Until now, we have been graphing quadratic equations of the form $y = x^2 + bx + c$, with leading coefficient 1. Of course, not all quadratic expressions are like this. In this section, we will learn how to graph parabolas with a leading coefficient different from 1.

Example 1. Graph the parabola $y = x^2 - 6x + 5$ and the equation $y = 2\left(x^2 - 6x + 5\right)$ in the same coordinate system.

Solution: We will start with $y = x^2 - 6x + 5$. We first factor it by completing the square to obtain the coordinates of vertex and $x-$intercepts.

$$\begin{aligned}
y &= x^2 - 6x + 5 \\
y &= \underbrace{x^2 - 6x + 9} - 9 + 5 \\
y &= (x-3)^2 - 4 \implies \text{vertex } V(3,-4) \\
y &= (x-3)^2 - 2^2 \\
y &= (x-3+2)(x-3-2) \\
y &= (x-1)(x-5) \\
&\implies x\text{-intercepts } (1,0) \text{ and } (5,0)
\end{aligned}$$

The standard form, $y = (x-3)^2 - 4$ tells us that the vertex is $(3,-4)$. Recall that the vertex of a parabola occurs when the complete square part is zero.

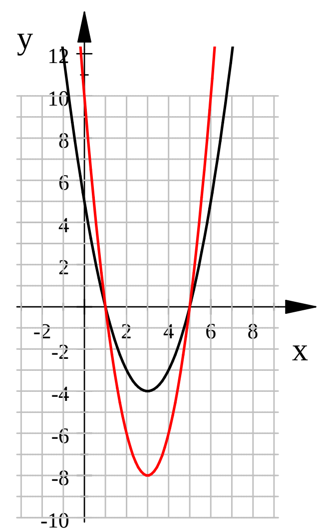
The factored form tells us where the $x-$intercepts are: $(1,0)$ and $(5,0)$.

We can obtain additional points by substituting any value of x into the equation $y = x^2 - 6x + 5$.

x	-1	0	1	2	3	4	5	6	7
y	12	5	0	-3	-4	-3	0	5	12

To graph the second equation, $y = 2\left(x^2 - 6x + 5\right)$, we can simply use the same points we just collected in the table. All we have to do is to mutiply the $y-$coordinate by 2. For example, if $x = 4$, then $y = x^2 - 6x + 5 = -3$, and also $y = 2\left(x^2 - 6x + 5\right) = 2(-3) = -6$.

x	-1	0	1	2	3	4	5	6	7
$y = x^2 - 6x + 5$	12	5	0	-3	-4	-3	0	5	12
$y = 2\left(x^2 - 6x + 5\right)$	24	10	0	-6	-8	-6	0	10	24



To obtain a point on the new graph, take any point on the parabola, keep the $x-$coordinate, and double the $y-$coordinate. We should repeat this process for every point on the original parabola.

The red graph is the new graph, it is also a parabola.
The $x-$intercepts remain the same. This makes sense: if we double zero, we do get zero again. Also, the equations $(x-1)(x-5) = 0$ and $2(x-1)(x-5) = 0$ have the same solutions.

The $y-$intercept was $(0,5)$. The new $y-$intercept is $(0,10)$.

The vertex is also affected by the multiplication. The $x-$coordinate is the same, but the $y-$coordinate is doubled. But if -4 is the lowest value of the expression $x^2 - 6x + 5$, then -8 will be the lowest value of $2\left(x^2 - 6x + 5\right)$.

Let us see how these properties show up in the algebra. Suppose we have to factor $y = 2x^2 - 12x + 10$.

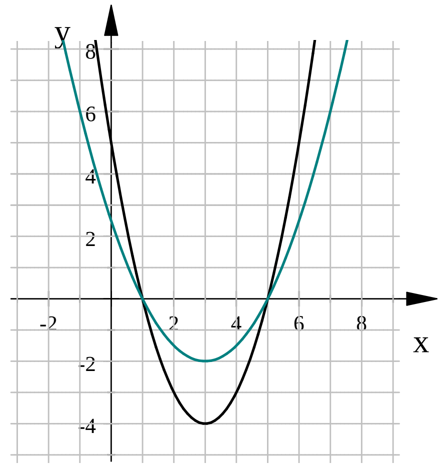
$$\begin{aligned}
y &= 2x^2 - 12x + 10 \implies \text{the } y\text{-intercept is } (0,10) \\
y &= 2\left(x^2 - 6x + 5\right) \\
y &= 2\left(x^2 - 6x + 9 - 9 + 5\right) \\
y &= 2\left((x-3)^2 - 4\right) = \boxed{2(x-3)^2 - 8} \implies \text{the vertex is } (3,-8) \\
y &= 2(x-3+2)(x-3-2) \\
y &= 2(x-1)(x-5) \implies \text{the } x\text{-intercepts are } (1,0) \text{ and } (5,0)
\end{aligned}$$

