leaves Chicago, heading East at 17 $\frac{\text{mi}}{\text{h}}$. How long will to take for the second bicycle to overtake the first bicycle?

20.) Milwaukee, WI and Albuquerque, NM are about 1500 miles apart. A plane leaves Milwaukee to Albuquerque at the same time as a train leaves Albuquerque for Milwaukee. The plane is 330 $\frac{\text{mi}}{\text{h}}$ faster than the train. Find the speed of the plane if it takes 3 hours until the plane and train meet.

Mixture Problems

21.) How many gallons of 3% acid solution must be mixed with 60 gallons of 10% acid solution to obtain an acid solution that is 8%?

22.) How many gallons of each of a 4% and an 11% salt solutions should be mixed to obtain 35 gallons of a 7% solution?

23.) How much water should we add to 20 gallons of 15% acid solution to dilute it to a concentration of 12%?

Investment Problems

24.) We invested \$10 000 into two bank accounts. One account earns 14% per year, the other account earns 8% per year. How much did we invest into each account if after the first year, the combined interest from the two accounts is \$1238?

25.) We invested \$7000 into two bank accounts. One account earns 14% per year, the other account earns 9% per year. How much did we invest into each account if after the first year, the combined interest from the two accounts is \$840?

Practice Problems

1.) Mary bought four less than three times the number of books that Jose did. Together they bought sixteen books. How many did Jose buy?

2.) A school purchases tickets to a show. A child ticket costs \$8 and an adult ticket costs \$14. The school has paid a total of \$610 for tickets. The number of child tickets was 5 greater than three times the number of adult ticket. How many of the tickets were for adults?

3.) Julia is 5 years younger than her brother, Tom. How old are they if the sum of their ages is 43?

4.) One side of a rectangle is 6 in shorter than the other side. Find the sides of the rectangle if its perimeter is 120 in.

5.) One side of a rectangle is 6 in shorter than twice the other side. Find the sides of the rectangle if its perimeter is 120 in.

6.) The largest angle in a triangle is three times as large as the smallest angle. The middle angle is 35° larger than the smallest angle. Find the angles in the triangle.

7.) What a great ceremony! We had 150% more guests this year than last year. If the number of guests this year is 1875, how many guests were there last year?

8.) The sum of two numbers is 27. Their difference is 11. Find these numbers.

9.) The sum of two numbers is 11. Their difference is 27. Find these numbers.

10.) The sum of two numbers is -11. Their difference is 27. Find these numbers.

11.) The sum of two consecutive odd integers is 92. Find these numbers.

12.) The sum of five times a number and -10 is 8 less than six times the sum of 7 and the opposite of the number. Find this number.

13.) Lisa took 5 exams. The first 4 received scores of 72, 93, 86, and 82. How much did she score on the fifth exam if her average score is 74 points?

Motion Problems

14.) We traveled for nine hours. Then we increased our velocity by 7 miles per hour and traveled an additional six hours. What was our original velocity if all together we have traveled 582 miles?

15.) A bicycle leaves Chicago, heading East at 12 $\frac{\text{mi}}{\text{h}}$. Two hours later, a second bicycle leaves Chicago, heading East at 15 $\frac{\text{mi}}{\text{h}}$. How long will to take for the second bicycle to overtake the first bicycle?

16.) Town A and town B are located 68 miles apart. A jogger starts in town A and jogs toward town B. At the same time, a bicycle starts in town B and travels toward town A. The difference between the speed of the jogger and that of the bicycle is 5 miles per hour. Find the speeds if the jogger and the bicycle meet exactly 4 hours after the start.

17.) Ann headed south at 45 miles per hour. Two hours later Sue followed her, at 55 miles per hour. How long until Sue catches up with Ann?

18.) The cop was chasing the crook who had a 60 ft head start. The velocity of the cop was 14 feet per second while that of the crook was 9 feet per second. How long until the cop catches up with him?

19.) A plane leaves an airport and flies south at 400 miles per hours. Later, a second plane leaves the same airport and flies south at 480 miles per hour. If the second plane overtakes the first one in 1 hour, how much earlier did the first plane leave?

