## Sample Problems

1. $\sqrt{3x-2}=x$
2. $x+2\sqrt{2x+16}=-2$
3. $10+\sqrt{4x-7}=7$
4. $5+\sqrt{x+15}=x$
5. $2\sqrt{x-1}=x-4$
6. $x=2\sqrt{x+8}$
7. $5\sqrt{x}+1=3\sqrt{x}+17$
8. $\sqrt{2x+5}+5=x$
9. $2\sqrt{2x+29}=x+12$
10. $\sqrt{x+10}+10=x$

## Practice Problems

1. $\sqrt{3x-5}=4$
2. $2\sqrt{a-1}+7=1$
3. $3\sqrt{7x+1}+2=20$
4. $\sqrt{x+3}=x-9$
5. $2\sqrt{x+5}=x-3$
6. $p+1=3\sqrt{p+1}$
7. $2\sqrt{x+5}=x+6$
8. $\sqrt{18+x}=x-2$
9. $w+13=2\sqrt{3w+30}$
10. $3\sqrt{4x-39}=x-3$
11. $-4a=3\sqrt{2a+22}$
12. $2y-3=\sqrt{2y+3}$
13. $\sqrt{b-2}+b=8$
14. $\sqrt{3x+1}-x=-1$
15. $2\sqrt{x+1}=x+2$
16. $m-4=2\sqrt{2m-8}$
17. $-k-1=-2\sqrt{k+16}$
18. $\sqrt{1-x}+1=x+12$
19. $2=\sqrt{x^2+1}-x$
20. $3\sqrt{4t-35}=t-6$

 

## Sample Problems – Answers

1.) $1, 2$ 2.) $-6$ 3.) no real solution 4.) $10$ 5.) $10$ 6.) $8$ 7.) $64$ 8.) $10$

9.) $-2$ 10.) $15$

## Practice Problems – Answers

1.) $7$ 2.) no real solution 3.) $5$ 4.) $13$ 5.) $11$ 6.) $-1, 8$ 7.) $-4$ 8.) $7$ 9.) $-7$

10.) $12, 30$ 11.) $-3$ 12.) $3$ 13.) $6$ 14.) $5$ 15.) $0$ 16.) $4, 12$ 17.) $9$

18.) $-8$ 19.) $-\frac{3}{4}$ 20.) $9, 39$

 

## Sample Problems – Solutions

1. $\sqrt{3x-2}=x$
   Solution:

$$\begin{aligned} \sqrt{3x-2} &= x && \text{square} \\ 3x-2 &= x^2 && \text{reduce one side to zero} \\ 0 &= x^2-3x+2 && \text{factor} \\ 0 &= (x-2)(x-1) && \Longrightarrow \quad x_1=2 \quad \text{and} \quad x_2=1 \end{aligned}$$

   We check: if $x=2$, then

$$\begin{aligned} \text{LHS} &= \sqrt{3(2)-2}=\sqrt{4}=2 \\ \text{RHS} &= 2 \end{aligned}$$

   and if $x=1$, then

$$\begin{aligned} \text{LHS} &= \sqrt{3(1)-2}=\sqrt{1}=1 \\ \text{RHS} &= 1 \end{aligned}$$

   Thus the solution set is: $\{1,2\}$

2. $x+2\sqrt{2x+16}=-2$
   Solution:

$$\begin{aligned} x+2\sqrt{2x+16} &= -2 && \text{add 2 and subtract } 2\sqrt{2x+16} \\ x+2 &= 2\sqrt{2x+16} && \text{square both sides} \\ (x+2)^2 &= \left(2\sqrt{2x+16}\right)^2 \\ x^2+4x+4 &= 4(2x+16) \\ x^2+4x+4 &= 8x+64 && \text{subtract } (8x+64) \\ x^2-4x-60 &= 0 \\ (x-10)(x+6) &= 0 \end{aligned}$$

$$x_1=10 \text{ and } x_2=-6$$

   We check: if $x=10$, then

$$\begin{aligned} \text{LHS} &= 10+2\sqrt{2\cdot 10+16}=10+2\sqrt{36}=10+12=22 \\ \text{RHS} &= -2 \end{aligned}$$

   Thus $x=10$ is NOT a solution. If $x=-6$, then

$$\begin{aligned} \text{LHS} &= -6+2\sqrt{2(-6)+16}=-6+2\sqrt{4}=-6+2\cdot 2=-2 \\ \text{RHS} &= -2 \end{aligned}$$

   Thus the only solution is $-6$.