The **standard form** of this parabola is $y = 2(x-3)^2 - 8$. This indicates that the vertex is at $(3, -8)$. Recall that the vertex is the point where the complete square part is zero. The smallest possible value of $(x-3)^2$ is zero, when $x = 3$. Then, if all the values are multplied by 2, the smallest value is still zero. Then the smallest value of $2(x-3)^2 - 8$ is -8.

Example 2. Graph the given equations in the same coordinate system. $y = x^2 - 6x + 5$ and $y = \frac{1}{2}\left(x^2 - 6x + 5\right)$

Solution: We have already performed the computations and graphed the first equation. Let us then focus on the second equation. We can use the table of values to graph the equation, $y = \frac{1}{2}\left(x^2 - 6x + 5\right)$. This time we will mutiply the $y-$coordinate of each point by $\frac{1}{2}$.

x	-1	0	1	2	3	4	5	6	7
$y = x^2 - 6x + 5$	12	5	0	-3	-4	-3	0	5	12
$y = \frac{1}{2}\left(x^2 - 6x + 5\right)$	24	10	0	-6	-8	-6	0	10	24



To obtain a point on the new graph, take any point on the parabola, keep the $x-$coordinate, and multiply the $y-$coordinate by $\frac{1}{2}$. We should repeat this process for every point on the original parabola.

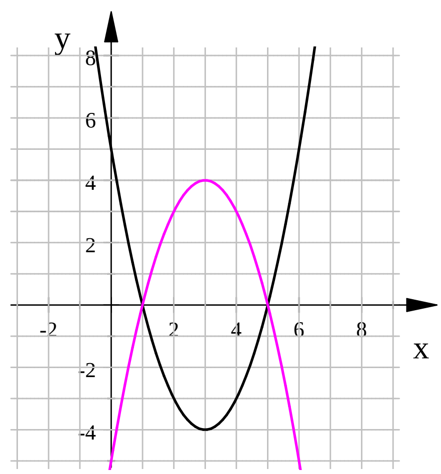
The blue graph is the new graph, it is also a parabola. Again, the $x-$intercepts remain the same, and every other point is affected. The $y-$intercept, that used to be $(0, 5)$ is now $\left(0, \frac{5}{2}\right)$. The vertex, that used to be $(3, -4)$ is now $(3, -2)$.

Example 3. Graph the given equations in the same coordinate system. $y = x^2 - 6x + 5$ and $y = -\left(x^2 - 6x + 5\right)$

Solution: We can use the table of values to graph the equation $y = -\left(x^2 - 6x + 5\right)$. This time we will mutiply the $y-$coordinate of each point by -1.

x	-1	0	1	2	3	4	5	6	7
$y = x^2 - 6x + 5$	12	5	0	-3	-4	-3	0	5	12
$y = -\left(x^2 - 6x + 5\right)$	24	10	0	-6	-8	-6	0	10	24

$$
\begin{aligned}
y &= -x^2 + 6x - 5 \quad \Longrightarrow \quad \text{the } y-\text{intercept is } (0, -5) \\
y &= -\left(x^2 - 6x + 5\right) \\
y &= -\left(x^2 - 6x + 9 - 9 + 5\right) \\
y &= -\left((x-3)^2 - 4\right) = \boxed{-(x-3)^2 + 4} \quad \Longrightarrow \quad \text{the vertex is } (3, 4) \\
y &= -(x - 3 + 2)(x - 3 - 2) \\
y &= -(x-1)(x-5) \quad \Longrightarrow \quad \text{the } x-\text{intercepts are } (1, 0) \text{ and } (5, 0)
\end{aligned}
$$



Multiplication by -1 causes a reflection of the graph to the $x-$axis. We will call such a parabola a downward opening parabola. This time the vertex is the highest point on the graph. If we look at the standard form as $y = 4 - (x-3)^2$, we see that now we are *subtracting* a non-negative number from 4. We obtain the greatest value when we subtract the smallest value from 4.