20.) Chicago, IL and Montpelier, VT are about 1000 miles apart. A car leaves Chicago to Montpelier at the same time as a train leaves Montpelier for Chicago. The train is $40\frac{\text{mi}}{\text{h}}$ faster than the car. Find the speed of the car if it takes 5 hours until the train and car meet.

21.) A bicycle leaves Chicago, heading West at 15 $\frac{\text{mi}}{\text{h}}$. Two hours later, a second bicycle leaves Chicago, heading West at $20\frac{\text{mi}}{\text{h}}$. How long will to take for the second bicycle to overtake the first bicycle?

22.) Milwaukee, WI and Albuquerque, NM are about 1500 miles apart. A plane leaves Milwaukee to Albuquerque at the same time as a train leaves Albuquerque for Milwaukee. The plane is $120\frac{\text{mi}}{\text{h}}$ faster than the train. Find the speed of the plane if it takes 3 hours until the plane and train meet.

Mixture Problems

23.) How many liters of a 15% acid solution should be mixed with 10 liters of a 36% acid solution to obtain a mixture that is 20%?

24.) How many liters of a 17% acid solution should be mixed with 8 liters of an 11% acid solution to obtain a mixture that is 15%?

25.) How many gallons of each of a 7% and a 23% salt solutions should be mixed to obtain 32 gallons of a 12% solution?

26.) How many liters of each of a 12% and a 33% alcohol solutions should be mixed to obtain 21 liters of a 25% solution?

27.) How much water should we add to 5 gallons of 18% acid solution to dilute it to a concentration of 10%?

28.) How much water should we add to 10 gallons of 30% acid solution to dilute it to a concentration of 25%?

29.) How much pure alcohol should we add to 6 gallons of 10% acid solution to obtain a solution that is 40%?

30.) How much pure alcohol should we add to 18 gallons of 35% acid solution to obtain a solution that is 55%?

Investment Problems

31.) We invested \$6000 into two bank accounts. One account earns 7% per year, the other account earns 11% per year. How much did we invest into each account if after the first year, the combined interest from the two accounts is \$520?

32.) We invested \$7600 into two bank accounts. One account earns 9% per year, the other account earns 13% per year. How much did we invest into each account if after the first year we have a total of \$8508 in the accounts?

Sample Problems - Answers

1.) 7 big, 17 small 2.) 11 cm by 48 cm 3.) -86 and -84 4.) 41 dimes and 54 quarters

5.) 17, 19, and 21 6.) 21 cm by 81 cm 7.) -6 8.) 25 9.) 10 blue and 43 red pens

10.) 26 women and 29 men 11.) Betty paid \$18 and Ann paid \$20 12.) $50\frac{\text{mi}}{\text{h}}$

13.) 15 hours 14.) $4\frac{\text{mi}}{\text{h}}$ and $7\frac{\text{mi}}{\text{h}}$ 15.) 7 hours 16.) 25 seconds 17.) 30 minutes earlier

18.) $75\frac{\text{mi}}{\text{h}}$ 19.) 14 hours 20.) $85\frac{\text{mi}}{\text{h}}$ and $415\frac{\text{mi}}{\text{h}}$

21.) 24 gallons of 3% solution with 60 gallons of 10% solution

22.) 20 gallons of 4% solution with 15 gallons of 11% solution 23.) 5 gallons

24.) \$7300 at 14% and \$2700 at 8% 25.) \$4200 at 14% and \$2800 at 9%

Practice Problems - Answers

1.) 5 2.) 15 3.) Julia is 19 and Tom is 24 4.) 27 in and 33 in

5.) 22 in and 38 in 6.) $29°$, $64°$, $87°$ 7.) 750 8.) 8 and 19 9.) -8 and 19

10.) -19 and 8 11.) 45 and 47 12.) 4 13.) 37 14.) $36\frac{\text{mi}}{\text{h}}$

15.) 8 hours after the second bicylce started

16.) The speed of the jogger is $6\frac{\text{mi}}{\text{h}}$ and that of the bicycle is $11\frac{\text{mi}}{\text{h}}$ 17.) 9 hours

18.) 12 seconds 19.) $\frac{1}{5}$ hour or 12 minutes earlier 20.) $80\frac{\text{mi}}{\text{h}}$ 21.) 6 hours

22.) The speed of the train is $190\frac{\text{mi}}{\text{h}}$ and that of the plane is $310\frac{\text{mi}}{\text{h}}$

