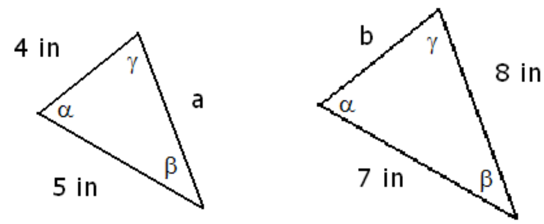
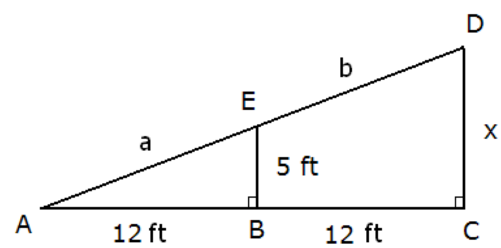
Sample Problems

1. The triangles shown below are similar. Find the exact values of a and b shown on the picture below.



2. Consider the picture shown below.



   a) Use the Pythagorean Theorem to find the value of a.

   b) Prove that the triangles ABE and ACD are similar.

   c) Use similar triangles to find the value of x.

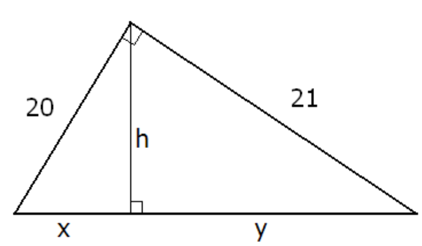
   d) Find the value of b.

3. a) A person is standing 40 ft away from a street light that is 30 ft tall. How tall is he if his shadow is 10 ft long?

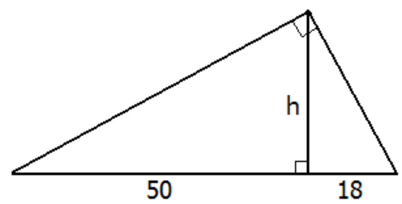
   b) A 6 ft tall person is standing 24 ft away from a street light that is 15 ft tall. How long is her shadow?

4. Prove the following statement. Let ABC be any right triangle, the right angle at point C. The altitude drawn from C to the hypotenuse splits the triangle into two right triangles that are similar to each other and to the original triangle.

5. Find x, y, and h based on the picture below.

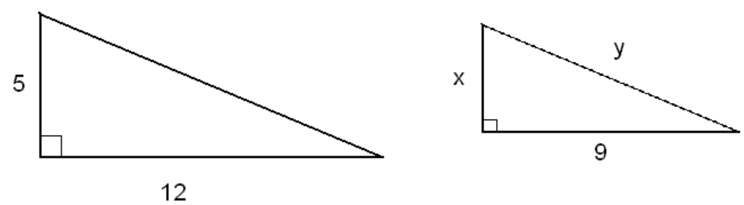


6. The picture below shows a right triangle. Find the length of h, the height drawn to the hypotenuse.

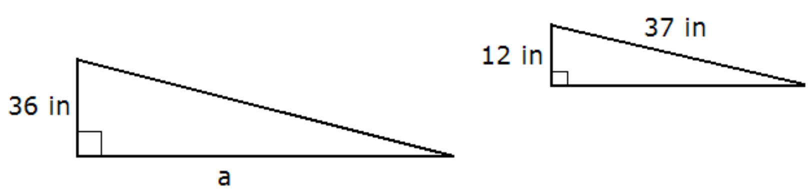


Practice Problems

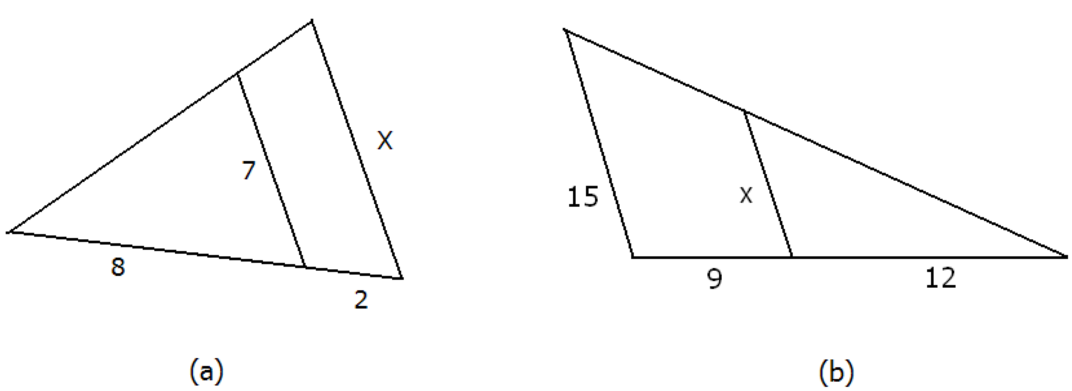
1. The picture below shows two similar right triangles. Find the exact values of x and y.



