Sample Problems

Solve each of the following inequalities.

1. $\dfrac{x+3}{x-4} \geq 2$
2. $\dfrac{x+7}{x-3} > 0$
3. $\dfrac{5t-1}{t-2} \leq 3$

Practice Problems

Solve each of the following inequalities.

1. $\dfrac{a-1}{a} > 0$
2. $\dfrac{3x+6}{2x-12} \leq 0$
3. $\dfrac{-x+8}{x-2} \geq 5$
4. $\dfrac{b+3}{5-2b} \leq 4$
5. $\dfrac{3x-1}{x} \leq -1$
6. $\dfrac{-2x+5}{x+6} > -2$
7. $\dfrac{2}{p-1} \geq \dfrac{3}{4}$
8. $\dfrac{2}{m+3} \leq 1$

Sample Problems - Answers

1.) $4 < x \leq 11$ - in interval notation: $(4, 11]$

2.) $x < -7$ or $x > 3$ - in interval notation: $(-\infty, -7) \cup (3, \infty)$

3.) $-\dfrac{5}{2} \leq x < 2$ - in interval notation: $\left[-\dfrac{5}{2}, 2\right)$

Practice Problems - Answers

1.) $a < 0$ or $a > 1$ - in interval notation: $(-\infty, 0) \cup (1, \infty)$

2.) $-2 \leq x < 6$ - in interval notation: $[-2, 6)$

3.) $2 < x \leq 3$ - in interval notation: $(2, 3]$

4.) $b \leq \dfrac{17}{9}$ or $b > \dfrac{5}{2}$ - in interval notation: $\left(-\infty, \dfrac{17}{9}\right] \cup \left(\dfrac{5}{2}, \infty\right)$

5.) $0 < x \leq \dfrac{1}{4}$ - in interval notation: $\left(0, \dfrac{1}{4}\right]$

6.) $x > -6$ - in interval notation: $(-6, \infty)$

7.) $1 < p \leq \dfrac{11}{3}$ - in interval notation: $\left(1, \dfrac{11}{3}\right]$

8.) $m < -3$ or $m \geq -1$ - in interval notation: $(-\infty, -3) \cup [-1, \infty)$

Sample Problems - Solutions

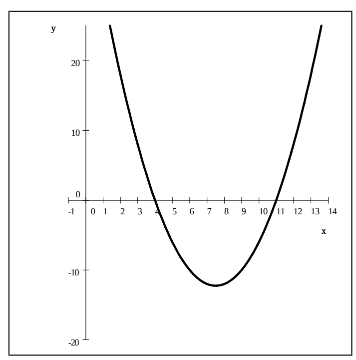
Solve each of the following inequalities.

1. $\dfrac{x+3}{x-4} \geq 2$

   Solution: This brilliant method is from Ahyoung Kim. We would like to multiply both sides by $x-4$ to clear the denominators, but we have to be careful with that. The problem is that $x-4$ may be positive or negative depending on the value of x. So when multiplying by $x-4$, we do not know whether to keep the inequality sign or reverse it. One possibilty is however, to multiply both sides by $(x-4)^2$. The expression $(x-4)^2$ is zero when $x=4$, and is positive for all other real values of x. Since $x-4$ is in the denominator, $x=4$ can not be a solution. We need to remember that. For all other values of x, $(x-4)^2$ is positive and so we may multiply by it, and we need to keep the inequality sign as it is.

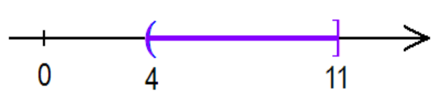
$$\begin{aligned}
\frac{x+3}{x-4} &\geq 2 && \text{multiply by } (x-4)^2 \\
\left(\frac{x+3}{x-4}\right)(x-4)^2 &\geq 2(x-4)^2 \\
(x+3)(x-4) &\geq 2(x-4)^2 \\
0 &\geq 2(x-4)^2-(x+3)(x-4) && \text{factor out } (x-4) \\
0 &\geq (x-4)(2(x-4)-(x+3)) \\
0 &\geq (x-4)(2x-8-x-3) \\
0 &\geq (x-4)(x-11)
\end{aligned}$$

Recall how we solve quadratic inequalities. If we graph $y=(x-4)(x-11)$, the graph will be an upward opening parabola with $x-$intercepts $(4,0)$ and $(11,0)$. We sketch that graph and look for the points satisfying $0 \geq y$. That is the part of the parabola that is on or below the $x-$axis. The $x-$coordinate of these points lie in the interval between the $x-$intercepts. Thus, the quadratic inequality's solution set is $[4,11]$. Our inequality, the rational inequality does not allow for $x=4$ and so the solution is $4<x\leq 11$ - or in interval notation: $(4,11]$



$y=(x-4)(x-11)$

That means that the final answer is $4<x\leq 11$ or, in interval notation, $(4,11]$.