23.) 32 liters 24.) 16 liters 25.) 10 gallons of 23% solution with 22 gallons of 7% solution

26.) 13 liters of 33% solution with 8 liters of 12% solution 27.) 4 gallons 28.) 2 gallons

29.) 3 gallons 30.) 8 gallons 31.) \$3500 at 7% and \$2500 at 11%

32.) \$2000 at 9% and \$5600 at 13%

Sample Problems - Solutions

1.) Small ones weigh 3 lb, big ones weigh 4 lb. The number of small ones is 3 more than twice the number of big ones. All together, they weigh 79 lb. How many small ones are there?
Solution: Let us denote the number of big ones by x. Then the number of small ones is $2x+3$. We obtain the equation expressing the total weight:

$$\begin{aligned} 3(2x+3)+4x &= 79 && \text{distribute} \\ 6x+9+4x &= 79 && \text{combine like terms} \\ 10x+9 &= 79 && \text{subtract 9} \\ 10x &= 70 && \text{divide by 10} \\ x &= 7 \end{aligned}$$

The number of big ones is then 7, and so the number of small ones is $2(7)+3=17$. We check: the number of small ones, 17 is indeed 3 more than twice the number of big ones, 7. The total weight is $7(4)+17(3)=28+51=79$. Thus the solution is 7 big, 17 small.

2.) One side of a rectangle is 7 cm shorter than five times the other side. Find the length of the sides if the perimeter of the rectangle is 118 cm.
Solution: Let us denote the shorter side by x. Then the longer side is $5x-7$. We obtain the equation for the perimeter:

$$\begin{aligned} 2x+2(5x-7) &= 118 && \text{distribute} \\ 2x+10x-14 &= 118 && \text{combine like terms} \\ 12x-14 &= 118 && \text{add 14} \\ 12x &= 132 && \text{divide by 12} \\ x &= 11 \end{aligned}$$

Thus the shorter side is 11 cm, the longer side is $5(11\,\text{cm})-7\,\text{cm}=48\,\text{cm}$. We check: the perimeter is $2(11\,\text{cm})+2(48\,\text{cm})=118$ cm and 48 is indeed 7 shorter than five times 11. Thus the solution is: 11 cm by 48 cm.

3.) The sum of two consecutive even integers is -170. Find these numbers.
Solution: Let us denote the smaller number by x. Then the larger number is $x+2$. The equation expresses the sum of the numbers.

$$\begin{aligned} x+x+2 &= -170 && \text{combine like terms} \\ 2x+2 &= -170 && \text{subtract 2} \\ 2x &= -172 && \text{divide by 2} \\ x &= -86 \end{aligned}$$

Then the larger number must be $-86+2=-84$. Thus the numbers are are -86 and -84.

4.) We have a jar of coins, all quarters and dimes. All together, they are worth \$17.60 We have 13 more quarters than dimes. How many quarters, how many dimes?
Solution: Let us denote the number of dimes by x. Then the number of quarters must be $x + 13$. We obtain the equation by expressing the total value, in pennies:

$$\begin{aligned} 10x + 25(x + 13) &= 1760 && \text{distribute} \\ 10x + 25x + 325 &= 1760 && \text{combine like terms} \\ 35x + 325 &= 1760 && \text{subtract 325} \\ 35x &= 1435 && \text{divide by 35} \\ x &= 41 \end{aligned}$$

Thus we have 41 dimes and $41 + 13 = 54$ quarters. We check: $41(0.1) + 54(0.25) = 4.1 + 13.5 = 17.6$. Thus the solution is 41 dimes and 54 quarters.

5.) The sum of three consecutive odd integers is 57. Find these numbers.
Let us denote the smallest number by x. Then the other two numbers must be $x + 2$ and $x + 4$. The equation expresses the sum of the three numbers.

$$\begin{aligned} x + x + 2 + x + 4 &= 57 && \text{combine like terms} \\ 3x + 6 &= 57 && \text{subtract 6} \\ x &= 51 && \text{divide by 3} \\ x &= 17 \end{aligned}$$

Thus the three numbers are 17, and $17 + 2 = 19$, and $17 + 4 = 21$. We check: indeed, $17 + 19 + 21 = 57$. Thus the solution is 17, 19, and 21.

