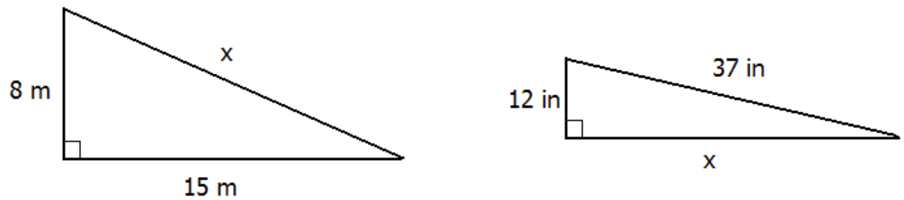
Sample Problems

1. Could the three line segments given below be the three sides of a right triangle? Explain your answer.

   a) 6 cm, 10 cm, and 8 cm b) 7 ft, 15 ft, and 50 ft c) 4 m, 5 m, and 6 m

2. Find the missing sides of the right triangles shown on the picture below.

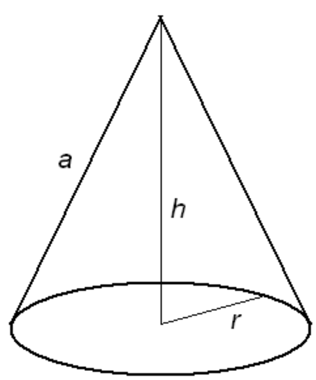


a) b)

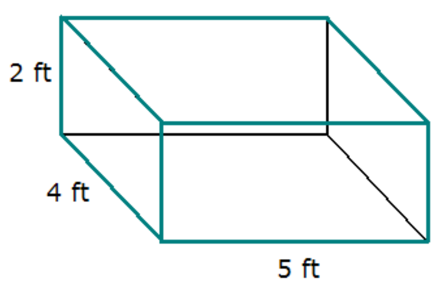
3. Find the distance between $(3, 8)$ and $(8, -4)$.

4. The sides of an isosceles triangle are 42 units, 29 units, and 29 units long. Find the length of the height drawn to the 42 units long side.

5. Find the height h of the cone shown on the picture below, if the base has a radius of 10 m and $a = 26$ m.

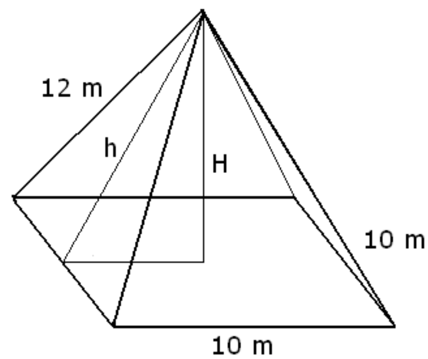


6. Find the length of the longest line segment (called the main diagonal) in the rectangular prism shown on the picture below.



 Last revised: September 26, 2012

7. A pyramid (shown on the picture below) has a square base with sides 10 m (meters) long. The other faces of the pyramid are isosceles triangles with sides 10 m, 12 m, and 12 m.



a) Find the exact value of h, the length of the height in a triangular face.

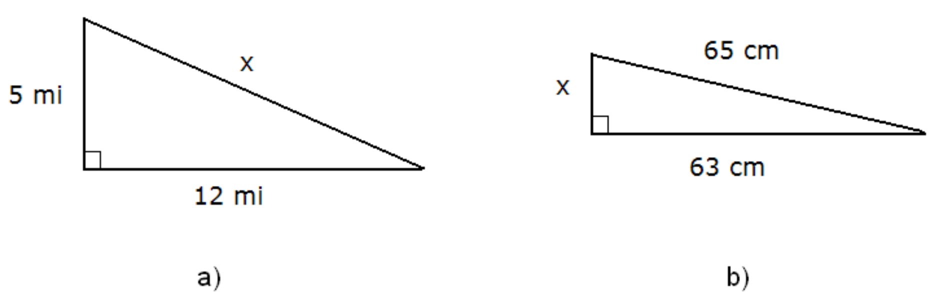
b) Find the exact value of H, the length of the height of the pyramid.