2. The picture below shows two similar right triangles. Find the exact value of a.



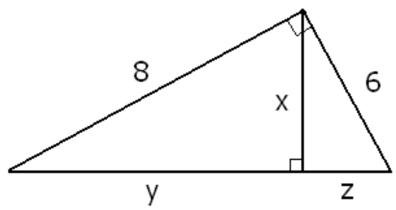
3. Find the value of x based on the figures below.



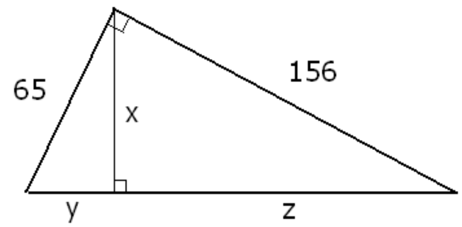
4. a) A person is standing 24 ft away from a street light that is 25 ft tall. How tall is he if his shadow is 6 ft long?

   b) A 5.2 ft tall person is standing 20 ft away from a street light that is 15.6 ft tall. How long is her shadow?

5. Find the exact value of x, y, and z, based on the figures shown below.

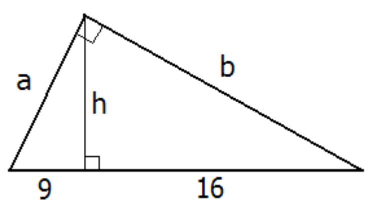


(a)

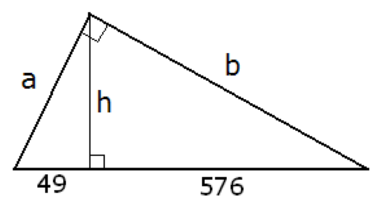


(b)

6. Find the exact value of a, b, and h, based on the picture shown below.



(a)



(b)

Sample Problems - Answers

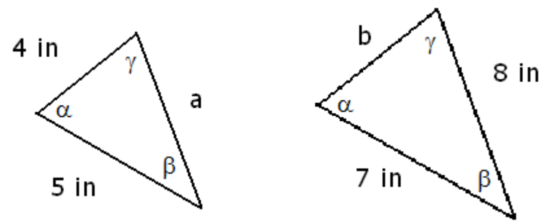
1. $a = \dfrac{40}{7}\,\text{in}, \quad b = \dfrac{28}{5}\,\text{in}$
2. a) 13 ft b) see solutions c) 10 ft d) 13 ft
3. a) 6 ft b) 16 ft
4. see solutions
5. $h = \dfrac{420}{29}, \quad x = \dfrac{400}{29}, \quad y = \dfrac{441}{29}$
6. 30 units

Practice Problems - Answers

1. $x = \dfrac{15}{4} = 3.75, \quad y = \dfrac{39}{4} = 9.75$
2. 105 in
3. a) $\dfrac{35}{4} = 8.75$ b) $\dfrac{60}{7}$
4. a) 5 ft b) 10 ft
5. a) $x = \dfrac{24}{5} = 4.8, \quad y = \dfrac{32}{5} = 6.4, \quad z = \dfrac{18}{5} = 3.6$ b) $x = 60, \quad y = 25, \quad z = 144$
6. a) $h = 12, \quad a = 15, \quad b = 20$ b) $h = 168, \quad a = 175, \quad b = 600$

Sample Problems - Solutions

1. The triangles shown below are similar. Find the exact values of a and b shown on the picture below.



Solution: In similar triangles, the ratios of corresponding sides are preserved. To find a, we write the ratio $\dfrac{\text{side opposite angle } \alpha}{\text{side opposite angle } \gamma}$ for both triangles.

$$\frac{\text{side opposite angle } \alpha}{\text{side opposite angle } \gamma} = \frac{a}{5} = \frac{8}{7}$$

We now solve the equation for a.

$$\begin{aligned} \frac{a}{5} &= \frac{8}{7} && \text{multiply both sides by 35} \\ 7a &= 40 && \text{divide by 7} \\ a &= \frac{40}{7} \end{aligned}$$

Similarly, we can find b by writing the ratio $\dfrac{\text{side opposite angle } \beta}{\text{side opposite angle } \gamma}$ for both triangles.