6.) One side of a rectangle is 3 cm shorter than four times the other side. Find the sides if the perimeter of the rectangle is 204 cm.
Solution: Let us denote the shorter side by x. Then the longer side is $4x - 3$. We obtain the equation for the perimeter:

$$\begin{aligned} 2x + 2(4x - 3) &= 204 && \text{distribute} \\ 2x + 8x - 6 &= 204 && \text{combine like terms} \\ 10x - 6 &= 204 && \text{add 6} \\ 10x &= 210 && \text{divide by 10} \\ x &= 21 \end{aligned}$$

Thus the shorter side is 21 cm, the longer side is $4(21) - 3 = 81$ cm. We check: the perimeter is $2(21) + 2(81) = 42 + 162 = 204$ cm and 81 is indeed 3 shorter than four times 21. Thus the solution is: 21 cm by 81 cm.

7.) The opposite of a number is 18 more than twice the number. Find this number.
Solution: Let us denote the number by x. The the two things that we are comparing are:

$$\begin{aligned} &\text{the opposite of the number is} && -x \\ &\text{twice the number is} && 2x \end{aligned}$$

Now we make these two equal by adding the difference to the SMALLER number. Since twice the number is 18 less than the opposite of the number, they will be equal once we add 18 to the smaller one.

$$\begin{aligned} -x &= 2x + 18 && \text{add } x \\ 0 &= 3x + 18 && \text{subtract } 18 \\ -18 &= 3x && \text{divide by } 3 \\ -6 &= x \end{aligned}$$

Thus the number is -6. Indeed, twice -6 is -12 which is 18 less than 6, the opposite of -6. Thus the number is -6.

8.) Two times a number is 5 less than the sum of 80 and the opposite of the number. Find this number.
Solution: let us denote the number by x. The two things we are comparing:

$$\begin{aligned} &\text{two times a number :} && 2x \\ &\text{the sum of 80 and the opposite of the number} && : \quad 80 + (-x) = 80 - x \end{aligned}$$

We make these two equal by adding the difference to the smaller one:

$$\begin{aligned} 2x + 5 &= 80 - x && \text{add } x \\ 3x + 5 &= 80 && \text{subtract } 5 \\ 3x &= 75 && \text{divide by } 3 \\ x &= 25 \end{aligned}$$

Thus the number is 25. We check: twice 25 is 50 and the sum of 80 and the opposite of 25 is 55. 55 is indeed 5 more than 50. Thus the solution is: the number is 25.

9.) Red pens cost \$1 each, blue ones cost \$1.50 each. We bought some pens. The number of red pens is 7 less than five times the number of blue pens. How many of each did we buy if we paid \$58?
Solution: Let us denote the number of blue pens by x. Then the number of red pens is $5x - 7$. The equation will express the total cost of the pens:

$$\begin{aligned} 1\,(5x - 7) + 1.50\,(x) &= 58 && \text{distribute} \\ 5x - 7 + 1.5x &= 58 && \text{combine like terms} \\ 6.5x - 7 &= 58 && \text{add } 7 \\ 6.5x &= 65 && \text{divide by } 6.5 \\ x &= 10 \end{aligned}$$

Thus we bought 10 blue and $5\,(10) - 7 = 43$ red pens. We check:

$$\begin{aligned} 43 &= 5\,(10) - 7 \\ 1\,(43) + 1.50\,(10) &= 43 + 15 = 58 \end{aligned}$$

Thus our solution is correct; we bought 10 blue and 43 red pens.

10.) 55 people showed up on the party. There were 3 less women than men. How many men were there?
Solution: Let us denote the number of women by x. Then $x+3$ men showed up. The equation expresses the number of people:

$$\begin{aligned} x+x+3 &= 55 && \text{combine like terms} \\ 2x+3 &= 55 && \text{subtract 3} \\ 2x &= 52 && \text{divide by 2} \\ x &= 26 \end{aligned}$$

Thus there were 26 women and 29 men on the party.

