Sample Problems

Determine the domain for each of the following functions.

1. $f(x) = \dfrac{1}{x+5}$
2. $g(x) = \sqrt{x+5}$
3. $h(x) = \log_2(x+5)$
4. $p(x) = \dfrac{1}{x^2-6x}$
5. $m(x) = \sqrt{x^2-6x}$
6. $n(x) = \ln\left(x^2-6x\right)$
7. $f(x) = \dfrac{1}{\sqrt{x+2}-3}$
8. $t(x) = \dfrac{1}{\sqrt{9-x^2}}$
9. $f(x) = \dfrac{3}{\log_{10}(2x-x^2)}$
10. $g(x) = \sqrt{x^2+4} - \dfrac{x+7}{x^2+1}$
11. $f(x) = \log_3\left(x^2+1\right)$
12. $f(x) = \sqrt{x^2-1} + \sqrt{9-x^2}$
13. $F(x) = \ln\left(x^2-16\right)$
14. $f(x) = \ln(x+4) + \ln(x-4)$
15. $f(x) = \dfrac{1}{\ln(x-3)}$
16. $f(x) = \dfrac{1}{\log_3(2x-1)-4}$
17. $f(x) = \dfrac{1}{x^2-25} + \sqrt{x^2-4x+3}$

Practice Problems

Determine the domain for each of the following functions.

1. $f(x) = \dfrac{1}{x-4}$
2. $g(x) = \sqrt{x-4}$
3. $h(x) = \log_2(x-4)$
4. $f(x) = \sqrt{x-7} + \sqrt{7-x}$
5. $p(x) = \dfrac{1}{4x-x^2}$
6. $p(x) = \sqrt{4x-x^2}$
7. $m(x) = \sqrt{x-5} - \dfrac{x+1}{x-10}$
8. $f(x) = \dfrac{1}{\log_2(x-2) + \log_2(x-4)}$
9. $f(x) = \dfrac{3x-1}{2x^2+7}$
10. $t(x) = \dfrac{2x-3}{3x+5}$
11. $f(x) = \sqrt{4x-x^2-3}$
12. $f(x) = \log_5\left(-x^2+10x-23\right)$
13. $t(x) = \dfrac{1}{\sqrt{x^2-4}-3}$
14. $g(x) = \sqrt{12-x} - \dfrac{2x+1}{x-8}$
15. $f(x) = \dfrac{\sqrt{1-x}}{\sqrt{x-5}}$
16. $h(x) = \dfrac{x-3}{x^2+9}$
17. $f(x) = \dfrac{\sqrt{9-x^2}}{x}$
18. $f(x) = \log_2\left(x^2-6x+8\right)$
19. $f(x) = \dfrac{1}{\log_2(x^2-6x+8)}$
20. $f(x) = \ln\left(\dfrac{x^2-6x}{4-x^2}\right)$
21. $f(x) = \dfrac{\ln\left(x^2-6x\right)}{\ln(4-x^2)}$
22. $t(x) = \ln\left(x^2-6x\right) - \ln\left(4-x^2\right)$

Sample Problems - Answers

1.) $\{x|x\in\mathbb{R}, x\neq -5\}$ or $\mathbb{R}\setminus\{-5\}$ or $(-\infty,-5)\cup(-5,\infty)$ 2.) $[-5,\infty)$ or $\{x|x\in\mathbb{R}, x\geq -5\}$

3.) $(-5,\infty)$ or $\{x|x\in\mathbb{R}, x>-5\}$ 4.) $\{x|x\in\mathbb{R}, x\neq 0,6\}$ or $\mathbb{R}\setminus\{0,6\}$ or $(-\infty,0)\cup(0,6)\cup(6,\infty)$

5.) $(-\infty,0]\cup[6,\infty)$ or $\{x|x\in\mathbb{R},\ x\leq 0 \text{ or } x\geq 6\}$ 6.) $(-\infty,0)\cup(6,\infty)$ or $\{x|x\in\mathbb{R},\ x<0 \text{ or } x>6\}$

7.) $[-2,\infty)\setminus\{7\}$ or $\{x|x\in\mathbb{R},\ x\geq -2,\ \ x\neq 7\}$ 8.) $(-3,3)$ or $\{x|x\in\mathbb{R},\ -3<x<3\}$

9.) $0<x<2$ but $x\neq 1$ or in interval notation, $(0,2)\setminus\{1\}$ (Also correct: $(0,1)\cup(1,2)$)

10.) $\mathbb{R}$ 11.) $\mathbb{R}$ or $(-\infty,\infty)$ 12.) $-3\leq x\leq -1$ or $1\leq x\leq 3$, or $[-3,-1]\cup[1,3]$