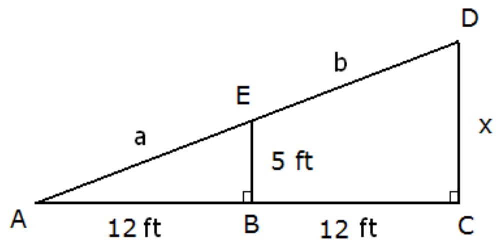
$$\frac{\text{side opposite angle } \beta}{\text{side opposite angle } \gamma} = \frac{4}{5} = \frac{b}{7}$$

We now solve the equation for b.

$$\begin{aligned} \frac{4}{5} &= \frac{b}{7} && \text{multiply both sides by 35} \\ 28 &= 5b && \text{divide by 5} \\ b &= \frac{28}{5} \end{aligned}$$

Thus $a = \dfrac{40}{7}$ in and $b = \dfrac{28}{5}$ in.

2. Consider the picture shown below.



a) Use the Pythagorean Theorem to find the value of a.
Solution: The shorter sides are 5 ft and 12 ft long. The hypotenuse is a. We state the Pythagorean Theorem for this triangle and solve the equation for a.

$$\begin{aligned} 5^2 + 12^2 &= a^2 \\ 25 + 144 &= a^2 \\ 169 &= a^2 \\ \pm 13 &= a \end{aligned}$$

Since distances can never be negative, $a = -13$ is ruled out. Thus $a = 13$ ft.

b) Prove that the triangles ABE and ACD are similar.
Solution: First, angles ABE and ACD are both right angles. Second, the two triangles literally share angle EAB (or angle DAC). Finally, if two triangles agree in the measure of two of their angles, the third angles must be equal since in every triangle, the three angles add up to $180°$. The two triangles are similar because they have identical angles.

c) Use similar triangles to find the value of x.
Solution: The triangles $\triangle ABE$ and $\triangle ACD$ are similar. To find x, we write the ratio $\dfrac{\text{side opposite point } A}{\text{horizontal side}}$ for both triangles.

$$\frac{\text{side opposite point } A}{\text{horizontal side}} = \frac{5}{12} = \frac{x}{24}$$

and solve the equation for x.

$$\begin{aligned} \frac{5}{12} &= \frac{x}{24} && \text{multiply both sides by 24} \\ 10 &= x \end{aligned}$$

Thus x is 10 ft. Indeed, once we established that the triangles are similar, and noticed that the horizontal side was doubled from 12 ft to 24 ft, we could easily predict this answer.

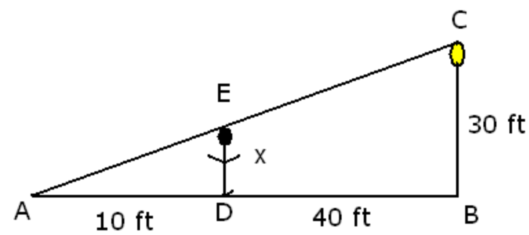
d) Find the value of b.
Solution: We can either use similar triangles or the Pythagorean Theorem to find the side AD. Either way, we easily get that 26 ft. However, the length of side AD is not b, but $a + b$. From part a), we know that $a = 13$ ft.

$$\begin{aligned} 13 + b &= 26 \\ b &= 13 \end{aligned}$$

Thus $b = 13$ ft.

3. a) A person is standing 40 ft away from a street light that is 30 ft tall. How tall is he if his shadow is 10 ft long?

Solution: After we draw a picture, we see that this problem is very similar to the previous one.



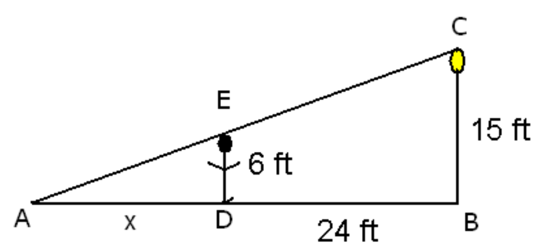
Triangles $\triangle ADE$ and $\triangle ABC$ are similar. We use the ratio $\dfrac{DE}{AD} = \dfrac{BC}{AB}$ and solve for x.

$$\begin{aligned} \frac{x}{10} &= \frac{30}{50} && \text{multiply both sides by 50} \\ 5x &= 30 && \text{divide by 5} \\ x &= 6 \end{aligned}$$

Thus the person is 6 ft tall. Notice that the number 40 did not occur in the equation. It is a common error to use 40 instead of 50.

b) A 6 ft tall person is standing 24 ft away from a street light that is 15 ft tall. How long is her shadow?