11.) Ann and Betty dine together. The total bill is \$38. Ann paid \$2 more than Betty. How much did Betty pay?
Solution: Let us denote by x the amount that Betty paid. Then Ann paid $x+2$. The equation expresses the total amount paid:

$$\begin{aligned} x+x+2 &= 38 && \text{combine like terms} \\ 2x+2 &= 38 && \text{subtract 2} \\ 2x &= 36 && \text{divide by 2} \\ x &= 18 \end{aligned}$$

Thus Betty paid \$18 and Ann paid \$20.

12.) We traveled for nine hours. Then we increased our velocity by 10 miles per hour and traveled an additional five hours. What was our original velocity if all together we have traveled 750 miles?
Solution: Let us denote our velocity in the first nine hours by x. During the second part of the trip our velocity was $x+10$.

	$v\ \left(\frac{\text{mi}}{\text{h}}\right)$	$t\ (\text{h})$	$s\ (\text{mi})$
Part 1	x	9	$9x$
Part 2	$x+10$	5	$5\,(x+10)$

The distance traveled in the first part and the distance traveled in the second part add up to 750 miles.

$$\begin{aligned} 9x+5\,(x+10) &= 750 \\ 9x+5x+50 &= 750 \\ 14x+50 &= 750 \\ 14x &= 700 \\ x &= 50 \end{aligned}$$

Thus the original velocity was $50\dfrac{\text{mi}}{\text{h}}$.

13.) A bicycle leaves Chicago, heading East at $10\frac{\text{mi}}{\text{h}}$. Three hours later, a second bicycle leaves Chicago, heading East at $12\frac{\text{mi}}{\text{h}}$. How long will to take for the second bicycle to overtake the first bicycle?
Solution: Let x denote the time that the second bicycle spent traveling. Then the first bicycle had $x+3$ hours to travel.

	$v\ \left(\frac{\text{mi}}{\text{h}}\right)$	t (h)	s (mi)
bicycle 1	10	$x+3$	$10(x+3)$
bicycle 2	12	x	$12x$

The distances are equal. They both start at Chicago and meet at the meeting point.

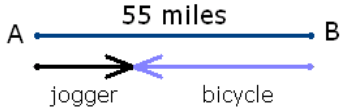
$$\begin{aligned} 10(x+3) &= 12x \\ 10x+30 &= 12x \\ 30 &= 2x \\ 15 &= x \end{aligned}$$

Thus it will take 15 hours after the second bicylce started.

14.) Town A and town B are located 55 miles apart. A jogger starts in town A and jogs toward town B. At the same time, a bicycle starts in town B and travels toward town A. The difference between the speed of the jogger and that of the bicycle is 3 miles per hour. Find the speeds if the jogger and the bicycle meet exactly 5 hours after the start.
Solution: Let x denote the speed of the jogger. Then the speed of the bicycle is $x+3$.

	$v\ \left(\frac{\text{mi}}{\text{h}}\right)$	t (h)	s (mi)
jogger	x	5	$5x$
bicycle	$x+3$	5	$5(x+3)$

The equation expresses that when they meet, the jogger and the bicycle has covered the 55 miles between A and B.



$$\begin{aligned} 5x+5(x+3) &= 55 \\ 5x+5x+15 &= 55 \\ 10x+15 &= 55 \\ 10x &= 40 \\ x &= 4 \end{aligned}$$

Thus the speed of the jogger is $4\frac{\text{mi}}{\text{h}}$ and that of the bicycle is $7\frac{\text{mi}}{\text{h}}$.

15.) Ann headed south at 35 miles per hour. Two hours later Sue followed her, at 45 miles per hour. How long until Sue catches up with Ann?
Solution: Let x denote the time that Sue has traveled, measured in hours. Since she started two hours earlier, Ann traveled for $x+2$ hours.

	$v\ \left(\frac{\text{mi}}{\text{h}}\right)$	t (h)	s (mi)
Ann	35	$x+2$	$35(x+2)$
Sue	45	x	$45x$

The equation expresses that the distances traveled are equal.

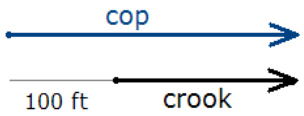
$$\begin{aligned} 35(x+2) &= 45x \\ 35x+70 &= 45x \\ 70 &= 10x \\ 7 &= x \end{aligned}$$

Thus it will take 7 hours.