13.) $x<-4$ or $x>4$ or in interval notation, $(-\infty,-4)\cup(4,\infty)$

14.) $x>4$ or in interval notation, $(4,\infty)$ 15.) $x>3$ but $x\neq 4$ or $(3,\infty)\setminus\{4\}$ (Also correct: $(3,4)\cup\ (4,\infty)$)

16.) $x>\dfrac{1}{2}$ but $x\neq 41$ or in interval notation, $\left(\dfrac{1}{2},\infty\right)\setminus\{41\}$ (Also correct: $\left(\dfrac{1}{2},41\right)\cup(41,\infty)$)

17.) $(-\infty,1]\cup[3,\infty)\setminus\{-5,5\}$ or $(-\infty,-5)\cup(-5,1]\cup[3,5)\cup(5,\infty)$

Practice Problems - Answers

1.) $\{x|x\in\mathbb{R}, x\neq 4\}$ or $\mathbb{R}\setminus\{4\}$ 2.) $[4,\infty)$ or $\{x|x\in\mathbb{R}, x\geq 4\}$ 3.) $(4,\infty)$ or $\{x|x\in\mathbb{R}, x>4\}$

4.) $\{7\}$ 5.) $\{x|x\in\mathbb{R}, x\neq 0,4\}$ or $\mathbb{R}\setminus\{0,4\}$ 6.) $[0,4]$ or $\{x|x\in\mathbb{R}, 0\leq x\leq 4\}$

7.) $[5,\infty)\setminus\{10\}$ 8.) $x>4$ and $x\neq 3+\sqrt{2}$ - in interval notation: $(4,\infty)\setminus\left\{3+\sqrt{2}\right\}$ 9.) $\mathbb{R}$

10.) $\mathbb{R}\setminus\left\{-\dfrac{5}{3}\right\}$ or $\left\{x|x\in\mathbb{R}, x\neq -\dfrac{5}{3}\right\}$ 11.) $[1,3]$ or $\{x|x\in\mathbb{R}, 1\leq x\leq 3\}$

12.) $5-\sqrt{2}<x<5+\sqrt{2}$ - in interval notation: $\left(5-\sqrt{2},5+\sqrt{2}\right)$

13.) $(-\infty,-2]\cup[2,\infty)\setminus\left\{\pm\sqrt{13}\right\}$ or $\left\{x|x\in\mathbb{R}, x\leq -2 \text{ but } x\neq -\sqrt{13} \text{ or } x\geq 2 \text{ but } x\neq\sqrt{13}\right\}$

14.) $(-\infty,12]\setminus\{8\}$ or $\{x|x\in\mathbb{R}, x\leq 12 \text{ but } x\neq 8\}$ 15.) $\varnothing$ 16.) $\mathbb{R}$

17.) $[-3,3]\setminus\{0\}$ or $\{x|x\in\mathbb{R}, -3\leq x\leq 3 \text{ but } x\neq 0\}$

18.) $x<2$ or $x>4$ - in interval notation: $(-\infty,2)\cup(4,\infty)$

19.) $x<2$ and $x\neq 3-\sqrt{2}$ or $x>4$ and $x\neq 3+\sqrt{2}$ in interval notation: $(-\infty,2)\cup(4,\infty)\setminus\left\{3-\sqrt{2},3+\sqrt{2}\right\}$

20.) $-2<x<0$ or $2<x<6$ -in interval notation: $(-2,0)\cup(2,6)$

21.) $-2<x<0$ and $x\neq -\sqrt{3}$ in interval notation: $(-2,0)\setminus\left\{-\sqrt{3}\right\}$

22.) $-2<x<0$ in interval notation: $(-2,0)$

Sample Problems - Solutions

Determine the domain for each of the following functions.

1.) $f(x) = \dfrac{1}{x+5}$

Solution: We have to rule out the value(s) of x that would result in division by zero. We solve the equation $x + 5 = 0$ and obtain $x = -5$. Thus the domain of this function is all real numbers except for -5. There are several notations available to express this:

$$\{x | x \in \mathbb{R}, x \neq -5\} \text{ or } \mathbb{R} \setminus \{-5\} \text{ or } (-\infty, -5) \cup (-5, \infty)$$

