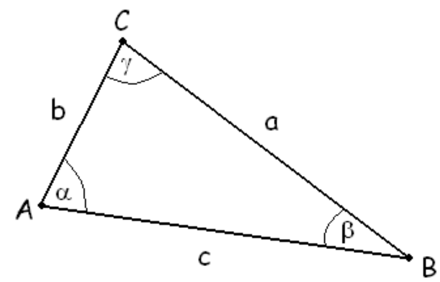
The perimeter of any triangle is simply the sum of the lengths of its three sides. We will have to learn much more for a discussion of the area of right triangles. But before we do that, let us agree first on a method of notation that avoids confusion and lengthy explanations in geometry problems. This agreement is called *standard labeling*, and it establishes a connection between the labels of sides, vertices, and angles in triangles. Every triangle has three of the following three components.

1. **vertices** (singular: vertex)
   Points are usually denoted by uppercase letters. In case of triangles, we often use A, B, and C.

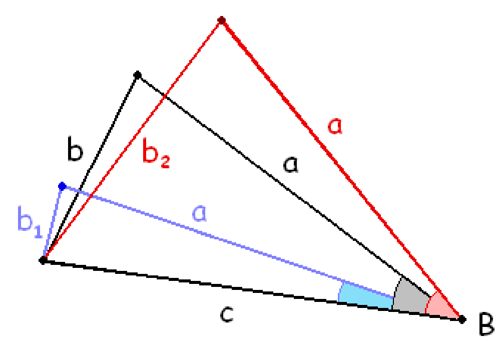
2. **angles**
   Angles are usually denoted by lowercase Greek letters. In case of triangles, we often use α (alpha), β (beta), and γ (gamma).

3. **sides**
   Lines and line segments are usually denoted by lowercase letters. In case of triangles, we often use a, b, and c.



In case of standard labeling, we automatically associate sides, vertices, and angles. A vertex is associated with the angle located at that vertex. These two are associated with the side opposite these. For example, angle α is always assumed to be located at point A, and side a is always assumed to be the side oppsite to point A and angle α. Point B, angle β, and side b are similarly grouped. Unless otherwise indicated, we should always assume standard labeling when presented with data that uses these letters.

Standard labeling is a smart approach to triangles, because there is a natural connection between an angle in a triangle and the side opposite that angle. Consider, for example, the triangle shown above with standard labeling. What if we fixed sides a and c and only modified angle β? Imagine that we have two rods in the lengths of a and c attached to each other at one end and we can freely change the angle between them. If we increase the angle between sides a and c (see the red lines), the side opposite will also increase. If we decrease the angle between sides a and c (see the blue lines), the side opposite will also decrease. So, there seems to be a natural correspondance between side b and angle β.



Theorem: In any triangle ABC, there is a correspondance between the length of a side and the measure of the angle opposite that side:
The longest side is opposite the greatest angle, and vica versa: the greatest angle is opposite the longest side. The shortest side is opposite the smallest angle, and vica versa: the smallest angle is opposite the shortest side.

So, the order between the three sides is the same as the order between the corresponding angles, and vica versa. We recommend that sides in triangles are tracked by their corresponding sides. This is because we can perceive the difference in angles much better than in side lengths.

Example 1. Suppose that ABC is a triangle with $\alpha = 82°$ and $\gamma = 39°$. List the length of the sides of the triangle in an increasing order.

Solution: Recall that the three angles in a triangle add up to $180°$. This means that if two angles are given, we can compute the third one. $\beta = 180° - (82° + 39°) = 180° - 121° = 59°$. Now we can see the order between the angles. γ is the smallest angle, β is in the middle, and α is the greatest angle. In short: $\gamma < \beta < \alpha$. The order between the lengths of the sides is the same: c is the shortest side, b is in the middle, and a is the longest side. In short: $c < b < a$.

There is an easy but important consequence of this property.

Theorem: In any triangle ABC, if two angles have equal measure, then the sides opposite them have equal length. If two sides are equally long, then the angles opposite those sides have equal mesaures. Such a triangle is called **isosceles**.

Theorem: (The Triangle Inequality) If a, b, and c are sides of a triangle, then

$$\begin{aligned} a+b &> c \quad \text{and} \\ a+c &> b \quad \text{and} \\ b+c &> a \end{aligned}$$

Example 2. Three sides of a triangle have lengths 12 units, 23 units, and x units. What are the possible values of x?

Solution: We will state the triangle inequality and solve the inequalities for x.

$$\begin{aligned} 12+23 &> x & x &< 35 \\ 12+x &> 23 & x &> 11 \\ 23+x &> 12 & x &> -11 \end{aligned}$$

For the triangle to exist, all three inequalities must be true. This gives us $11 < x < 35$