Practice Problems

1. Could the three line segments given below be the three sides of a right triangle? Explain your answer.

a) 2 cm, 7 cm, and 1 cm b) 37 ft, 12 ft, and 35 ft c) 6 m, 7 m, and 8 m

2. Find the missing sides of the right triangles shown on the picture below.



3. Find the length of the diagonal in a rectangle with sides 20 ft and 21 ft long.

4. Find the length of the diagonal of a square with sides 1 unit long.

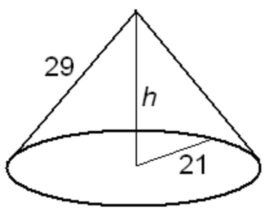
5. The sides of an isosceles triangle are 25 m, 25 m, and 14 m long. Find the length of the height drawn to the 14 m long side.

6. Two sides of a right triangle are 8 cm and 17 cm long. Find the length of the missing side.

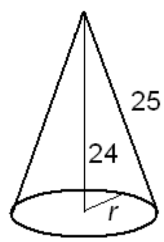
7. Find the distance between the points given

   a) $(-2,-3)$ and $(3,1)$ b) $(-9,-3)$ and $(15,4)$.

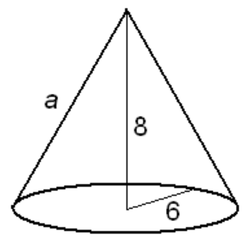
8. Find the missing lengths indicated of the picture below. Dimensions are in meters.



a)



b)



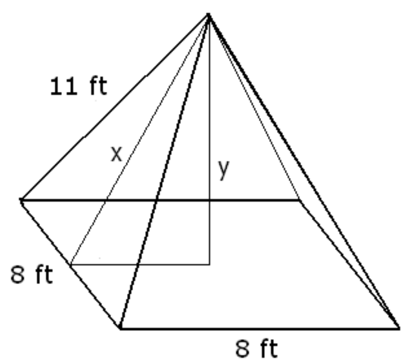
c)

9. Find the length of the main diagonal in a rectangular prism with sides

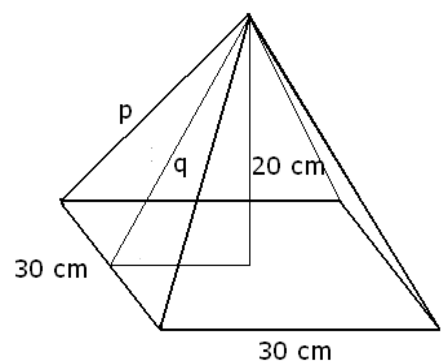
   a) 2 m, 10 m, and 11 m b) 5 ft, 7 ft, and 1 ft

10. Find the exact value of the missing lengths, labeled

    a) x and y b) p and q



(a)



(b)

Sample Problems - Answers

1.) a) There is a right angle opposite the 10 cm long side

b) not even a triangle c) not a right triangle

2.) a) 17 m b) 35 inches 3.) 13 units 4.) 20 units 5.) 24 m 6.) $\sqrt{45}$ ft

7.) $h = \sqrt{119}$ m $H = \sqrt{94}$ m

Practice Problems - Answers

1.) a) not even a triangle b) There is a right angle opposite the 37 ft long side.

c) not a right triangle

2.) a) 13 mi b) 16 cm 3.) 29 ft 4.) $\sqrt{2}$ units 5.) 24 m 6.) 15 cm or $\sqrt{353}$ cm

7.) a) $\sqrt{41}$ units b) 25 units 8.) a) 20 m b) 7 m c) 10 m 9.) a) 15 m b) $\sqrt{75}$ ft

10.) a) $x = \sqrt{105}$ ft $y = \sqrt{89}$ ft b) $q = 25$ cm, $p = \sqrt{850}$ cm $= 5\sqrt{34}$ cm

