1. Simplify each of the following expressions.

   a) $\dfrac{a^2-9}{a+2} \div \left(1-\dfrac{5}{a+2}\right)$

   b) $\dfrac{9a-3b}{9a^2-b^2} \cdot \dfrac{15a+5b}{3}$ where $|3a| \neq |b|$

   c) $\dfrac{1-\dfrac{x^2}{x^2-1}}{2+\dfrac{3x-1}{1-x}}$ where $|x| \neq 1$

2. Find the exact value of each of the following expressions if $x = 2$, $y = \sqrt{3}$, and $z = 0.2009$

   a) $\dfrac{1}{(x-y)(x-z)} + \dfrac{1}{(z-x)(z-y)} + \dfrac{1}{(y-x)(y-z)}$

   b) $\dfrac{1}{x(x+z)} + \dfrac{1}{z(x+z)} + \dfrac{1}{x(x-z)} + \dfrac{1}{z(z-x)}$

   c) $\left(\dfrac{2x^2+x}{x^3-1} - \dfrac{x+1}{x^2+x+1}\right)\left(1+\dfrac{x+1}{x} - \dfrac{x^2+5x}{x^2+x}\right)$

   d) $\dfrac{(x+y)^2-(x-y)^2}{4xy}$

   e) $\dfrac{\left(x^2-y^2-z^2-2yz\right)(x+y-z)}{(x+y+z)(x^2+z^2-2xz-y^2)}$

3. Simplify each of the following expressions.

   a) $\dfrac{4-a^2-2ab-b^2}{2+a+b}$ where $a+b \neq -2$

   b) $\dfrac{a^2+b^2-c^2+2ab}{a^2-b^2+c^2+2ac}$ where $|a+c| \neq |b|$

4. Prove that if $a+b+c=0$, then $a^3+a^2c+b^2c-abc+b^3=0$

5. Simplify each of the following expressions.

   a) $\sqrt{12}+\sqrt{75}-\sqrt{147}$

   b) $\sqrt{28}+\sqrt{7}-\sqrt{63}$

   c) $\sqrt{\sqrt{41}+4\sqrt{2}} \cdot \sqrt{\sqrt{41}-\sqrt{32}}$

   d) $\sqrt{5\sqrt{3}+\sqrt{59}} \cdot \sqrt{\sqrt{75}-\sqrt{59}}$

   e) $\left(\sqrt{6+\sqrt{11}}+\sqrt{6-\sqrt{11}}\right)^2$

   f) $\sqrt{7+2\sqrt{6}}-\sqrt{7-2\sqrt{6}}$

   g) $\sqrt{7-4\sqrt{3}}-\sqrt{7+\sqrt{48}}$

   h) $\sqrt[3]{7+5\sqrt{2}}$

   i) $\sqrt[3]{20+14\sqrt{2}}+\sqrt[3]{20-14\sqrt{2}}$

   j) $\sqrt[3]{10+6\sqrt{3}}+\sqrt[3]{10-6\sqrt{3}}$

   k) $\sqrt[4]{7-4\sqrt{5}}$

6. Simplify each of the following expressions.

   a) $\dfrac{3-\sqrt{5}}{3+\sqrt{5}} + \dfrac{3+\sqrt{5}}{3-\sqrt{5}}$

   b) $\left(\dfrac{8}{\sqrt{7}+\sqrt{3}} + \dfrac{12}{\sqrt{7}-\sqrt{3}}\right)\left(5\sqrt{7}-\sqrt{3}\right)$

7. Rationalize the denominator in each of the following expressions.

   a) $\dfrac{3}{\sqrt{5}-\sqrt{2}}$

   b) $\dfrac{a}{\sqrt{a}+\sqrt{b}}$ where $a, b > 0$

   c) $\dfrac{\sqrt{7}-\sqrt{2}}{\sqrt{7}+\sqrt{2}}$

8. Which one is greater?

   a) $2\sqrt{7}$ or $\dfrac{1}{\sqrt{7}-\sqrt{6}}$

   b) $\sqrt[4]{4}$ or $\sqrt[5]{5}$

   c) $\dfrac{7}{5-3\sqrt{2}}$ or $\sqrt{72}$

   d) $2\sqrt{3}$ or $\dfrac{1}{\sqrt{3}-\sqrt{2}}$

9. For what values of k can we factor out $x+3$ from the polynomial $2x^2+x+k$?

10. Find the exact value of the following expression.

$$\frac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}-\sqrt{3-2\sqrt{2}}$$

11. We divided a line segment into two parts so that the ratio between the shorter and longer part is the same as the ratio between the longer part and the entire line segment. If R represents this ratio, find the exact value of the following expression.

$$R^{\left(R^{\left(R^2+R^{-1}\right)}+R^{-1}\right)}+R^{-1}$$

12. Find the integer part in $\left(\sqrt{3}+\sqrt{2}\right)^6$.

13. If p, q, and r are solutions of the equation x^3-x^2+x-2, the find the exact value of $p^3+q^3+r^3$.

14. Is the number $\sqrt[3]{7+4\sqrt{3}}+\sqrt[3]{7-4\sqrt{3}}$ a solution of the equation $x^3-3x-14=0$?