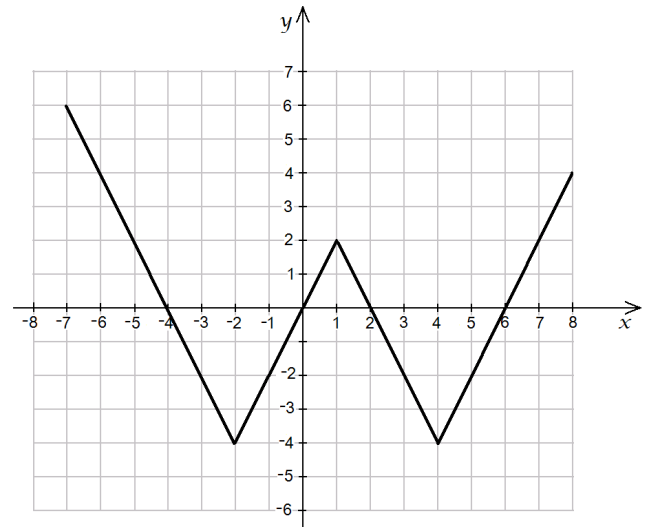


In the following problems, we will solve equations and inequalities graphically. This means that the formulas will not be given, only graphs. We are asked to find solutions of equations and inequalities based only on the graphs given.

Sample Problems

1. Consider the graph shown on the picture. Assume that x is between -7 and 8 (in short, $-7 \leq x \leq 8$). Find all values of x so that the point $P(x, y)$ is on the graph and

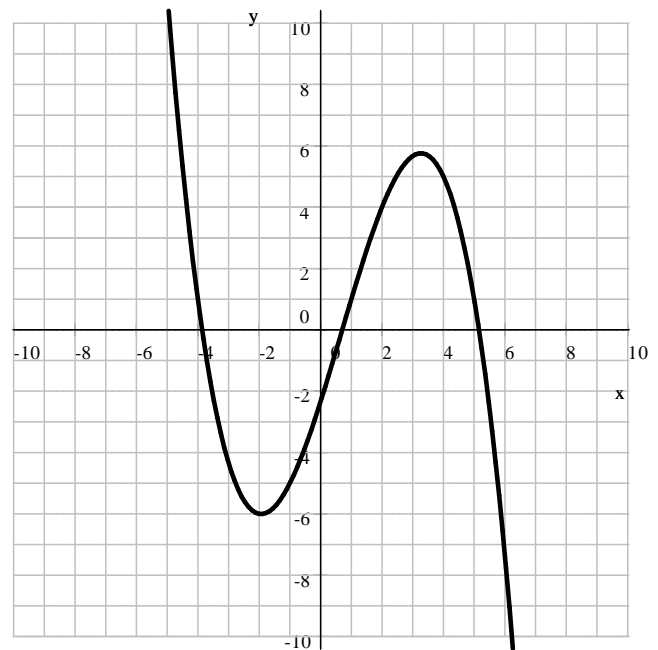
- a) $y = -2$ c) $y = -5$
 b) $y = 2$ d) $y \leq -2$



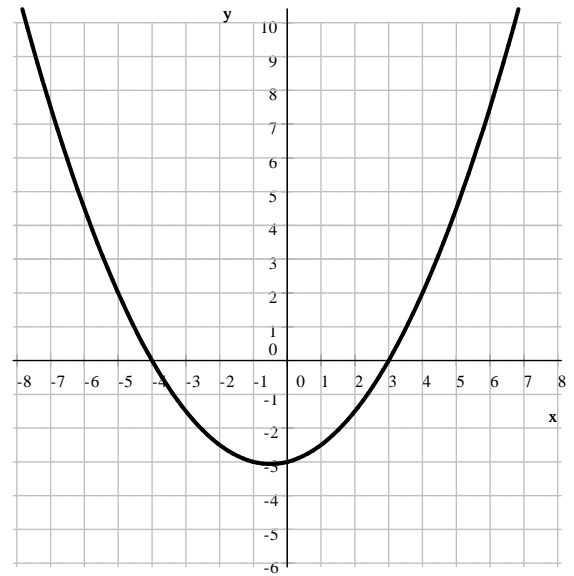
Practice Problems

1. Consider the graph shown on the picture. Assume that x is between -5 and 6 (in short, $-5 \leq x \leq 6$). Find all values of x so that the point $P(x, y)$ is on the graph and