Sample Problems - Solutions

1. Could the three line segments given below be the three sides of a right triangle? Explain your answer.

a) 6 cm, 10 cm, and 8 cm
Solution: The longest side is 10 cm long. Thus, only this side can be the hypotenuse. First we check the triangle-inequality: the two shorter sides should add up to a number greater than the logest side. $6+8=14$ and $14>10$, so this triangle does exist. Now we use the Pythagorean theorem to check for a right angle:

$$6^2+8^2 \stackrel{?}{=} 10^2$$

We get that the two quantities are equal, thus this triangle has a right angle. As always, it is opposite the longest side that, in this case, is 10 cm long.

b) 7 ft, 15 ft, and 50 ft
Solution: The longest side is 50 ft long. Thus, only this side can be the hypotenuse. First we check the triangle-inequality: the two shorter sides should add up to a number greater than the logest side. $7+15=22$ and $22 \not> 50$, so this triangle does not even exist, let alone has a right angle.

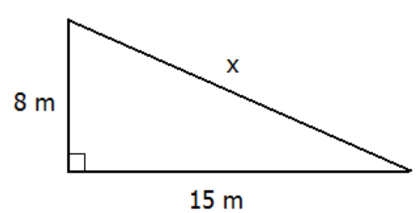
c) 4 m, 5 m, and 6 m
Solution: The longest side is 6 m long. Thus, only this side can be the hypotenuse. First we check the triangle-inequality: the two shorter sides should add up to a number greater than the logest side. $4+5=9$ and $9>6$, so this triangle does exist. Now we use the Pythagorean theorem to check for a right angle:

$$4^2+5^2 \stackrel{?}{=} 6^2$$

$$\begin{aligned} \text{LHS} &= 4^2+5^2=16+25=41 \\ \text{RHS} &= 6^2=36 \qquad\qquad \text{LHS} \neq \text{RHS} \end{aligned}$$

We get that the two quantities are not equal, thus this triangle does not have a right angle.

2. a) Find the hypotenuse of the triangle shown on the figure below.

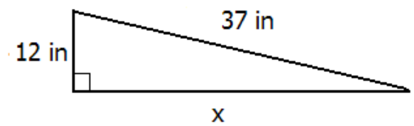


Solution: We apply the Pythagorean theorem. The longest side is always the one opposite the right angle.

$$\begin{aligned} 8^2+15^2 &= x^2 \\ 289 &= x^2 \\ x &= \pm 17 \end{aligned}$$

Since distance can not be negative, -17 is ruled out. The answer is $17\,\mathrm{m}$.

b) Find the missing leg of the right triangle shown on the picture below.

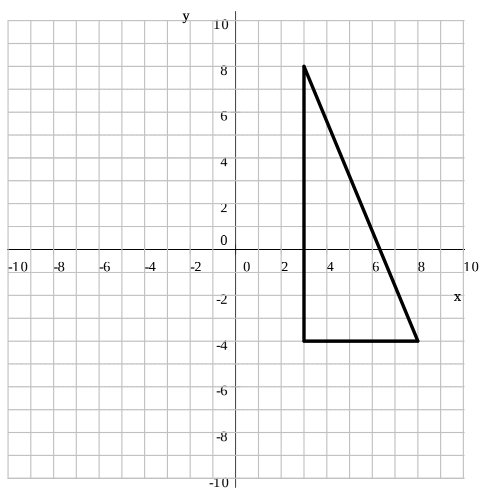


Solution: We apply the Pythagorean theorem. The longest side is always the one opposite the right angle.

$$\begin{aligned} 12^2 + x^2 &= 37^2 \\ x^2 + 144 &= 1369 && \text{subtract } 144 \\ x^2 &= 1225 && \sqrt{1225} = 35 \\ x &= \pm 35 \end{aligned}$$

Since distance can not be negative, -35 is ruled out. The answer is 35 inches.