3. $10+\sqrt{4x-7}=7$
   Solution:

$$\begin{aligned} 10+\sqrt{4x-7} &= 7 && \text{subtract 10} \\ \sqrt{4x-7} &= -3 \end{aligned}$$

   Since the square root of no real number is negative, there is no real solution.

4. $5+\sqrt{x+15}=x$
Solution:

$$\begin{array}{rcll} 5+\sqrt{x+15} &=& x & \text{subtract 5} \\ \sqrt{x+15} &=& x-5 & \text{square both sides} \\ x+15 &=& (x-5)^2 & \text{FOIL right hand side} \\ x+15 &=& x^2-10x+25 & \text{reduce one side to zero} \\ 0 &=& x^2-11x+10 & \text{factor} \\ 0 &=& (x-1)(x-10) & \Longrightarrow \quad x_1=1 \quad \text{and } x_2=10 \end{array}$$

We check: If $x=1$, then

$$\begin{array}{rcl} \text{LHS} &=& 5+\sqrt{1+15}=5+\sqrt{16}=5+4=9 \\ \text{RHS} &=& 1 \\ \text{RHS} &\neq& \text{LHS} \end{array}$$

Thus $x=1$ is NOT a solution.

If $x=10$, then

$$\begin{array}{rcl} \text{LHS} &=& 5+\sqrt{10+15}=5+\sqrt{25}=5+5=10 \\ \text{RHS} &=& 10 \\ \text{RHS} &=& \text{LHS} \end{array}$$

Thus $x=10$ is the only solution.

5. $2\sqrt{x-1}=x-4$
Solution:

$$\begin{array}{rcll} 2\sqrt{x-1} &=& x-4 & \text{square both sides} \\ 4(x-1) &=& (x-4)^2 & \text{FOIL, distribute} \\ 4x-4 &=& x^2-8x+16 & \text{reduce one side to zero} \\ 0 &=& x^2-12x+20 & \text{factor} \\ 0 &=& (x-2)(x-10) & \Longrightarrow \quad x_1=2 \quad \text{and} \quad x_2=10 \end{array}$$

We check: If $x=2$, then

$$\begin{array}{rcl} \text{LHS} &=& 2\sqrt{2-1}=2\sqrt{1}=2(1)=2 \\ \text{RHS} &=& 2-4=-2 \\ \text{RHS} &\neq& \text{LHS} \end{array}$$

Thus $x=2$ is NOT a solution.
If $x=10$, then

$$\begin{array}{rcl} \text{LHS} &=& 2\sqrt{10-1}=2\sqrt{9}=2(3)=6 \quad \text{and} \quad \text{RHS}=10-4=6 \\ \text{RHS} &=& \text{LHS} \end{array}$$

Thus $x=10$ is the only solution.

6. $x = 2\sqrt{x+8}$
Solution:

$$\begin{aligned} x &= 2\sqrt{x+8} && \text{square both sides} \\ (x)^2 &= \left(2\sqrt{x+8}\right)^2 \\ x^2 &= 4(x+8) \\ x^2 - 4x - 32 &= 0 \\ (x-8)(x+4) &= 0 \end{aligned}$$

$$x_1 = 8 \text{ and } x_2 = -4$$

We check: If $x = 8$, then

$$\text{LHS} = 8 \quad \text{and} \quad \text{RHS} = 2\sqrt{8+8} = 2\sqrt{16} = 2 \cdot 4 = 8$$

Thus $x = 8$ is indeed a solution.
If $x = -4$, then

$$\begin{aligned} \text{LHS} &= -4 \\ \text{RHS} &= 2\sqrt{-4+8} = 2\sqrt{4} = 2 \cdot 2 = 4 \\ \text{RHS} &\neq \text{LHS} \end{aligned}$$

Thus $x = -4$ is NOT a solution. The only solution is $x = 8$.