2.) $g(x) = \sqrt{x+5}$

Solution: In this case, we express what expressions are meaningful in a square-root: numbers greater than or equal to zero. We solve the inequality $x + 5 \geq 0$ and obtain $x \geq -5$. There are several notations available to express this:

$$[-5, \infty) \text{ or } \{x | x \in \mathbb{R}, x \geq -5\}$$

3.) $h(x) = \log_2 (x+5)$

Solution: In this case, we express what expressions are meaningful in the argument of a logarithm: only positive numbers. We solve the inequality $x + 5 > 0$ and obtain $x \geq -5$. There are several notations available to express this:

$$[-5, \infty) \text{ or } \{x | x \in \mathbb{R}, x \geq -5\}$$

4.) $p(x) = \dfrac{1}{x^2 - 6x}$

Solution: We have to rule out the value(s) of x that would result in division by zero. We solve the equation $x^2 - 6x = 0$ and obtain $x = 0$ or $x = 6$. Thus the domain of this function is all real numbers except for 0 and 6. There are several notations available to express this:

$$\{x | x \in \mathbb{R}, x \neq 0, 6\} \text{ or } \mathbb{R} \setminus \{0, 6\} \text{ or } (-\infty, 0) \cup (0, 6) \cup (6, \infty)$$

5.) $m(x) = \sqrt{x^2 - 6x}$

Solution: In this case, we express what expressions are meaningful in a square-root: numbers greater than or equal to zero. We state and then solve the inequality $x^2 - 6x \geq 0$. (For details on how to solve this inequality, see quadratic inequalities.) There are several notations available to express this:

$$(-\infty, 0] \cup [6, \infty) \text{ or } \{x | x \in \mathbb{R},\ x \leq 0 \text{ or } x \geq 6\}$$

6.) $n(x) = \ln\left(x^2 - 6x\right)$

Solution: In this case, we express what expressions are meaningful in the argument of a logarithm: only positive numbers. We state and then solve the inequality $x^2 - 6x > 0$ and obtain $x < 0$ or $x > 6$. There are several notations available to express this:

$$(-\infty, 0) \cup (6, \infty) \text{ or } \{x | x \in \mathbb{R},\ x < 0 \text{ or } x > 6\}$$

7.) $f(x) = \dfrac{1}{\sqrt{x+2}-3}$

Solution: First, we have to rule out the value(s) of x that would result in a negative number under the square root. For the expression $\sqrt{x+2}$ to be defined, we solve the inequality $x+2 \geq 0$ and obtain $x \geq -2$. Now that we have guarantee that the radical expression is defined, we still need to worry about division by zero. For the entire expression $\dfrac{1}{\sqrt{x+2}-3}$ to be defined, we need to rule out the value(s) of x for which $\sqrt{x+2}-3=0$. We solve this equation

$$\begin{aligned} \sqrt{x+2}-3 &= 0 \qquad\qquad & x+2 &= 9 \\ \sqrt{x+2} &= 3 & x &= 7 \end{aligned}$$

Thus x can not be 7. The domain is, in several notations:

$$[-2,\infty)\setminus\{7\} \text{ or } \{x|x\in\mathbb{R},\ x\geq -2,\ \ x\neq 7\}$$

8.) $t(x) = \dfrac{1}{\sqrt{9-x^2}}$

Solution: First, we have to rule out the value(s) of x that would result in a negative number under the square root. For the expression $\sqrt{9-x^2}$ to be defined, we solve the inequality $9-x^2 \geq 0$ and obtain $-3 \leq x \leq 3$. Now that we have guarantee that the radical expression is defined, we still need to worry about division by zero. For the entire expression $\dfrac{1}{\sqrt{9-x^2}}$ to be defined, we need to rule out the value(s) of x for which $\sqrt{9-x^2}=0$. We solve this equation

$$\begin{aligned} \sqrt{9-x^2} &= 0 \qquad\qquad & -(x+3)(x-3) &= 0 \\ 9-x^2 &= 0 & x_1 = -3 \qquad x_2 &= 3 \end{aligned}$$

We rule out these values of x. Thus the domain is $(-3,3)$ or $\{x|x\in\mathbb{R},\ -3<x<3\}$.

9.) $f(x) = \dfrac{3}{\log_{10}(2x-x^2)}$

Solution: for $\log_{10}(2x-x^2)$ to be defined, $2x-x^2>0$ needs to be true. We solve this inequality and get that $0<x<2$. Even if the logarithm is defined, we still need to worry about division by zero. We have to rule out all values of x for which $\log_{10}(2x-x^2)=0$. We solve the equation

$$\begin{aligned} \log_{10}(2x-x^2) &= 0 \qquad\qquad & 0 &= x^2-2x+1 \\ 2x-x^2 &= 10^0 & 0 &= (x-1)^2 \\ 2x-x^2 &= 1 & x &= 1 \end{aligned}$$

So the domain of this expression is $\{x|\ \ 0<x<2 \text{ but } x\neq 1\}$ or in interval notation, $(0,2)\setminus\{1\}$.

