Sample Problems

Determine the domain for each of the following functions.

1.) $f(x) = \dfrac{1}{x+5}$

2.) $g(x) = \sqrt{x+5}$

3.) $p(x) = \dfrac{1}{x^2 - 6x}$

4.) $m(x) = \sqrt{x^2 - 6x}$

5.) $f(x) = \dfrac{1}{\sqrt{x+2} - 3}$

6.) $t(x) = \dfrac{1}{\sqrt{9 - x^2}}$

7.) $f(x) = \dfrac{1}{x^2 - 25} + \sqrt{x^2 - 4x + 3}$

8.) $g(x) = \sqrt{x^2 + 4} - \dfrac{x+7}{x^2+1}$

9.) $f(x) = \sqrt{x^2 - 1} + \sqrt{9 - x^2}$

Practice Problems

Determine the domain for each of the following functions.

1.) $f(x) = \dfrac{1}{x-4}$

2.) $g(x) = \sqrt{x-4}$

3.) $f(x) = \sqrt{x-7} + \sqrt{7-x}$

4.) $p(x) = \dfrac{1}{4x - x^2}$

5.) $p(x) = \sqrt{4x - x^2}$

6.) $m(x) = \sqrt{x-5} - \dfrac{x+1}{x-10}$

7.) $f(x) = \dfrac{3x-1}{2x^2+7}$

8.) $t(x) = \dfrac{2x-3}{3x+5}$

9.) $f(x) = \sqrt{4x - x^2 - 3}$

10.) $t(x) = \dfrac{1}{\sqrt{x^2-4} - 3}$

11.) $g(x) = \sqrt{12 - x} - \dfrac{2x+1}{x-8}$

12.) $f(x) = \dfrac{\sqrt{1-x}}{\sqrt{x-5}}$

13.) $h(x) = \dfrac{x-3}{x^2+9}$

14.) $f(x) = \dfrac{\sqrt{9-x^2}}{x}$

Sample Problems - Answers

1.) $\{x|x \in \mathbb{R}, x \neq -5\}$ or $\mathbb{R} \setminus \{-5\}$ or $(-\infty, -5) \cup (-5, \infty)$ 2.) $[-5, \infty)$ or $\{x|x \in \mathbb{R}, x \geq -5\}$

3.) $\{x|x \in \mathbb{R}, x \neq 0, 6\}$ or $\mathbb{R} \setminus \{0, 6\}$ or $(-\infty, 0) \cup (0, 6) \cup (6, \infty)$

4.) $(-\infty, 0] \cup [6, \infty)$ or $\{x|x \in \mathbb{R},\ x \leq 0 \text{ or } x \geq 6\}$ 5.) $[-2, \infty) \setminus \{7\}$ or $\{x|x \in \mathbb{R},\ x \geq -2,\ \ x \neq 7\}$

6.) $(-3, 3)$ or $\{x|x \in \mathbb{R},\ -3 < x < 3\}$

7.) $(-\infty, 1] \cup [3, \infty) \setminus \{-5, 5\}$ or $(-\infty, -5) \cup (-5, 1] \cup [3, 5) \cup (5, \infty)$ 8.) $\mathbb{R}$

9.) $-3 \leq x \leq -1$ or $1 \leq x \leq 3$, $[-3, -1] \cup [1, 3]$

Practice Problems - Answers

1.) $\{x|x \in \mathbb{R}, x \neq 4\}$ or $\mathbb{R} \setminus \{4\}$ 2.) $[4, \infty)$ or $\{x|x \in \mathbb{R}, x \geq 4\}$ 3.) $\{7\}$

4.) $\{x|x \in \mathbb{R}, x \neq 0, 4\}$ or $\mathbb{R} \setminus \{0, 4\}$ 5.) $[0, 4]$ or $\{x|x \in \mathbb{R}, 0 \leq x \leq 4\}$

6.) $[5, \infty) \setminus \{10\}$ 7.) $\mathbb{R}$ 8.) $\mathbb{R} \setminus \left\{-\dfrac{5}{3}\right\}$ or $\left\{x|x \in \mathbb{R}, x \neq -\dfrac{5}{3}\right\}$ 9.) $[1, 3]$ or $\{x|x \in \mathbb{R}, 1 \leq x \leq 3\}$

10.) $(-\infty, -2] \cup [2, \infty) \setminus \left\{\pm\sqrt{13}\right\}$ or $\left\{x|x \in \mathbb{R}, x \leq -2 \text{ but } x \neq -\sqrt{13} \text{ or } x \geq 2 \text{ but } x \neq \sqrt{13}\right\}$

11.) $(-\infty, 12] \setminus \{8\}$ or $\{x|x \in \mathbb{R}, x \leq 12 \text{ but } x \neq 8\}$ 12.) $\varnothing$ 13.) $\mathbb{R}$

14.) $[-3, 3] \setminus \{0\}$ or $\{x|x \in \mathbb{R}, -3 \leq x \leq 3 \text{ but } x \neq 0\}$

Sample Problems - Solutions

Determine the domain for each of the following functions.