16.) The cop was chasing the crook who had a 100 ft head start. The velocity of the cop was 15 feet per second while that of the crook was 11 feet per second. How long until the cop catches up with him?
Solution: Let x denote the time they both had to travel.

	$v\ \left(\frac{\text{ft}}{\text{s}}\right)$	t (s)	s (ft)
crook	11	x	$11x$
cop	15	x	$15x$

The cop had to run as much as the crook ran, in addition to the headstart.



$$\begin{aligned} 11x+100 &= 15x \\ 100 &= 4x \\ 25 &= x \end{aligned}$$

Thus it will take 25 seconds.

17.) A plane leaves an airport and flies south at 300 miles per hours. Later, a second plane leaves the same airport and flies south at 450 miles per hour. If the second plane overtakes the first one in 1 hour, how much earlier did the first plane leave?
Solution: Let x denote the time that the first plane traveled before the second plane started. Then the faster plane traveled for one hour, and the slower on has traveled $x+1$ hour until they meet.

	$v\ \left(\frac{\text{mi}}{\text{h}}\right)$	t (h)	s (mi)
slower plane	300	$x+1$	$300(x+1)$
faster plane	450	1	450

The equation expresses that when they meet, the distances traveled by tha planes are equal.

$$\begin{aligned} 300\,(x+1) &= 450 \\ 300x + 300 &= 450 \\ 300x &= 150 \\ x &= \frac{150}{300} = \frac{1}{2} \end{aligned}$$

Thus the first plane left $\frac{1}{2}$ hour, or 30 minutes earlier.

18.) Chicago, IL and Montpelier, VT are about 1000 miles apart. A car leaves Chicago to Montpelier at the same time as a train leaves Montpelier for Chicago. The train is $50\frac{\text{mi}}{\text{h}}$ faster than the car. Find the speed of the car if it takes 5 hours until the train and car meet.
Solution: Let x denote the speed of the car, measured in miles per hour. Then the speed of the train is $x + 50$.

	velocity $\left(\frac{\text{mi}}{\text{h}}\right)$	time (h)	distance (mi)
car	x	5	$5x$
train	$x + 50$	5	$5\,(x + 50)$

The equation expresses that the distances traveled are equal.

$$\begin{aligned} 5x + 5\,(x+50) &= 1000 \\ 5x + 5x + 250 &= 1000 \quad \text{subtract } 250 \\ 10x &= 750 \quad \text{divide by } 10 \\ x &= 75 \end{aligned}$$

Then the speed of the car is $75\frac{\text{mi}}{\text{h}}$.

19.) A bicycle leaves Chicago, heading East at $14\frac{\text{mi}}{\text{h}}$. Three hours later, a second bicycle leaves Chicago, heading East at $17\frac{\text{mi}}{\text{h}}$. How long will to take for the second bicycle to overtake the first bicycle?
Solution: Let us denote by x the time it took for the second bicycle to overtake the first bicycle. Then the other bicycle had $x + 3$ hours, since it started three hours earlier.

	velocity $\left(\frac{\text{mi}}{\text{h}}\right)$	time (h)	distance (mi)
slower	14	$x + 3$	$14\,(x + 3)$
faster	17	x	$17x$

$$\begin{aligned} 14\,(x+3) &= 17x \\ 14x + 42 &= 17x \quad \text{subtract } 14x \\ 42 &= 3x \quad \text{divide by } 3 \\ 14 &= x \end{aligned}$$

so it takes 14 hours.

20.) Milwaukee, WI and Albuquerque, NM are about 1500 miles apart. A plane leaves Milwaukee to Albuquerque at the same time as a train leaves Albuquerque for Milwaukee. The plane is $330\frac{\text{mi}}{\text{h}}$ faster than the train. Find the speed of the plane if it takes 3 hours until the plane and train meet.
Solution:

	velocity $\left(\frac{\text{mi}}{\text{h}}\right)$	time (h)	distance (mi)
train	x	3	$3x$
plane	$x+330$	3	$3(x+330)$

$$\begin{aligned} 3x+3(x+330) &= 1500 \\ 3x+3x+990 &= 1500 \\ 6x+990 &= 1500 \\ 6x &= 510 \\ x &= 85 \end{aligned}$$

Thus the speed of the train is $85\frac{\text{mi}}{\text{h}}$ and that of the plane is $415\frac{\text{mi}}{\text{h}}$.