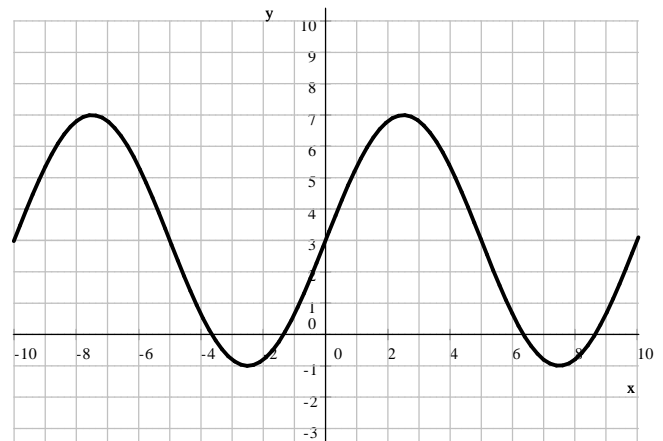
- a) $y = 1$ b) $y > 1$



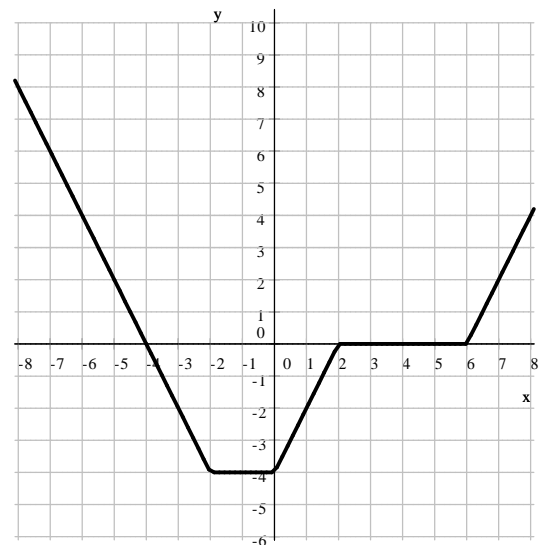
2. Consider the graph shown on the picture. Find all values of x so that the point $P(x, y)$ is on the graph and
- a) $y > 0$ b) $y < 0$



3. Consider the graph shown on the picture. Assume that x is between -10 and 10 (in short, $-10 \leq x \leq 10$). Find all values of x so that the point $P(x, y)$ is on the graph and
- a) $y = 3$ b) $y < 3$



4. Consider the graph shown on the picture. Find all values of x so that the point $P(x, y)$ is on the graph and
- a) $y = 2$ c) $y > 0$
 b) $y \leq -2$ d) $y \geq 0$



Sample Problems - Answers

1. a) $x = -3, -1, 3, 5$ We can also represent this as a set: $\{-3, -1, 3, 5\}$
- b) $x = -5, 1, 7$ We can also represent this as a set: $\{-5, 1, 7\}$
- c) There is no such value of x . We can also represent this as a set. The empty set is a set that contains no element. Our solution set in this case is the empty set, which is denoted by \emptyset .
- d) The x -values are between -3 and -1 (endpoints included) and between 3 and 5 (endpoints included). This can be represented using inequality notation: $-3 \leq x \leq -1$ or $3 \leq x \leq 5$. The solution set can also be given using interval notation: $[-3, -1] \cup [3, 5]$

