Sample Problems

Solve each of the following inequalities.

1.) $4x + x^2 < 21$ 2.) $33 - x^2 \leq 8x$ 3.) $x^2 - 10x + 20 \geq -9$ 4.) $6x - x^2 \geq 0$

Practice Problems

Solve each of the following inequalities.

1.) $2x + x^2 \geq 35$ 2.) $-12x - 2x^2 > 20$ 3.) $2x^2 - 4 < 7x$ 4.) $(x+3)^2 \leq 25$

5.) $x^2 - 14x \leq -49$ 6.) $x^2 - 10x \geq 1$ 7.) $x^2 + 30x + 1 > 4x - 170$

Sample Problems - Answers

1.) $-7 < x < 3$ - in interval notation: $(-7, 3)$

2.) $x \leq -11$ or $x \geq 3$ - in interval notation: $(-\infty, -11] \cup [3, \infty)$

3.) $\mathbb{R}$ (all real numbers are solution)

4.) $0 \leq x \leq 6$ - in interval notation: $[0, 6]$

Practice Problems - Answers

1.) $x \leq -7$ or $x \geq 5$ - in interval notation: $(-\infty, -7] \cup [5, \infty)$ 2.) There is no solution

3.) $-\frac{1}{2} < x < 4$ - in interval notation: $\left(-\frac{1}{2}, 4\right)$ 4.) $-8 \leq x \leq 2$ - in interval notation: $[-8, 2]$

5.) $x = 7$ 6.) $x \leq 5 - \sqrt{26}$ or $x \geq 5 + \sqrt{26}$ - in interval notation: $\left(-\infty, 5 - \sqrt{26}\right] \cup \left[5 + \sqrt{26}, \infty\right)$

7.) $\mathbb{R}$ (all numbers are solution)

 Last revised: August 26, 2009

Sample Problems - Solutions

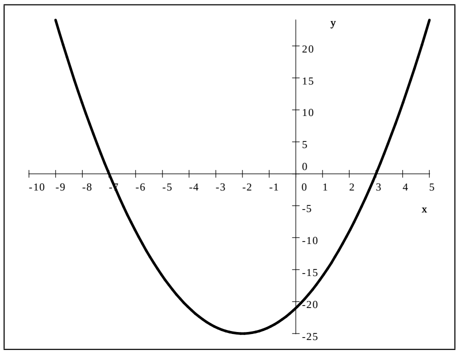
1.) Solve the inequality $4x + x^2 < 21$

Solution: We reduce one side to zero first and then factor. (There are several factoring techniques possible, we will factor by completing the square.)

$$\begin{aligned} 4x + x^2 &< 21 \\ x^2 + 4x - 21 &< 0 \\ \underbrace{x^2 + 4x + 4} - 4 - 21 &< 0 \\ (x+2)^2 - 25 &< 0 \\ (x+2+5)(x+2-5) &< 0 \\ (x+7)(x-3) &< 0 \end{aligned}$$

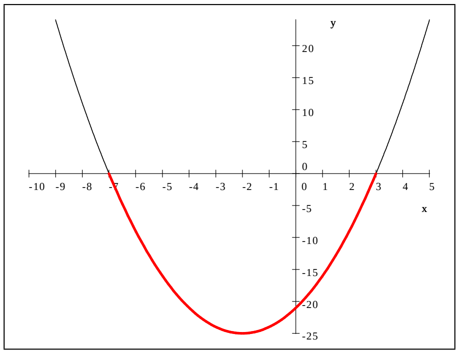
$$(x+2)^2 = x^2 + 4x + 4$$

The left-hand side is a quadratic expression with a positive leading coefficient. The graph of such an expression is a regular (or upward opening) parabola, with x-intercepts at $x = -7$ and $x = 3$. If we plot the graph of this expression we get this picture



The graph of $y = (x+7)(x-3)$

Now, consider the inequality to be solved: $(x+7)(x-3) < 0$. We need to find the $x-$values for which $y = (x+7)(x-3)$ is negative. In other words, the $x-$coordinates of all points on the parabola that lie below the $x-$axis. That's easy: it's the part between the $x-$intercepts.



The graph of $y = (x+7)(x-3)$

The $x-$coordinate of these points range from -7 to 3. Thus the solution is:

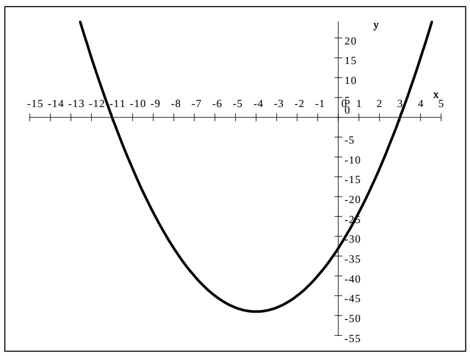
$$-7 < x < 3 \quad \text{or in interval notation,} \quad (-7, 3)$$

2.) Solve the inequality $33 - x^2 \le 8x$

Solution: We reduce one side to zero first and then factor. (There are several factoring techniques possible, we will complete the square.)