Theorem: Suppose that b, and c are any real numbers and a is any non-zero real number.
Then the graph of $y = ax^2 + bx + c$ is a parabola.
If $a > 0$, the parabola opens upward and the vertex is the lowest point on the graph.
If $a < 0$, the parabola opens downward and the vertex is the highest point on the graph.
By completing the square, we can obtain the standard form of the parabola, which is
$y = a(x-h)^2 + k$. The vertex of this parabola is (h,k).

Example 4. Graph the parabola $y = 20x - 5x^2 + 60$. Clearly label the coordinates of five points of the parabola, including vertex and intercepts.

Solution: We arrange the terms by degree to obtain the **polynomial form,** $y = -5x^2 + 20x + 60$. From the polynomial form, we obtain the $y-$intercept by substituting $x = 0$. If $x = 0$, then $y = 60$. And so the $y-$intercept is $(0, 60)$.

We factor out the leading coefficient and complete the square to obtain the standard form of the equation.

$$
\begin{aligned}
y &= -5x^2 + 20x + 60 \\
y &= -5\left(x^2 - 4x - 12\right) && (x-2)^2 = x^2 - 4x + 4 \\
y &= -5\left(\underbrace{x^2 - 4x + 4} - 4 - 12\right) \\
y &= -5\left((x-2)^2 - 16\right) && \text{distribute } -5 \\
y &= -5(x-2)^2 + 80
\end{aligned}
$$

The equation $y = -5(x-2)^2 + 80$ is the **standard form** of the equation of the parabola. It is the form we use to determine the vertex of the parabola. The vertex has to do with the complete square being zero. That happens when $x = 2$. And so the $x-$coordinate of the vertex is 2. And if the complete square is zero, then the $y-$coordinate of the vertex can be easily found: $y = -5(x-2)^2 + 80 = -5 \cdot 0 + 80 = 80$. So the vertex is $(2, 80)$.

Since $(x-2)^2$ is always positive or zero, the expression $-5(x-2)^2$ is always negative or zero, and so now 80 is the greatest value that the expression $-5(x-2)^2 + 80$ achieves. We can look at the standard form as $80 - 5(x-2)^2$ where we *subtract* a non-negative quantity from 80. Indeed, **since the leading coefficient is negative, the parabola opens downward and so is vertex is where the greatest value of y is achieved**.

For the $x-$intercepts, we factor via the difference of squares theorem.

$$
\begin{aligned}
-5\left((x-2)^2 - 16\right) &= 0 \\
-5\left((x-2)^2 - 4^2\right) &= 0 \\
-5(x-2+4)(x-2-4) &= 0 \\
-5(x+2)(x-6) &= 0 \quad \Longrightarrow \quad x_1 = -2 \quad x_2 = 6
\end{aligned}
$$

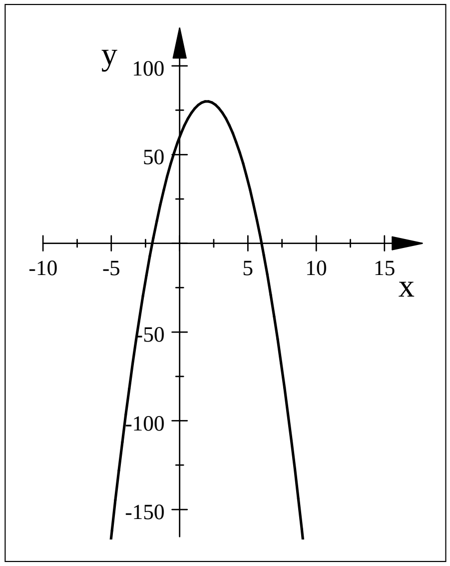
Thus, there are two $x-$intercepts, $(-2, 0)$ and $(6, 0)$. We will compute a few more points before graphing the parabola. We will work with $x-$coordinates close to that of the vertex, and use the standard form of the parabola, $y = -5(x-2)^2 + 80$.