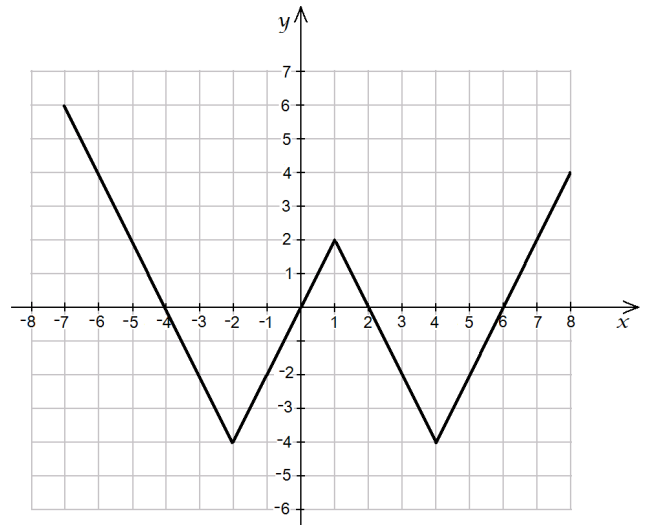
Practice Problems - Answers

1. a) $x = -4, 1, \text{ and } 5$. We can also represent this as a set: $\{-4, 1, 5\}$
- b) $-5 < x < -4$ or $1 < x < 5$. The solution set can also be given using interval notation: $(-5, -4) \cup (1, 5)$
2. a) $x < -4$ or $x > 3$ or in interval notation: $(-\infty, -4) \cup (3, \infty)$
- b) $-4 < x < 3$ or in interval notation: $(-4, 3)$
3. a) $x = -10, -5, 0, 5, 10$ or as a set: $\{-10, -5, 0, 5, 10\}$
- b) $-5 < x < 0$ or $5 < x < 10$ in interval notation: $(-5, 0) \cup (5, 10)$
4. a) $x = -5$ or 7 The solution set: $\{-5, 7\}$
- b) $-3 \leq x \leq 1$ in interval notation: $[-3, 1]$
- c) $x < -4$ or $x > 6$ in interval notation: $(-\infty, -4) \cup (6, \infty)$
- d) $x \leq -4$ or $x \geq 2$ in interval notation: $(-\infty, -4] \cup [2, \infty)$

Sample Problems - Solutions

1. Consider the graph shown on the picture. Assume that x is between -7 and 8 (in short, $-7 \leq x \leq 8$). Find all values of x so that the point $P(x, y)$ is on the graph and

- a) $y = -2$ c) $y = -5$
 b) $y = 2$ d) $y \leq -2$

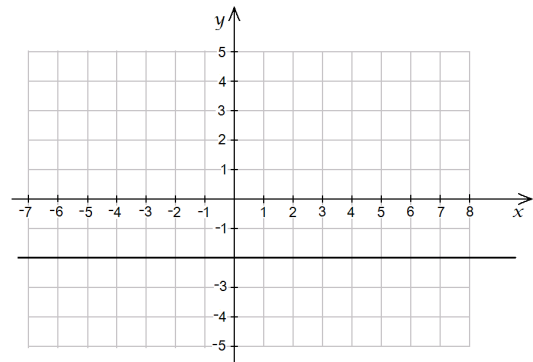


To solve an equation or inequality graphically, we will need three steps:

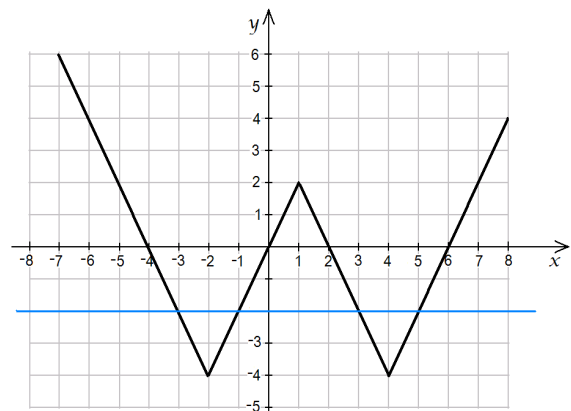
- Step 1. Interpret the equation or inequality graphically.
 Step 2. Find all points on the graph that are solutions of the equation or inequality.
 Step 3. Find the x -coordinates of all solution points.