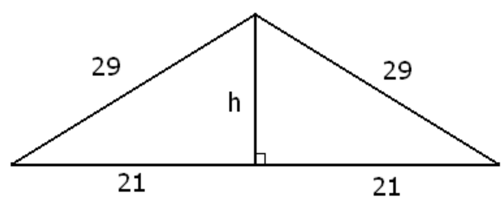
3. Find the distance between $(3, 8)$ and $(8, -4)$.
Solution: We graph the points, they determine a right triangle as shown below. We can compute the distance as the hypotenuse of the right triangle. How long are the legs?
Algebra: $8 - 3 = 5$ and $8 - (-4) = 12$.
The difference will alwys work. Even if we get -5 instead of 5, it will not matter since we will square it in the Pythagorean theorem.
Geometry: From 3 to 8 we have to step 5 units up. From -4 to 8 : first we step 4 to get from -4 to 0. Then another 8 steps to 8, and so $4 + 8 = 12$ steps. The message here is that the algebra and geometry will always agree.
The legs are 5 and 12 units long, and we need to find the hypotenuse.



$$\begin{aligned} 5^2 + 12^2 &= x^2 \\ 25 + 144 &= x^2 \\ 169 &= x^2 \\ 0 &= x^2 - 13^2 \\ 0 &= (x+13)(x-13) \\ x_1 &= -13 \quad \text{and} \quad x_2 = 13 \end{aligned}$$

Since distances are never negative, -13 is ruled out and so the answer is 13 units.

4. The sides of an isosceles triangle are 42 units, 29 units, and 29 units long. Find the length of the height drawn to the 42 units long side.
Solution: In case of isosceles triangles, the height drawn to the base splits the triangle into two identical right triangles as shown on the picture below.

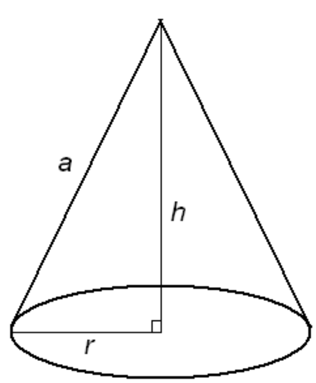


The height now can be easily computed via the Pythagorean theorem.

$$\begin{aligned} 21^2 + h^2 &= 29^2 \\ 441 + h^2 &= 841 \\ h^2 &= 400 \\ h &= \pm 20 \quad \Longrightarrow \quad h = 20 \end{aligned}$$

The height belonging to the base is 20 units long.

5. Find the height h of the cone shown on the picture below, if the base has a radius of 10 m and $a = 26$ m.
Solution: Consider the right triangle shown on the figure.

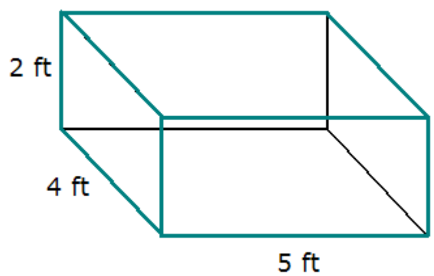


By the Pythagorean theorem, $r^2 + h^2 = a^2$. We can use this to solve for h.

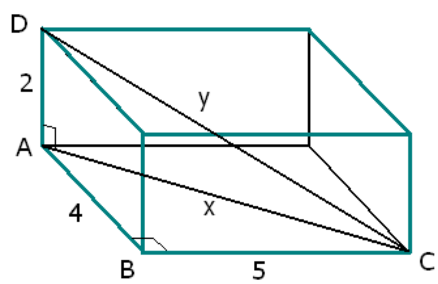
$$\begin{aligned} r^2 + h^2 &= a^2 \\ (10\,\mathrm{m})^2 + h^2 &= (26\,\mathrm{m})^2 \\ 100\,\mathrm{m}^2 + h^2 &= 676\,\mathrm{m}^2 \\ h^2 &= 576\,\mathrm{m}^2 \\ h &= \pm\sqrt{576\,\mathrm{m}^2} = \pm 24\,\mathrm{m} \quad \Longrightarrow \quad h = 24\,\mathrm{m} \end{aligned}$$

Thus the height of the cone is 24 m.