10.) $g(x) = \sqrt{x^2+4} - \dfrac{x+7}{x^2+1}$

Solution: First, we have to rule out the value(s) of x that would result in a negative number under the square root. For the expression $\sqrt{x^2+4}$ to be defined, we solve the inequality $x^2+4\geq 0$. This inequality is true for all real numbers. Now that we have guarantee that the radical expression is defined, we still need to worry about division by zero. We now solve the equation $x^2+1=0$. Thus equation has no real solution and so we don't need to rule out any number. Thus the domain of this function is all real numbers, or in set notation, $\mathbb{R}$.

11.) $f(x) = \log_3(x^2+1)$

Solution: for this logarithm to be defined, $x^2+1>0$ needs to be true. Since this inequality is true for all real numbers, this expression's domain is the set of all real numbers, $\mathbb{R}$.

12.) $f(x) = \sqrt{x^2 - 1} + \sqrt{9 - x^2}$

Solution: We have to rule out the value(s) of x that would result in a negative number under the square root. For the expression $\sqrt{x^2 - 1}$ to be defined, we solve the inequality $x^2 - 1 \geq 0$ and obtain the solution $x \leq -1$ or $x \geq 1$. For the expression $\sqrt{9 - x^2}$ to be defined, we solve the inequality $9 - x^2 \geq 0$ and obtain the solution $-3 \leq x \leq 3$. Thus the domain is $-3 \leq x \leq -1$ or $1 \leq x \leq 3$ or, in interval notation, $[-3, -1] \cup [1, 3]$.

13.) $F(x) = \ln\left(x^2 - 16\right)$
Solution: We need to solve the inequality $x^2 - 16 > 0$. (If you need to review these, see Quadratic Inequalities.) The solution is $\{x | x < -4 \text{ or } x > 4\}$ or in interval notation, $(-\infty, -4) \cup (4, \infty)$.

14.) $f(x) = \ln(x + 4) + \ln(x - 4)$
Solution: We need to solve the inequalities $x + 4 > 0$ and $x - 4 > 0$.

$$\begin{aligned} x + 4 &> 0 \text{ and } x - 4 > 0 \\ x &> -4 \text{ and } x > 4 \quad \Longrightarrow \quad x > 4 \end{aligned}$$

Thus the domain is $\{x | x > 4\}$ or in interval notation, $(4, \infty)$.

15.) $f(x) = \dfrac{1}{\ln(x - 3)}$
Solution: for the expression $\ln(x - 3)$ to be defined, we need that $x - 3 > 0$, thus $x > 3$. Now if x is greater than 3, $\ln(x - 3)$ is defined but we still need to worry about division by zero. We have to rule out all values of x for which $\ln(x - 3) = 0$. So we solve the equation

$$\begin{aligned} \ln(x - 3) &= 0 & \qquad 1 &= x - 3 \\ e^0 &= x - 3 & \qquad 4 &= x \end{aligned}$$

Thus the domain is: $\{x | x > 3 \text{ but } x \neq 4\}$ or in interval notation, $(3, \infty) \setminus \{4\}$

16.) $f(x) = \dfrac{1}{\log_3(2x - 1) - 4}$

Solution: for $\log_3(2x - 1)$ to be defined, $2x - 1 > 0$ needs to be true. We solve this inequality and get that $x > \dfrac{1}{2}$. Even if the logarithm is defined, we still need to worry about division by zero. We have to rule out all values of x for which $\log_3(2x - 1) - 4 = 0$. We solve the equation

$$\begin{aligned} \log_3(2x - 1) - 4 &= 0 & \qquad 2x - 1 &= 81 \\ \log_3(2x - 1) &= 4 & \qquad 2x &= 82 \\ 2x - 1 &= 3^4 & \qquad x &= 41 \end{aligned}$$

So the domain of this expression is $\left\{x | x > \dfrac{1}{2} \text{ but } x \neq 41\right\}$ or in interval notation, $\left(\dfrac{1}{2}, \infty\right) \setminus \{41\}$.

17.) $f(x) = \dfrac{1}{x^2 - 25} + \sqrt{x^2 - 4x + 3}$

Solution: First, we have to rule out the value(s) of x that would result in division by zero. We solve the equation $x^2 - 25 = 0$ and obtain $x = \pm 5$. These values have to be ruled out. For the expression $\sqrt{x^2 - 4x + 3}$ to be defined, we solve the inequality $x^2 - 4x + 3 \geq 0$ and obtain $x \leq 1$ or $x \geq 3$. Thus the domain is $x \leq 1$ but $x \neq -5$ or $x \geq 3$ but $x \neq 5$. Or in interval notation,

$$(-\infty, 1] \cup [3, \infty) \setminus \{-5, 5\} \quad \text{or} \quad (-\infty, -5) \cup (-5, 1] \cup [3, 5) \cup (5, \infty)$$

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