a) $y = -2$

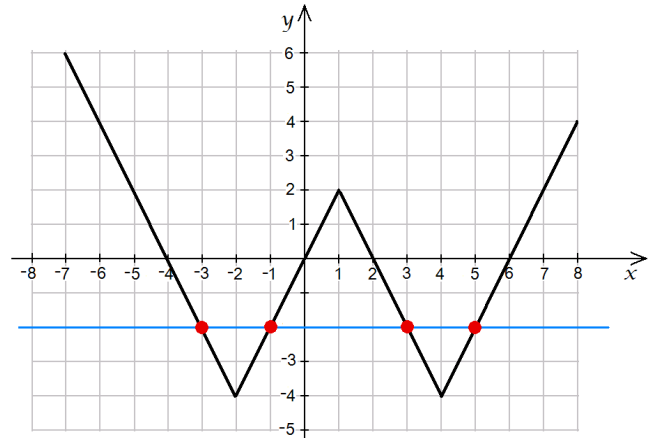
Step 1. Interpret the equation graphically. The equation $y = -2$ indicates all points with y -coordinate -2 . These points lie on a horizontal line.



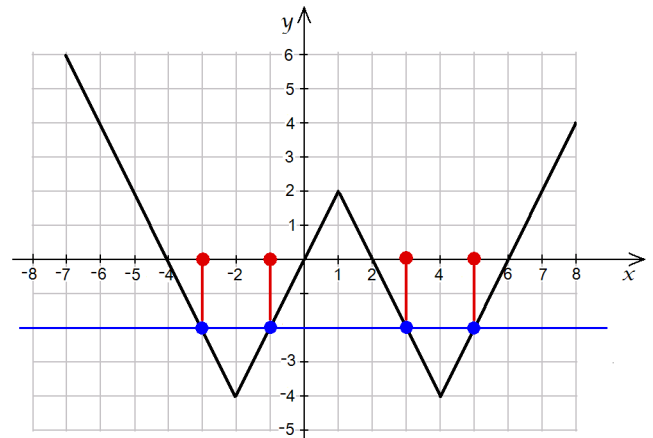
Step 2. Find all points on the graph that are solutions of the equation. We are looking for all points on the graph with y -coordinate -2 .



We find four points.

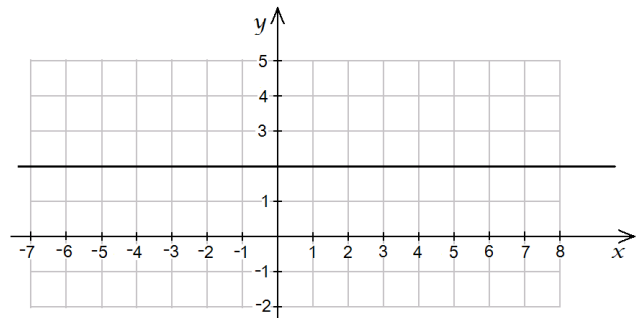


Step 3. Find the x -coordinates of all solution points. We project the four points to the x -axis and can easily read the x -values: $x = -3, -1, 3,$ and 5 . The solution can be also represented as a set; the solution set is $\{-3, -1, 3, 5\}$.