21.) How many gallons of 3% acid solution must be mixed with 60 gallons of 10% acid solution to obtain an acid solution that is 8%?
Solution: Let us denote the amount of 3% solution we use. Then

	Amount of Solution (gallons)	Percentage	Amount of Solvant (gallons)
Component 1	x	0.03	$0.03x$
Component 2	60	0.1	$60(0.1)$
Mixture	$x+60$	0.08	$0.08(x+60)=0.03x+60(0.1)$

We obtain the equation by stating that the amount of solvant in the components must add up to the amount of solvant. (In other words, the last entry in the third row can be written in two different ways: the product of $x+60$ and 8%; and the sum of $0.03x$ and $60(0.1)$

$$\begin{aligned} 0.08(x+60) &= 0.03x+60(0.1) \\ 0.08(x+60) &= 0.03x+6 && \text{multiply by 100 to make numbers 'nice'} \\ 8(x+60) &= 3x+600 && \text{distribute} \\ 8x+480 &= 3x+600 && \text{subtract } 3x \\ 5x+480 &= 600 && \text{subtract 480} \\ 5x &= 120 && \text{divide by 5} \\ x &= 24 \end{aligned}$$

Thus we need to mix 24 gallons of 3% solution with 60 gallons of 10% solution.
We check: suppose we mix the two solutions specified above. We need to find how much solution and how much solvant we have, hoping that the amount of solvant indeed will be 8% of the amount of mixture.

	Amount of Solution	Percentage	Amount of Solvant
Component 1	60 gallons of	10% solution	$0.1(60)=6$ gallons
Component 2	24 gallons of	3% solution	$0.03(24)=0.72$ gallons
	$\Downarrow$		$\Downarrow$
Mixture	84 gallons		6.72 gallons

8% of 84 is $0.08(84)=6.72$. Thus our solution has the right concentration.

22.) How many gallons of each of a 4% and an 11% salt solutions should be mixed to obtain 35 gallons of a 7% solution?
Solution: Let us denote by x the amount of 4% solution. Since we need to mix 35 gallons of a mixture, the amount of 11% solution must be $35 - x$ gallons.

	Amount of Solution (gallons)	Percentage	Amount of Solvant (gallons)
Component 1	x	0.04	$0.04x$
Component 2	$35 - x$	0.11	$0.11\,(35 - x)$
Mixture	35	0.07	

We obtain the equation by stating that the amount of solvant in the components must add up to the amount of solvant. (In other words, the last entry in the third row can be written in two different ways: the product of 35 and 7%; and the sum of $0.04x$ and $0.11\,(35 - x)$

$$\begin{aligned} 0.07\,(35) &= 0.04x + 0.11\,(35 - x) && \\ 2.45 &= 0.04x + 0.11\,(35 - x) && \text{multiply by 100 to make numbers 'nice'} \\ 245 &= 4x + 11\,(35 - x) && \text{distribute} \\ 245 &= 4x + 385 - 11x && \text{combine like terms} \\ 245 &= -7x + 385 && \text{subtract 385} \\ -140 &= -7x && \text{divide by } -7 \\ x &= 20 && \end{aligned}$$

If $x = 20$, then the other amount, denoted by $35 - x$ must be $35 - 20 = 15$. Thus we need to mix 20 gallons of 4% solution with 15 gallons of 11% solution.
We check: suppose we mix the two solutions specified above. We need to find how much solution and how much solvant we have, hoping that the amount of solvant indeed will be 8% of the amount of mixture.

	Amount of Solution	Percentage	Amount of Solvant
Component 1	20 gallons	of 4% solution	$0.04\,(20) = 0.8$ gallons
Component 2	15 gallons	of 11% solution	$0.11\,(15) = 1.65$ gallons
	$\Downarrow$		$\Downarrow$
	35 gallons		$0.8 + 1.65 = 2.45$ gallons

7% of 35 is $0.07\,(35) = 2.45$ Thus our solution has the right concentration.