if $x = -3$, then $y = -5(-3-2)^2 + 80 = -45 \quad \Longrightarrow$ found $(-3, -45)$

if $x = -1$, then $y = -5(-1-2)^2 + 80 = 35 \quad \Longrightarrow$ found $(-1, 35)$

if $x = 7$, then $y = -5(7-2)^2 + 80 = -45 \quad \Longrightarrow \quad$ found $(7, -45)$
if $x = 1$, then $y = -5(1-2)^2 + 80 = 75 \quad \Longrightarrow \quad$ found $(1, 75)$
if $x = 3$, then $y = -5(3-2)^2 + 80 = 75 \quad \Longrightarrow \quad$ found $(3, 75)$
if $x = 4$, then $y = -5(4-2)^2 + 80 = 60 \quad \Longrightarrow \quad$ found $(4, 60)$
if $x = 5$, then $y = -5(5-2)^2 + 80 = 35 \quad \Longrightarrow \quad$ found $(6, -5)$
We are ready to graph: we have the following points, listed left to right:

	$(-3, -45)$
$x-$intercept	$(-2, 0)$
	$(-1, 35)$
$y-$intercept	$(0, 60)$
	$(1, 75)$
vertex	$(2, 80)$
	$(3, 75)$
	$(4, 60)$
	$(5, 35)$
$x-$intercept	$(6, 0)$
	$(7, -45)$



Example 5. Graph the parabola $y = 6x^2 - 36x + 78$. Clearly label the coordinates of five points of the parabola, including vertex and intercepts.

Solution: Since the leading coefficient is positive, the graph will be an upward opening parabola. We first factor out the leading coefficient and then complete the square to obtain the standard form.

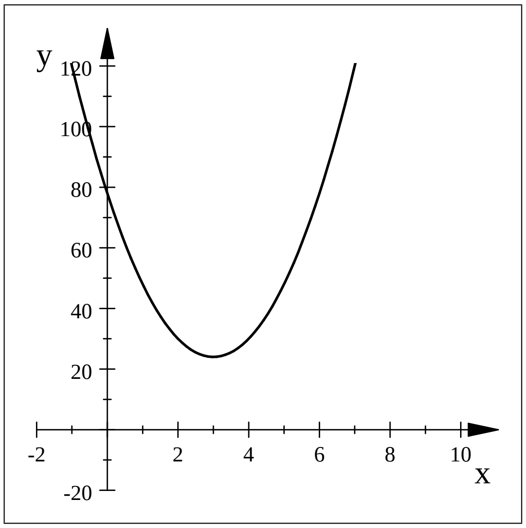
$$\begin{aligned} y &= 6x^2 - 36x + 78 && \Longrightarrow \quad y - \text{intercept: } (0, 78) \\ y &= 6\left(x^2 - 6x + 13\right) && \qquad (x-3)^2 = x^2 - 6x + 9 \\ y &= 6\left(\underbrace{x^2 - 6x + 9} - 9 + 13\right) \\ y &= 6\left((x-3)^2 + 4\right) = 6(x-3)^2 + 24 && \Longrightarrow \quad \text{vertex: } (3, 24) \end{aligned}$$

This parabola has no $x-$intercepts. This is because it is an upward opening parabola with its vertex above the $x-$axis. Indeed, the expression $6(x-3)^2 + 24$ is always at least 24. Consequently, this expression is never zero.

Algebraically, this means that the factored form can not exist, because linear factors would certainly guarantee $x-$intercepts. Indeed, the expression $6(x-3)^2 + 24$ parabola does not have a factored form. This is because the sum of squares in the parentheses in $6\left((x-3)^2 + 4\right)$ can not be factored. Recall that **the sum of two squares can never be factored.**

We find a few more points close to the vertex and graph the parabola.

	$(-1, 120)$
$y-$intercept	$(0, 78)$
	$(1, 48)$
	$(2, 30)$
vertex	$(3, 24)$
	$(4, 30)$
	$(5, 48)$
	$(6, 120)$



With the introduction of quadratic expresssions, we have discovered a new property: the existence of a smallest or greatest value of an expression. In real life, we can often describe situations using quadratic expressions. In such cases, it is natural to look for smallest or greatest values. For example, we want to find the smallest cost or smallest risk or the greatest profit. Problems in which we are looking for a greatest or smalles value (and not for a given number for value) are called **optimization** problems. Optimization will be a central topic in calculus, an important course in advanced mathematics. In case of quadratic expressions, completing the square enables us to find greatest and lowest values. In case of expressions of degree 3 or 4, or otherwise more complications, calculus develops methods to find these best values.