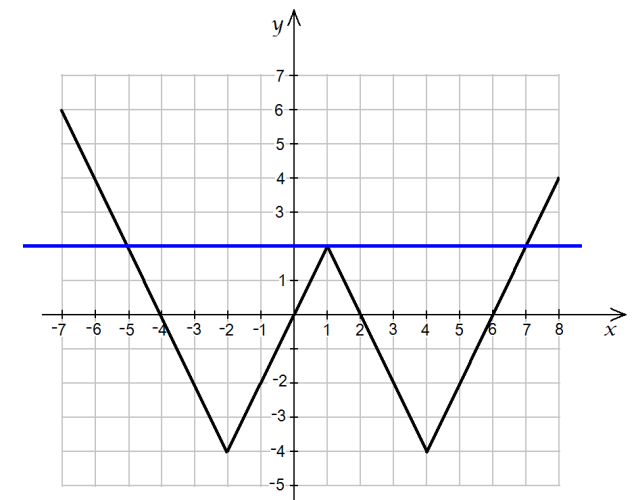


b) $y = 2$

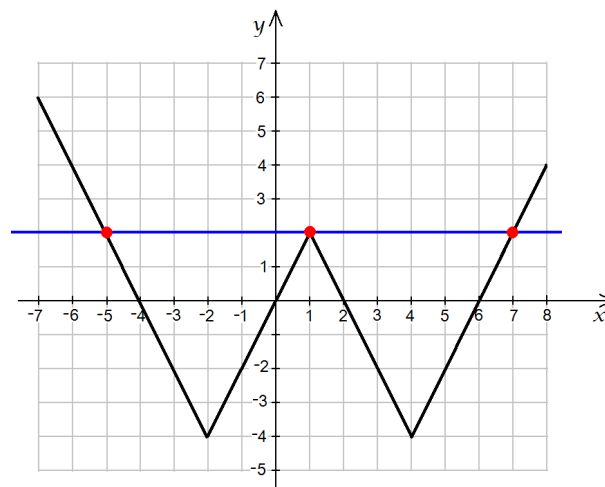
Step 1. Interpret the equation graphically. The equation $y = 2$ indicates all points with y -coordinate 2. These points lie on a horizontal line.



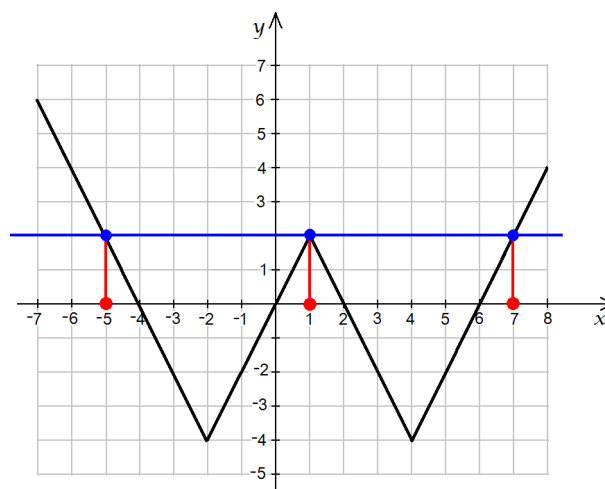
Step 2. Find all points on the graph that are solutions of the equation. We are looking for all points on the graph with y -coordinate 2.



We find three points.

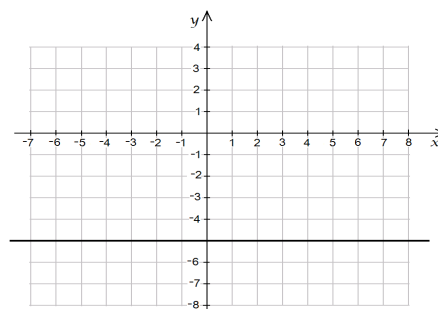


Step 3. Find the x -coordinates of all solution points. We project the three points to the x -axis and can easily read the x -values: $x = -5, 1$, and 7 . The solution can be also represented as a set; the solution set is $\{-5, 1, 7\}$.



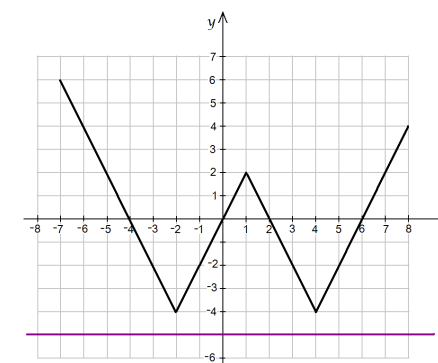
c) $y = -5$

Step 1. Interpret the equation graphically. The equation $y = -5$ indicates all points with y -coordinate -5 . These points lie on a horizontal line.