6. Find the length of the longest line segment (called the main diagonal) in the rectangular prism shown on the picture below.



Solution: We will apply the Pythagorean theorem twice. Let us label the points and sides we will use on the picture first.



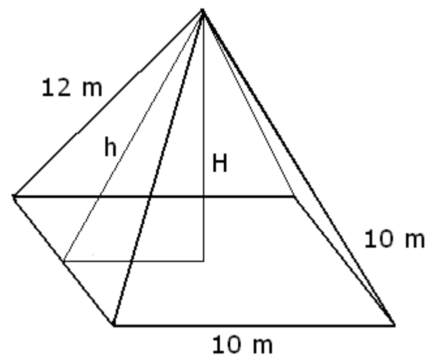
We will find x using the Pythagorean theorem in triangle ABC. Then we can find y using the Pythagorean theorem in triangle ACD.

$$\begin{aligned} 4^2 + 5^2 &= x^2 \\ 41 &= x^2 \\ \pm\sqrt{41} &= x \quad \Longrightarrow x = \sqrt{41} \end{aligned}$$

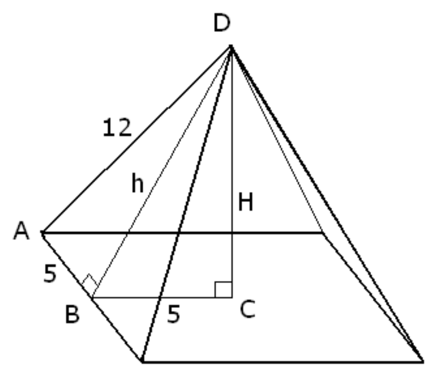
$$\begin{aligned} 2^2 + x^2 &= y^2 \\ 45 &= y^2 \\ \pm\sqrt{45} &= y \quad \Longrightarrow y = \sqrt{45} \end{aligned}$$

Note: Our result is actually $\sqrt{2^2 + 4^2 + 5^2}$. Indeed, we can see that the length of the main diagonal in a rectangular prism.with sides x, y, and z is $L = \sqrt{x^2 + y^2 + z^2}$. This is sometimes called the 3-dimensional Pythagorean theorem.

7. A pyramid (shown on the picture below) has a square base with sides 10 m (meters) long. The other faces of the pyramid are isosceles triangles with sides 10 m, 12 m, and 12 m.



a) Find the exact value of h, the length of the height in a triangular face.
Solution: Let us label some of the vertices as shown on the figure below.



We can find the value of h by stating the Pythagorean theorem on the right triangle ABD. The hypotenuse is the side opposite the right angle.

$$\begin{aligned} (5\,\mathrm{m})^2 + h^2 &= (12\,\mathrm{m})^2 \\ 25\,\mathrm{m}^2 + h^2 &= 144\,\mathrm{m}^2 \\ h^2 &= 119\,\mathrm{m}^2 \\ h &= \pm\sqrt{119\,\mathrm{m}^2} \quad \Longrightarrow \quad h = \sqrt{119}\,\mathrm{m} \end{aligned}$$

b) Find the exact value of H, the length of the height of the pyramid.
Solution: Let use the labels shown on the figure above. We can find the value of H by stating the Pythagorean theorem on the right triangle BCD. The hypotenuse is the side opposite the right angle.

$$\begin{aligned} (5\,\mathrm{m})^2 + H^2 &= h^2 \\ (5\,\mathrm{m})^2 + H^2 &= \left(\sqrt{119}\,\mathrm{m}\right)^2 \\ 25\,\mathrm{m}^2 + H^2 &= 119\,\mathrm{m}^2 \\ H^2 &= 94\,\mathrm{m}^2 \\ H &= \pm\sqrt{94\,\mathrm{m}^2} \quad \Longrightarrow \quad H = \sqrt{94}\,\mathrm{m} \end{aligned}$$