The following example is about a true to life formula about how objects move due to the gravitational force exerted on them by Earth.

Example 6. We are standing on the top of a $720\,\mathrm{ft}$ tall building and throw a small object upward. The object's distance from the ground, measured in feet, after t seconds is

$$y = -16t^2 + 192t + 720$$

What is the highest point that the object reaches?

Solution: $y = -16t^2 + 192t + 720$ is quadratic, with a negative leading coefficient. Consequently, the graph is a downward opening parabola. The highest point is the vertex. We will find the vertex by completing the square.

$$\begin{aligned} y &= -16t^2 + 192t + 720 = -16\left(t^2 - 12t - 45\right) && (t-6)^2 = t^2 - 12t + 36 \\ y &= -16\left(t^2 - 12t + 36 - 36 - 45\right) \\ y &= -16\left((t-6)^2 - 81\right) && \text{distribute } -16 \\ y &= -16\,(t-6)^2 + 1296 \end{aligned}$$

This indicates that the parabola has its vertex at $(6, 1296)$. Thus the highest point is reached 6 seconds after we threw the object, and it is at a height of 1296 feet.

Example 7. A company finds that if they price their product at $\$60$, they can sell 500 items of it. For every dollar increase in the price, the number of items sold will decrease by 5.

a) Let x be the increase in price from $\$60$. Define the revenue function, R to be the sales revenue that results in such pricing. Find a formula for R.

b) What price would guarantee an income of $\$31\,500$?

c) Find the price that guarantees the maximum revenue.

d) Find the maximum revenue.

Solution: a) Let x be the increase in price from $\$60$. Define the revenue function, R to be the sales revenue (revenue is the same as income) that results in such pricing. Find a formula for R.

The revenue from sales of this product can be found by multiplying the price by the number of items sold. $R = (\text{ Price}) \cdot (\text{ Number of items sold })$ For example, if the price was $\$60$, then 500 items are sold and the revenue would be $\$60 \cdot 500 = \$30\,000$. If we increase the price to $\$61$, we can only sell 495 items. Then the revenue is $\$61 \cdot 495 = \$30\,195$. And so on, we should see a few examples to understand how the revenue works.

price	items sold	Revenue
60	500	30 000
61	495	30 195
62	490	30 380
$60 + x$	$500 - 5x$	$(60 + x)(500 - 5x)$

If we denote the change in the price from $\$60$, then the price is $60 + x$. With this price, we can sell $5x$ less items, that is, $500 - 5x$.

$$\begin{aligned} R &= (60 + x)(500 - 5x) && \text{rearrange} \\ &= (x + 60)(-5x + 500) && \text{factor out } -5 \\ &= -5(x + 60)(x - 100) \\ &= -5\left(x^2 - 100x + 60x - 6000\right) \\ &= -5\left(x^2 - 40x - 6000\right) \end{aligned}$$

So $\boxed{R = (x + 60)(-5x + 500) = -5x^2 + 200x + 30\,000}$.

b) To determine the price that would guarantee an income of $\$31\,500$, we need to solve the equation: $x = ?$ so that $R = 31\,500$.

$$\text{Solve} \quad -5x^2 + 200x + 30\,000 = 31500 \quad \text{for } x$$

Since the equation is quadratic, we will reduce one side to zero. We will avoid a negative leading coefficient by reducing the left-hand side to zero.

$$\begin{aligned} -5x^2 + 200x + 30\,000 &= 31500 \\ 0 &= 5x^2 - 200x + 1500 \end{aligned}$$

We will factor by completing the square.

$$\begin{aligned}
5x^2-200x+1500 &= 0 \quad \text{factor out } 5 \\
5\left(x^2-40x+300\right) &= 0 \qquad (x-20)^2=x^2-40x+400 \\
5\left(\underbrace{x^2-40x+400}-400+300\right) &= 0 \\
5\left((x-20)^2-100\right) &= 0 \\
5\left((x-20)^2-10^2\right) &= 0 \\
5(x-20+10)(x-20-10) &= 0 \\
5(x-10)(x-30) &= 0 \implies x_1=10 \qquad x_2=30
\end{aligned}$$