23.) How much water should we add to 20 gallons of 15% acid solution to dilute it to a concentration of 12%?
Solution: The trick is to think of pure water as a 0% solution. The rest of the problem goes as the previous problems. Let us denote the amount of 3% solution we use. Then

	Amount of Solution (gallons)	Percentage	Amount of Solvant (gallons)
Component 1	x	0	0
Component 2	20	0.15	$20\,(0.15) = 3$
Mixture	$x + 20$	0.12	$0.12\,(x + 20) = 20\,(0.15)$

$$\begin{aligned} 0.12\,(x+20) &= 20\,(0.15) && \\ 0.12\,(x+20) &= 3 && \text{multiply by 100 to make numbers 'nice'} \\ 12\,(x+20) &= 300 && \text{distribute} \\ 12x + 240 &= 300 && \text{subtract 240} \\ 12x &= 60 && \text{divide by 5} \\ x &= 5 && \end{aligned}$$

Thus we need to mix 5 gallons of water with 20 gallons of 15% solution. We check:

	Amount of Solution	Percentage	Amount of Solvant
Component 1	20 gallons of	15% solution	$0.15\,(20) = 3$ gallons
Component 2	5 gallons of	0% solution	0 gallons
	$\Downarrow$		$\Downarrow$
Mixture	25 gallons		3 gallons

12% of 25 is $0.12\,(25) = 3$ Thus our solution has the right concentration.

24.) We invested \$10 000 into two bank accounts. One account earns 14% per year, the other account earns 8% per year. How much did we invest into each account if after the first year, the combined interest from the two accounts is \$1238?
Solution: Let us denote the amount invested at 14% by x. Then the amount invested at 8% is $10\,000 - x$.

amount invested	interest paid
at 14%: x	$0.14x$
at 8% : $10\,000 - x$	$0.08\,(10\,000 - x)$

The equation will express the combined interest:

$$\begin{aligned} 0.14x + 0.08\,(10\,000 - x) &= 1238 && \text{multiply by 100} \\ 14x + 8\,(10\,000 - x) &= 123\,800 && \text{divide by 2} \\ 7x + 4\,(10\,000 - x) &= 61\,900 && \text{distribute} \\ 7x + 40\,000 - 4x &= 61\,900 && \text{combine like terms} \\ 3x + 40\,000 &= 61\,900 && \text{subtract } 40\,000 \\ 3x &= 21\,900 && \text{divide by 3} \\ x &= 7300 && \end{aligned}$$

Thus we invested \$7300 at 14%. The other amount is then $10\,000 - x = 10\,000 - 7300 = 2700$. We invested \$7300 at 14% and \$2700 at 8%. We check: the amounts add up to \$7300 + \$2700 = \$10 000. The interest from the accounts are

$$14\% \text{ of } 7300 \text{ is } 0.14\,(7300) = 1022 \text{ and } 8\% \text{ of } 2700 \text{ is } 0.08\,(2700) = 216$$

Since $1022 + 216 = 1238$, our solution is correct.

25.) We invested \$7000 into two bank accounts. One account earns 14% per year, the other account earns 9% per year. How much did we invest into each account if after the first year, the combined interest from the two accounts is \$840?

Solution: Let us denote the amount invested at 14% by x. Then the amount invested at 9% is $7000 - x$.

amount invested	interest paid
at 14%: x	$0.14x$
at 9% : $7000 - x$	$0.09\,(7000 - x)$

The equation will express the combined interest:

$$\begin{aligned} 0.14x + 0.09\,(7000 - x) &= 840 && \text{multiply by } 100 \\ 14x + 9\,(7000 - x) &= 84\,000 && \text{distribute} \\ 14x + 63\,000 - 9x &= 84\,000 && \text{combine like terms} \\ 5x + 63\,000 &= 84\,000 && \text{subtract } 63\,000 \\ 5x &= 21\,000 && \text{divide by } 5 \\ x &= 4200 \end{aligned}$$

Then $y = 7000 - x = 7000 - 4200 = 2800$. Thus we invested \$4200 at 14% and \$2800 at 9%. We check: the amounts add up to \$4200 + \$2800 = \$7000. The interest from the accounts are:

$$14\% \text{ of } 4200 \text{ is } 0.14\,(4200) = 588 \text{ and } 9\% \text{ of } 2800 \text{ is } 0.08\,(2700) = 252$$

Since $588 + 252 = 840$, our solution is correct.