Solution: After we draw a picture, write an equation expressing that triangles ADE and ABC are similar.



We can use the same ratio as before, $\dfrac{DE}{AD} = \dfrac{BC}{AB}$ and solve for x.

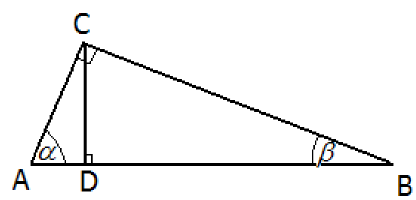
$$\begin{aligned} \frac{6}{x} &= \frac{15}{x+24} && \text{multiply both sides by } x\,(x+24) \\ 6\,(x+24) &= 15x && \text{distribute} \\ 6x+144 &= 15x && \text{subtract } 6x \\ 144 &= 9x && \text{divide by 9} \\ 16 &= x \end{aligned}$$

Thus her shadow is 16 ft long.

Note: If the first step, multiplying by $x(x+24)$ (same as cross-multiplying) is confusing, here is the break-down:

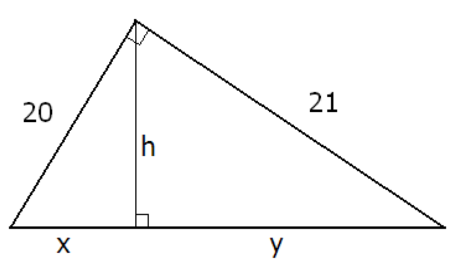
$$\begin{aligned}
\frac{6}{x} &= \frac{15}{x+24} && \text{multiply by } x(x+24)\\
x(x+24)\frac{6}{x} &= \frac{15}{x+24}\cdot x(x+24) && \text{expressing everything as a fraction}\\
\frac{x(x+24)}{1}\cdot\frac{6}{x} &= \frac{15}{x+24}\cdot\frac{x(x+24)}{1} && \\
\frac{x(x+24)6}{x} &= \frac{15x(x+24)}{x+24} && \text{cancel}\\
\frac{(x+24)6}{1} &= \frac{15x}{1} && \text{simplify}\\
(x+24)6 &= 15x
\end{aligned}$$

4. Prove the following statement. Let ABC be any right triangle, the right angle at point C. The altitude drawn from C to the hypotenuse splits the triangle into two right triangles that are similar to each other and to the original triangle.
Solution: Let us draw a picture and use standard labeling of points.

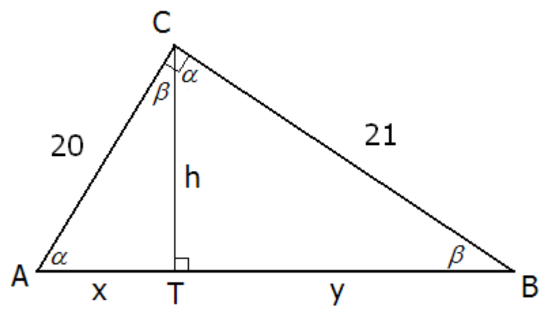


The two triangles created, $\triangle ADC$ and $\triangle DBC$ are both right triangles. $\triangle ADC$ is similar to the original triangle, because they agree in two angles: the right angle and α. $\triangle DBC$ is similar to the original triangle, because they agree in two angles: the right angle and β. Thus all three triangles are similar. Also, this will be very useful later: $\angle ACD = \beta$ and $\angle BCD = \alpha$.