We check. If $x=10$, the price is increased by $\$10$ and so it sells for $\$70$. We can then sell $500-5\cdot 10=450$ items, and so the income is $R=70\cdot 450=31\,500$. On the other hand, if $x=30$, the selling price is $\$90$. We then can sell $500-5\cdot 30=350$ items, and so the revenue is $R=90\cdot 350=31\,500$. So, a price of both $\boxed{\$70 \text{ and } \$90}$ would guarantee a revenue of $\$31\,500$.

c) To find the price that guarantees the maximum revenue, we need to find the greatest value of $-5x^2+200x+30\,000$. Since $y=-5x^2+200x+30\,000$ is clearly a downward turning parabola, its vertex is wherethe greatest value is taken. To find the vertex, we obtain the standard form by completing the square.

$$\begin{aligned}
R &= -5x^2+200x+30\,000 \\
&= -5\left(x^2-40x-6000\right) \qquad (x-20)^2=x^2-40x+400 \\
&= -5\left(\underbrace{x^2-40x+400}-400-6000\right) \\
&= -5\left((x-20)^2-6400\right) \qquad \text{distribute } -5 \\
&= -5(x-20)^2+32\,000
\end{aligned}$$

The vertex of the parabola is $(20,32\,000)$. Thus a $\$20$ increase in the price, implying a price of $\boxed{\$80}$, will guarantee a maximal income of $\$32\,000$.

d) The maximum revenue, (same as the greatest revenue), was found before, it is $\boxed{\$32\,000}$.

Practice Problems

Graph each of the parabolas given below. In each case, clearly label the coordinates of five points of the parabola, including vertex and intercepts.

1. $y = 336 - 16x^2 - 64x$
2. $y = 3x^2 - 30x + 75$
3. $y = 4x - 2x^2 - 20$
4. $y = 6x - x^2$
5. $y = 3x^2 - 12$
6. $y = 4x + \dfrac{1}{2}x^2 + 10$
7. $y = x(2 - x) - 1$
8. $y = -12x - 2x^2 - 10$
9. Find the greatest or smallest value for each of the given expressions. In each case, also state whether you found a greatest or a smallest value.

   a) $(x + 5)^2 - 49$

   b) $-(2x - 3)^2 - 1$

   c) $2x^2 - 20x + 84$

   d) $-x^2 - 6x$

   e) $-2x^2 + 12$

   f) $\dfrac{1}{3}x^2 - \dfrac{1}{2}x + 1$

   g) $(20 - x)(x + 80)$

10. We are standing on the top of a $720\,\mathrm{ft}$ tall building and throw a small object upward. The object's distance from the ground, measured in feet, after t seconds is

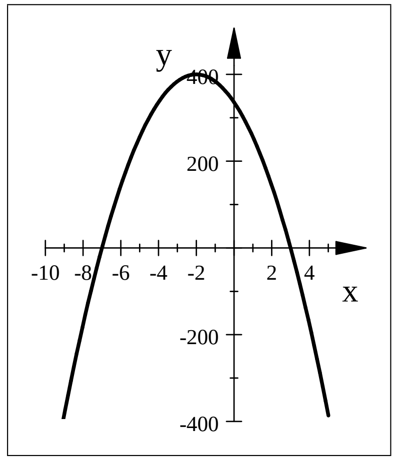
$$y = -16t^2 + 64t + 720$$

    What is the highest point that the object reaches?

11. A company finds that if they price their product at $\$20$, they can sell 3000 items of it. For every dollar increase in the price, the number of items sold will decrease by 10. Find the price that guarantees the maximum revenue. Find the maximum revenue possible.