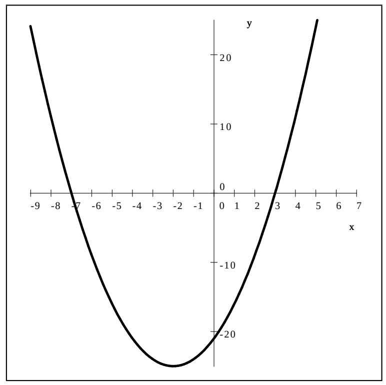


2. $\dfrac{x+7}{x-3} > 0$

Solution: We would like to multiply both sides by $x-3$ to clear the denominators, but we have to be careful with that. The problem is that $x-3$ may be positive or negative depending on the value of x. So when multiplying by $x-3$, we do not know whether to keep the inequality sign or reverse it. One possibilty is however, to multiply both sides by $(x-3)^2$. The expression $(x-3)^2$ is zero when $x=3$, and is positive for all other real values of x. Since $x-3$ is in the denominator, $x=3$ can not be a solution. We need to remember that. For all other values of x, $(x-3)^2$ is positive and so we may multiply by it, and we need to keep the inequality sign as it is.

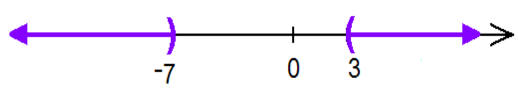
$$\begin{aligned} \frac{x+7}{x-3} &> 0 && \text{multiply by } (x-3)^2 \\ \left(\frac{x+7}{x-3}\right)(x-3)^2 &> 0\,(x-3)^2 \\ (x+7)(x-3) &> 0 \end{aligned}$$

Recall how we solve quadratic inequalities. If we graph $y=(x+7)(x-3)$, the graph will be an upward opening parabola with $x-$intercepts $(-7,0)$ and $(3,0)$. We sketch that graph and look for the points satisfying $y>0$. That is the part of the parabola that is above the $x-$axis. The $x-$coordinates of all of these points is the region to the left of the first $x-$intercept and the region to the right of the second $x-$intercept. Thus, the quadratic inequality's solution set is $(-\infty,-7)\cup(3,\infty)$. Our inequality, the rational inequality does not allow for $x=3$ but that does not affect our solution set - it already does not include $x=3$.



$y=(x+7)(x-3)$

The solution set is the union of these two sets, $x<-7$ or $x>-3$, or, in interval notation, $(-\infty,-7)\cup(3,\infty)$.

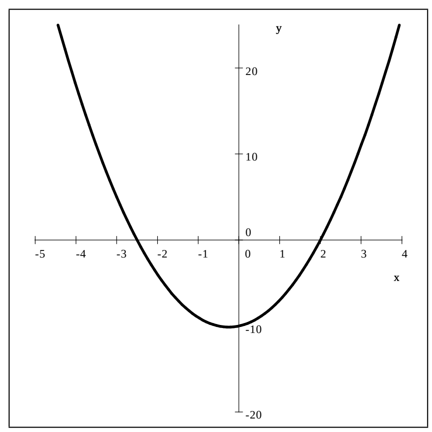


3. $\dfrac{5t-1}{t-2} \leq 3$

We would like to multiply both sides by $t-2$ to clear the denominators, but we have to be careful with that. The problem is that $t-2$ may be positive or negative depending on the value of t. So when multiplying by $t-2$, we do not know whether to keep the inequality sign or reverse it. One possibilty is however, to multiply both sides by $(t-2)^2$. The expression $(t-2)^2$ is zero when $t=2$, and is positive for all other real values of t. Since $t-2$ is in the denominator, $t=2$ can not be a solution. We need to remember that. For all other values of t, $(t-2)^2$ is positive and so we may multiply by it, and we need to keep the inequality sign as it is.

$$
\begin{aligned}
\frac{5t-1}{t-2} &\leq 3 && \text{multiply by } (t-2)^2 \\
\left(\frac{5t-1}{t-2}\right)(t-2)^2 &\leq 3(t-2)^2 \\
(5t-1)(t-2) &\leq 3(t-2)^2 && \text{subtract } 3(t-2)^2 \\
(5t-1)(t-2) - 3(t-2)^2 &\leq 0 && \text{factor out } (t-2) \\
(t-2)((5t-1) - 3(t-2)) &\leq 0 \\
(t-2)(5t-1-3t+6) &\leq 0 \\
(t-2)(2t+5) &\leq 0
\end{aligned}
$$

Recall how we solve quadratic inequalities. If we graph $y=(x-2)(2x+5)$, the graph will be an upward opening parabola with $x-$intercepts $(2,0)$ and $\left(-\dfrac{5}{2},0\right)$. We sketch that graph and look for the points satisfying $y \leq 0$. That is the part of the parabola that is on or below the $x-$axis. The $x-$coordinate of these points lie in the interval between the $x-$intercepts. Thus, the quadratic inequality's solution set is $\left[-\dfrac{5}{2},2\right]$. Our inequality, the rational inequality does not allow for $t=2$ and so the solution is $-\dfrac{5}{2} \leq t < 2$ - or in interval notation: $\left[-\dfrac{5}{2},2\right)$



$y=(x-2)(2x+5)$

That means that the final answer is $-\dfrac{5}{2} \leq t < 2$ or in interval notation: $\left[-\dfrac{5}{2},2\right)$.