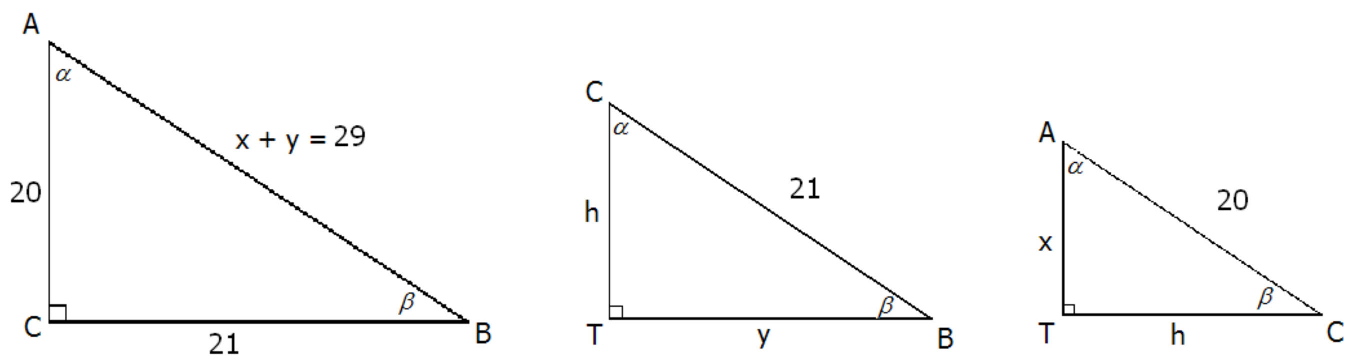
5. Find x, y, and h based on the picture below.



Solution: We can easily find the hypotenuse of this triangle via the Pythagorean Theorem. The hypotenuse, $x+y$ is $\sqrt{20^2+21^2} = 29$ units long. Next, let us first label the points, angles and sides in the triangle.



We now re-draw the three similar triangles in a separate figure, all three of them rotated and reflected into the same direction. This way, it is easy to realize what sides correspond to each other. (Hint: start with the anges, they are in the same location. Then identify the points, and finally the sides.)



We can find y using the following ratio in the first two triangles

$$\frac{\text{side opposite } \alpha}{\text{hypotenuse}} = \frac{21}{29} = \frac{y}{21}$$

$$\begin{aligned} \frac{21}{29} &= \frac{y}{21} && \text{multiply both sides by } 21 \cdot 29 \\ 441 &= 29y && \text{divide by } 29 \\ \frac{441}{29} &= y \end{aligned}$$

A different ratio in the same triangles can be used to obtain

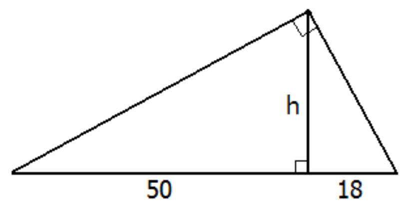
$$\frac{\text{side opposite } \beta}{\text{hypotenuse}} = \frac{20}{29} = \frac{h}{21}$$

$$\begin{aligned} \frac{20}{29} &= \frac{h}{21} && \text{multiply both sides by } 21 \cdot 29 \\ 420 &= 29h && \text{divide by } 29 \\ \frac{420}{29} &= h \end{aligned}$$

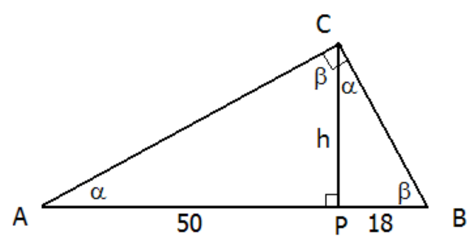
For x, we can simply use the fact that $x+y=29$ and we already computed $y=\dfrac{441}{29}$.

$$\begin{aligned} x+\frac{441}{29} &= 29 \\ x &= 29-\frac{441}{29}=\frac{400}{29} \end{aligned}$$

6. The picture below shows a right triangle. Find the length of h, the height drawn to the hypotenuse.



Solution: Let us first label the points, angles and sides in the triangle. As we proved it in the previous problem, the two new triangles are similar to the original triangle.



Consider now the ratio $\dfrac{\text{side opposite } \beta}{\text{side opposite } \alpha}$ in triangles $\triangle APC$ and $\triangle PBC$. Since these triangles are similar, this ratio is preserved.

$$\frac{\text{side opposite } \beta}{\text{side opposite } \alpha}=\frac{50}{h}=\frac{h}{18}$$

We solve this equation for h.

$$\begin{aligned} \frac{50}{h} &= \frac{h}{18} \\ 50\cdot 18 &= h^2 \\ 900 &= h^2 \\ h &= \pm 30 \end{aligned}$$

$h=-30$ is ruled out since distances can not be negative. Thus $h=30$.

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