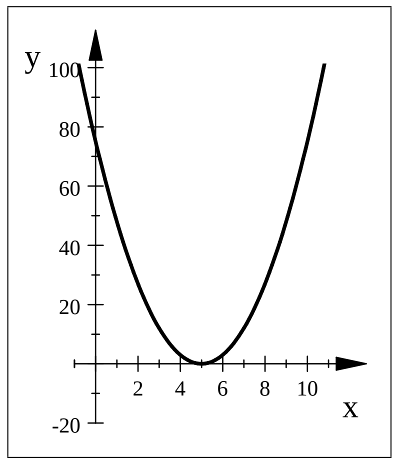
Answers

1. $y = 336 - 16x^2 - 64x$



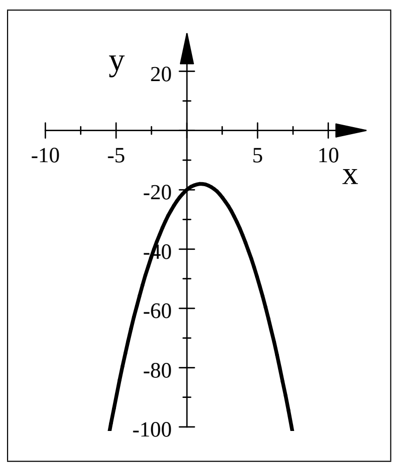
polynomial form	$y = -16x^2 - 64x + 336$ $\Rightarrow$ $y-$intercept: $(0, 336)$
standard form	$y = -16(x+2)^2 + 400$ $\Rightarrow$vertex: $(-2, 400)$
factored form:	$y = -16(x+7)(x-3)$ $x-$intercepts: $(-7, 0)$ and $(3, 0)$
additional points:	$(-4, 336)$, $(-3, 384)$, $(-1, 384)$, $(1, 256)$, $(2, 144)$, $(4, -176)$

2. $y = 3x^2 - 30x + 75$



polynomial form	$y = 3x^2 - 30x + 75$ $\Rightarrow$ $y-$intercept: $(0, 75)$
standard form	$y = 3(x-5)^2$ $\Rightarrow$vertex: $(5, 0)$
factored form	$y = 3(x-5)^2$ $\Rightarrow$ $x-$intercept: $(5, 0)$
additional points	$(1, 48)$, $(2, 27)$, $(3, 12)$ $(4, 3)$, $(6, 3)$, $(7, 12)$, $(8, 27)$

3. $y = 4x - 2x^2 - 20$



polynomial form	$y = -2x^2 + 4x - 20$ $\Rightarrow$ $y-$intercept: $(0, -20)$
standard form	$y = -2(x-1)^2 - 18$ $\Rightarrow$vertex: $(1, -18)$
factored form	doesn't exist $\Rightarrow$no $x-$intercepts
additional points	$(-3, -50)$, $(-2, -36)$, $(2, -20)$ $(3, -26)$, $(4, -36)$, $(5, -50)$

4. $y = 6x - x^2$

polynomial form $y = -x^2 + 6x$

$\Rightarrow y-$intercept: $(0, 0)$

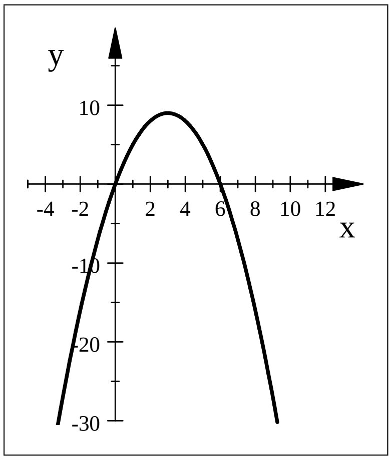
standard form $y = -(x-3)^2 + 9$

$\Rightarrow$vertex: $(3, 9)$

factored form $y = -x(x-6)$

$\Rightarrow x-$intercepts: $(0, 0)$ and $(6, 0)$

additional points $(-1, -7)$, $(1, 5)$, $(2, 8)$, $(4, 8)$
$(5, 5)$, $(7, -7)$, $(8, -16)$



5. $y = 3x^2 - 12$

polynomial form $y = 3x^2 - 12$

$y-$intercept: $(0, -12)$

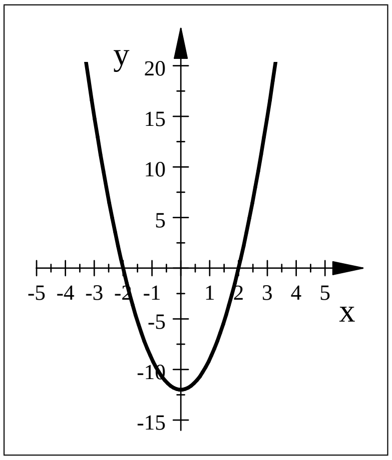
standard form $y = 3x^2 - 12$

$\Rightarrow$vertex: $(0, -12)$

factored form $y = 3(x+2)(x-2)$

$\Rightarrow x-$intercepts: $(-2, 0)$ and $(2, 0)$

additional points $(-4, 36)$, $(-3, 15)$, $(-1, -9)$,
$(1, -9)$, $(3, 15)$, $(4, 36)$, $(5, 63)$