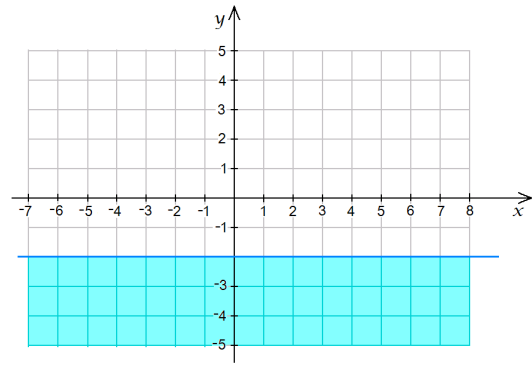
Step 2. Find all points on the graph that are solutions of the equation. We are looking for all points on the graph with y -coordinate -5 .

There is no such point. So this equation has no solution. We can also represent this as a set. The empty set is a set that contains no element. Our solution set in this case is the empty set, which is denoted by \emptyset .

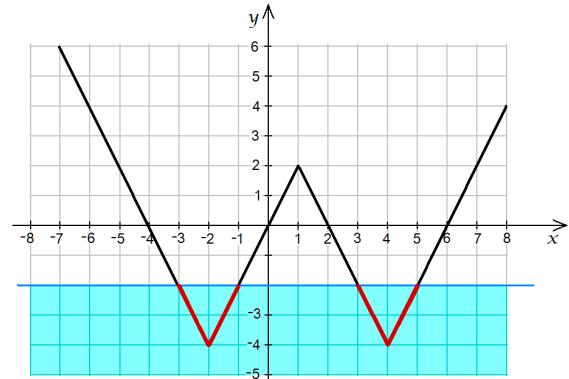


d) $y \leq -2$

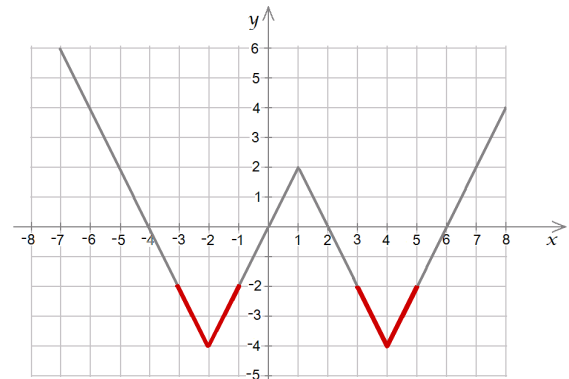
Step 1. Interpret the inequality graphically. The equation $y \leq -2$ indicates all points with y -coordinate -2 or less. These points lie on or below a horizontal line.



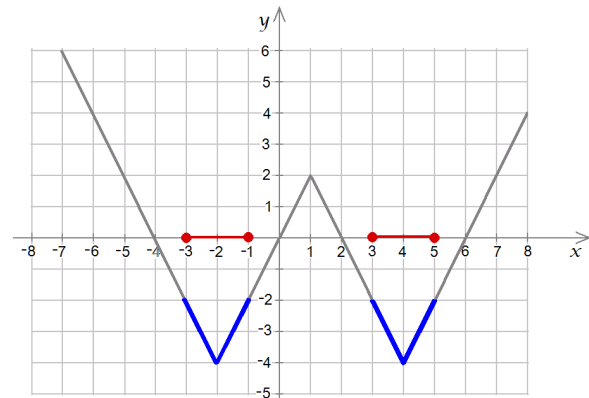
Step 2. Find all points on the graph that are solutions of the inequality. We are looking for all points on the graph with y -coordinate -2 or less.



We find two V-shaped line segment on the graph.



Step 3. Find the x -coordinates of all solution points. We project the line segments to the x -axis and can easily read the x -values: x is between -3 and -1 and between 3 and 5 . This can be presented using inequality notation: $-3 \leq x \leq -1$ or $3 \leq x \leq 5$. The solution can be also represented as a set, using interval notation: $[-3, -1] \cup [3, 5]$.



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