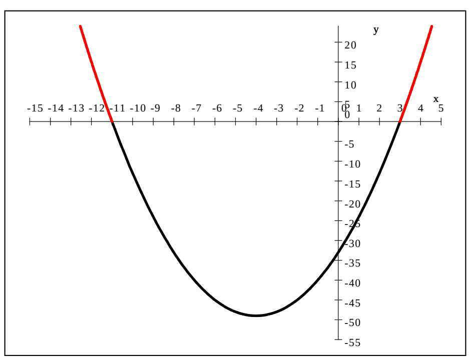
$$\begin{aligned} 33 - x^2 &\le 8x \\ 0 &\le x^2 + 8x - 33 \qquad\qquad (x+4)^2 = x^2 + 8x + 16 \\ 0 &\le \underbrace{x^2 + 8x + 16} - 16 - 33 \\ 0 &\le (x+4)^2 - 49 \\ 0 &\le (x+4+7)(x+4-7) \\ 0 &\le (x+11)(x-3) \end{aligned}$$

The right-hand side is a quadratic expression with a positive leading coefficient. The graph of such an expressions is a regular (or upward opening) parabola, with $x-$intercepts at $x = -11$ and $x = 3$. If we plot the graph of this expression we get this picture



The graph of $y = (x+11)(x-3)$

Now, consider the inequality to be solved: $(x+11)(x-3) \ge 0$. We need to find the $x-$values for which $y = (x+11)(x-3)$ is positive or zero. In other words, the $x-$coordinates of all points on the parabola that lie on or above the $x-$axis.



The graph of $y = (x+11)(x-3)$

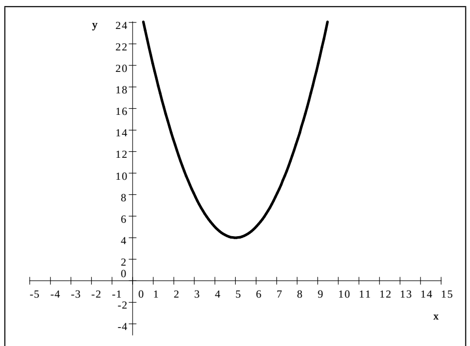
The $x-$coordinate of these points range from $-\infty$ to -11 or from 3 to ∞. Thus the solution is:

$$x \le -11 \quad \text{or} \quad x \ge 3 \quad \text{or in interval notation:} \ (-\infty, -11] \cup [3, \infty)$$

3.) Solve the inequality $x^2 - 10x + 20 \geq -9$
Solution: We reduce one side to zero first and then factor.

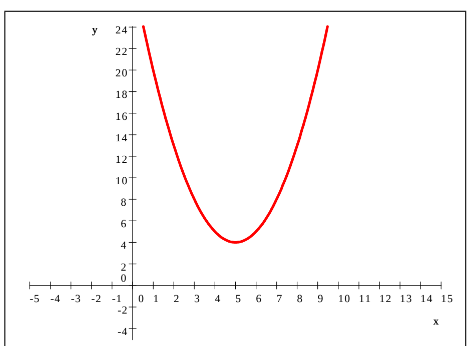
$$\begin{aligned} x^2 - 10x + 20 &\geq -9 \\ x^2 - 10x + 29 &\geq 0 \qquad\qquad (x-5)^2 = x^2 - 10x + 29 \\ \underbrace{x^2 - 10x + 25}\, -25 + 29 &\geq 0 \\ (x-5)^2 + 4 &\geq 0 \end{aligned}$$

The right-hand side is a quadratic expression that does not factor. However, in case of inequalities, this is NOT the end of the story. The right-hand side is a quadratic expression with a positive leading coefficient. The graph of such an expressions is a regular (or upward opening) parabola, with vertex at $(5,4)$. If we plot the graph of this expression we get this picture



The graph of $y = (x-5)^2 + 4$

and consider now the inequality to be solved: $(x-5)^2 + 4 \geq 0$. We need to find the $x-$values for which $y = (x-5)^2 + 4$ is positive or zero. In other words, the $x-$coordinates of all points on the parabola that lie on or above the $x-$axis.



The graph of $y = (x-5)^2 + 4$

This is clearly true for every point on the parabola. Thus the solution is:

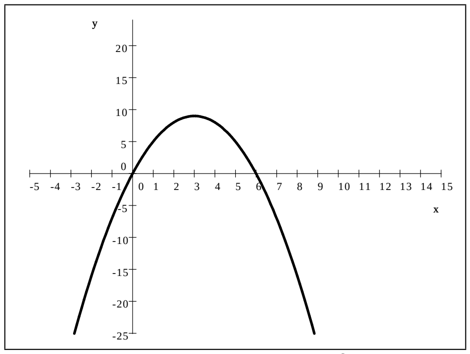
all real numbers: $\mathbb{R}$ or in interval notation: $(-\infty, \infty)$

4.) Solve the inequality $6x - x^2 \geq 0$

Solution: The inequality already has one side reduced to zero, so then we need to factor first.

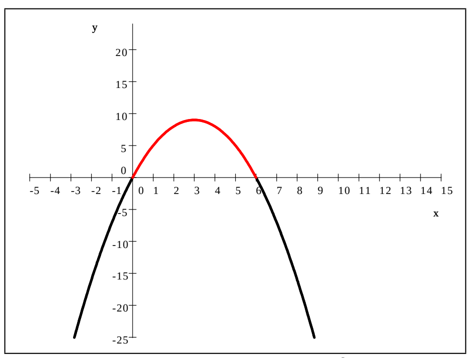
$$\begin{aligned} -x^2 + 6x &\geq 0 \\ -x(x-6) &\geq 0 \end{aligned}$$

The left-hand side is a quadratic expression with a negative leading coefficient. The graph of such an expressions is an upside down (or downward opening) parabola, with $x-$intercepts at $x = 0$ and $x = 6$. If we plot the graph of this expression we get this picture



The graph of $y = -x^2 + 6x$

Now, consider the inequality to be solved: $-x^2 + 6x \geq 0$. We need to find the $x-$values for which $y = -x^2 + 6x$ is positive or zero. In other words, the $x-$coordinates of all points on the parabola that lie on or above the $x-$axis.



The graph of $y = -x^2 + 6x$

The $x-$coordinate of these points range from 0 to 6. Thus the solution is:

$0 \leq x \leq 6$, or in interval notation, $[0, 6]$

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