6. $y = 4x + \frac{1}{2}x^2 + 10$

polynomial form $y = \frac{1}{2}x^2 + 4x + 10$

$\Rightarrow y-$intercept: $(0, 10)$

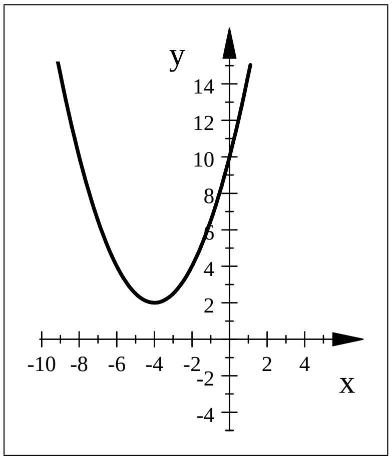
standard form $y = \frac{1}{2}(x+4)^2 + 2$

$\Rightarrow$ vertex: $(-4, 2)$

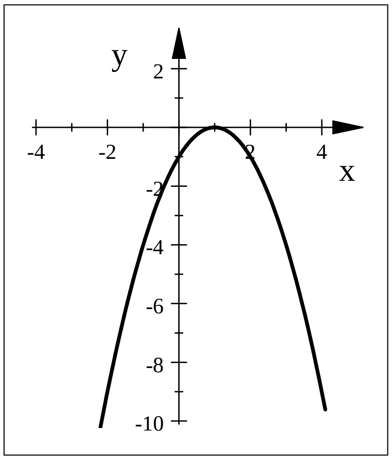
factored form doesn't exist

$\Rightarrow$ no $x-$intercepts

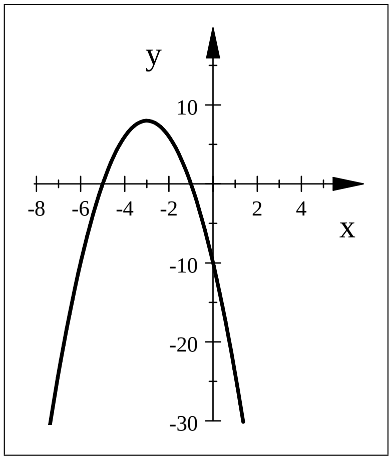
additional points: $\left(-7, \frac{13}{2}\right)$, $(-6, 4)$, $\left(-5, \frac{5}{2}\right)$, $\left(-3, \frac{5}{2}\right)$,
$(-2, 4)$, $\left(-1, \frac{13}{2}\right)$, $\left(1, \frac{29}{2}\right)$, $(2, 20)$



7. $y = x(2-x) - 1$
   polynomial form $y = -x^2 + 2x - 1$
   $\Rightarrow$ $y-$intercept: $(0, -1)$
   standard form $y = -(x-1)^2$
   $\Rightarrow$ vertex: $(1, 0)$
   factored form $y = -(x-1)^2$
   $\Rightarrow$ $x-$intercept: $(1, 0)$
   additional points $(-2, -9)$, $(-1, -4)$, $(2, -1)$, $(3, -4)$, $(4, -9)$, $(5, -16)$



8. $y = -12x - 2x^2 - 10$
   polynomial form $y = -2x^2 - 12x - 10$
   $\Rightarrow$ $y-$intercept: $(0, -10)$
   standard form $y = -2(x+3)^2 + 8$
   $\Rightarrow$ vertex: $(-3, 8)$
   factored form $y = -2(x+5)(x+1)$
   $\Rightarrow$ $x-$intercepts: $(-5, 0)$ and $(-1, 0)$
   additional points $(-7, -24)$, $(-6, -10)$, $(-4, 6)$, $(-2, 6)$, $(1, -24)$, $(2, -42)$



9. a) -49 is the smallest value b) -1 is the greatest value c) 34 is the smallest value d) 9 is the greatest value
   e) 12 is the greatest value f) $\dfrac{13}{16}$ is the greatest value g) 2500 is the greatest value

10. 784 feet 11. max income: $\$256\,000$ when the price is $\$160$

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