7. $5\sqrt{x} + 1 = 3\sqrt{x} + 17$
Solution:

$$\begin{aligned} 5\sqrt{x} + 1 &= 3\sqrt{x} + 17 && \text{subtract } 3\sqrt{x} \\ 2\sqrt{x} + 1 &= 17 && \text{subtract 1} \\ 2\sqrt{x} &= 16 && \text{divide by 2} \\ \sqrt{x} &= 8 && \text{square both sides} \\ x &= 64 \end{aligned}$$

We check: If $x = 64$, then

$$\text{LHS} = 5\sqrt{64} + 1 = 5(8) + 1 = 41 \quad \text{and} \quad \text{RHS} = 3\sqrt{64} + 17 = 3(8) + 17 = 24 + 17 = 41$$

Thus $x = 64$ is indeed a solution.

8. $\sqrt{2x+5} + 5 = x$
Solution:

$$\begin{aligned} \sqrt{2x+5} + 5 &= x && \text{subtract 5} \\ \sqrt{2x+5} &= x - 5 && \text{square} \\ 2x + 5 &= x^2 - 10x + 25 && \text{reduce one side to zero} \\ 0 &= x^2 - 12x + 20 && \text{factor} \\ 0 &= (x-2)(x-10) && \Longrightarrow \quad x_1 = 2 \quad \text{and} \quad x_2 = 10 \end{aligned}$$

We check: If $x = 2$, then

$$\begin{aligned} \text{LHS} &= \sqrt{2(2)+5} + 5 = \sqrt{4+5} + 5 = \sqrt{9} + 5 = 3 + 5 = 8 \quad \text{and} \quad \text{RHS} = 2 \\ \text{RHS} &\neq \text{LHS} \end{aligned}$$

Thus $x=2$ is NOT a solution.
If $x=10$, then

$$\begin{aligned} \text{LHS} &= \sqrt{2(10)+5}+5=\sqrt{20+5}+5=\sqrt{25}+5=5+5=10 \\ \text{RHS} &= 10 \\ \text{RHS} &= \text{LHS} \end{aligned}$$

Thus $x=10$ is the only solution.

9. $2\sqrt{2x+29}=x+12$
Solution:

$$\begin{aligned} 2\sqrt{2x+29} &= x+12 && \text{square both sides} \\ \left(2\sqrt{2x+29}\right)^2 &= (x+12)^2 \\ 4(2x+29) &= (x+12)^2 \\ 8x+116 &= x^2+24x+144 && \text{reduce one side to zero} \\ 0 &= x^2+16x+28 \\ 0 &= (x+14)(x+2) \implies x_1=-14 \quad \text{and} \quad x_2=-2 \end{aligned}$$

We check: if $x=-14$, then

$$\text{LHS}=2\sqrt{2(-14)+29}=2\sqrt{1}=2 \quad \text{and} \quad \text{RHS}=-14+12=-2$$

Since the two sides are not equal, $x=-14$ is NOT a solution. If $x=-2$, then

$$\text{LHS}=2\sqrt{2(-2)+29}=2\sqrt{25}=2\cdot 5=10 \quad \text{and} \quad \text{RHS}=-12+12=10$$

Thus $x=-2$ is the only solution.

10. $\sqrt{x+10}+10=x$
Solution:

$$\begin{aligned} \sqrt{x+10}+10 &= x && \text{subtract } 10 \\ \sqrt{x+10} &= x-10 && \text{square} \\ x+10 &= (x-10)^2 \\ x+10 &= x^2-20x+100 && \text{reduce one side to zero} \\ 0 &= x^2-21x+90 && \text{factor} \\ 0 &= (x-6)(x-15) \implies x_1=6 \quad \text{and} \quad x_2=15 \end{aligned}$$

We check: if $x=6$, then

$$\begin{aligned} \text{LHS} &= \sqrt{6+10}+10=\sqrt{16}+10=4+10=14 \\ \text{RHS} &= 6 \\ \text{LHS} &\neq \text{RHS} \end{aligned}$$

If $x=15$, then

$$\text{LHS}=\sqrt{15+10}+10=\sqrt{25}+10=5+10=15=\text{RHS}$$

since $x=6$ doesn't work, the only solution is 15. In set notation: the solution set is $\{15\}$.

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