1.) $f(x) = \dfrac{1}{x+5}$

Solution: We have to rule out the value(s) of x that would result in division by zero. We solve the equation $x+5=0$ and obtain $x=-5$. Thus the domain of this function is all real numbers except for -5. There are several notations available to express this:

$$\{x | x \in \mathbb{R}, x \neq -5\} \text{ or } \mathbb{R} \setminus \{-5\} \text{ or } (-\infty, -5) \cup (-5, \infty)$$

2.) $g(x) = \sqrt{x+5}$

Solution: We have to rule out the value(s) of x that would result in a negative number under the square root. We solve the inequality $x+5 \geq 0$ and obtain $x \geq -5$. There are several notations available to express this:

$$[-5, \infty) \text{ or } \{x | x \in \mathbb{R}, x \geq -5\}$$

3.) $p(x) = \dfrac{1}{x^2 - 6x}$

Solution: We have to rule out the value(s) of x that would result in division by zero. We solve the equation $x^2 - 6x = 0$ and obtain $x = 0$ or $x = 6$. Thus the domain of this function is all real numbers except for 0 and 6. There are several notations available to express this:

$$\{x | x \in \mathbb{R}, x \neq 0, 6\} \text{ or } \mathbb{R} \setminus \{0, 6\} \text{ or } (-\infty, 0) \cup (0, 6) \cup (6, \infty)$$

4.) $m(x) = \sqrt{x^2 - 6x}$

Solution: We have to rule out the value(s) of x that would result in a negative number under the square root. We solve the inequality $x^2 - 6x \geq 0$. (For details on how to solve this inequality, see quadratic inequalities.) There are several notations available to express this:

$$(-\infty, 0] \cup [6, \infty) \text{ or } \{x | x \in \mathbb{R},\ x \leq 0 \text{ or } x \geq 6\}$$

5.) $f(x) = \dfrac{1}{\sqrt{x+2} - 3}$

Solution: First, we have to rule out the value(s) of x that would result in a negative number under the square root. For the expression $\sqrt{x+2}$ to be defined, we solve the inequality $x+2 \geq 0$ and obtain $x \geq -2$. Now that we have guarantee that the radical expression is defined, we still need to worry about division by zero. For the entire expression $\dfrac{1}{\sqrt{x+2} - 3}$ to be defined, we need to rule out the value(s) of x for which $\sqrt{x+2} - 3 = 0$. We solve this equation

$$\begin{aligned} \sqrt{x+2} - 3 &= 0 \\ \sqrt{x+2} &= 3 \end{aligned} \qquad\qquad \begin{aligned} x+2 &= 9 \\ x &= 7 \end{aligned}$$

Thus x can not be 7. The domain is, in several notations:

$$[-2, \infty) \setminus \{7\} \text{ or } \{x | x \in \mathbb{R},\ x \geq -2,\ \ x \neq 7\}$$

6.) $t(x) = \dfrac{1}{\sqrt{9-x^2}}$

Solution: First, we have to rule out the value(s) of x that would result in a negative number under the square root. For the expression $\sqrt{9-x^2}$ to be defined, we solve the inequality $9-x^2 \geq 0$ and obtain $-3 \leq x \leq 3$. Now that we have guarantee that the radical expression is defined, we still need to worry about division by zero. For the entire expression $\dfrac{1}{\sqrt{9-x^2}}$ to be defined, we need to rule out the value(s) of x for which $\sqrt{9-x^2}=0$. We solve this equation

$$\begin{aligned} \sqrt{9-x^2} &= 0 \\ 9-x^2 &= 0 \end{aligned} \qquad\qquad \begin{aligned} -(x+3)(x-3) &= 0 \\ x_1 = -3 \qquad x_2 &= 3 \end{aligned}$$

We rule out these values of x. Thus the domain is $(-3,3)$ or $\{x | x \in \mathbb{R}, \ -3 < x < 3\}$.

7.) $f(x) = \dfrac{1}{x^2-25} + \sqrt{x^2-4x+3}$

Solution: First, we have to rule out the value(s) of x that would result in division by zero. We solve the equation $x^2-25=0$ and obtain $x = \pm 5$. These values have to be ruled out. For the expression $\sqrt{x^2-4x+3}$ to be defined, we solve the inequality $x^2-4x+3 \geq 0$ and obtain $x \leq 1$ or $x \geq 3$. Thus the domain is $x \leq 1$ but $x \neq -5$ or $x \geq 3$ but $x \neq 5$. Or in interval notation,

$$(-\infty,1] \cup [3,\infty) \setminus \{-5,5\} \quad \text{or} \quad (-\infty,-5) \cup (-5,1] \cup [3,5) \cup (5,\infty)$$

8.) $g(x) = \sqrt{x^2+4} - \dfrac{x+7}{x^2+1}$

Solution: First, we have to rule out the value(s) of x that would result in a negative number under the square root. For the expression $\sqrt{x^2+4}$ to be defined, we solve the inequality $x^2+4 \geq 0$. This inequality is true for all real numbers. Now that we have guarantee that the radical expression is defined, we still need to worry about division by zero. We now solve the equation $x^2+1=0$. Thus equation has no real solution and so we don't need to rule out any number. Thus the domain of this function is all real numbers, or in set notation, $\mathbb{R}$.

9.) $f(x) = \sqrt{x^2-1} + \sqrt{9-x^2}$

Solution: We have to rule out the value(s) of x that would result in a negative number under the square root. For the expression $\sqrt{x^2-1}$ to be defined, we solve the inequality $x^2-1 \geq 0$ and obtain the solution $x \leq -1$ or $x \geq 1$. For the expression $\sqrt{9-x^2}$ to be defined, we solve the inequality $9-x^2 \geq 0$ and obtain the solution $-3 \leq x \leq 3$. Thus the domain is $-3 \leq x \leq -1$ or $1 \leq x \leq 3$ or, in interval notation, $[-3,-1] \cup [1,3]$.

