Sample Problems

1. Compute each of the following limits.

a) $\lim\limits_{x\to\infty} 3x^4$ c) $\lim\limits_{x\to\infty} \left(-2x^5\right)$ e) $\lim\limits_{x\to\infty} \left(-\dfrac{2}{3}x^6\right)$ g) $\lim\limits_{x\to\infty} 4x^3$

b) $\lim\limits_{x\to-\infty} 3x^4$ d) $\lim\limits_{x\to-\infty} \left(-2x^5\right)$ f) $\lim\limits_{x\to-\infty} \left(-\dfrac{2}{3}x^6\right)$ h) $\lim\limits_{x\to-\infty} 4x^3$

2. Compute each of the following limits.

a) $\lim\limits_{x\to\infty} 2^x$ c) $\lim\limits_{x\to\infty} \left(\dfrac{2}{3}\right)^x$ e) $\lim\limits_{x\to\infty} \dfrac{2^{x+3}}{3^{x+1}}$ g) $\lim\limits_{x\to\infty} \dfrac{2^{2x+1}}{3^{x-1}}$

b) $\lim\limits_{x\to-\infty} 2^x$ d) $\lim\limits_{x\to-\infty} \left(\dfrac{2}{3}\right)^x$ f) $\lim\limits_{x\to-\infty} \dfrac{2^{x+3}}{3^{x+1}}$ h) $\lim\limits_{x\to-\infty} \dfrac{2^{2x+1}}{3^{x-1}}$

3. Compute each of the following limits.

a) $\lim\limits_{x\to\infty} \dfrac{1}{x}$ d) $\lim\limits_{x\to-\infty} \left(\dfrac{-5}{2x^3} - 7 + \dfrac{8}{x}\right)$ g) $\lim\limits_{x\to\infty} \dfrac{-5x^3 - 2x + 4}{x^2}$

b) $\lim\limits_{x\to-\infty} \dfrac{1}{x}$ e) $\lim\limits_{x\to\infty} \left(-2x^3 + 1 - \dfrac{5}{x} + \dfrac{12}{x^4}\right)$ h) $\lim\limits_{x\to\infty} \dfrac{-5x^3 - 2x + 4}{x^3}$

c) $\lim\limits_{x\to\infty} \dfrac{-5}{2x^3}$ f) $\lim\limits_{x\to-\infty} \dfrac{3x-2}{x}$ i) $\lim\limits_{x\to\infty} \dfrac{-5x^3 - 2x + 4}{x^4}$

4. Compute each of the following limits.

a) $\lim\limits_{x\to-\infty} \left(-2x^5 - 8x^4 + 7x^3 - 10\right)$ c) $\lim\limits_{x\to-\infty} \left(-2x^5 + 8x^6\right)$

b) $\lim\limits_{x\to\infty} \left(-2x^5 - 8x^4 + 7x^3 - 10\right)$ d) $\lim\limits_{x\to\infty} \left(-2x^5 + 8x^6\right)$

5. Compute each of the following limits.

a) $\lim\limits_{x\to-\infty} \dfrac{x + x^2 - 6}{6x + 5x^2 + 2x^3}$ b) $\lim\limits_{x\to\infty} \dfrac{x^2 + 9}{5x + 2x^2 - 3}$ c) $\lim\limits_{x\to-\infty} \dfrac{x^3 - 9x + 1}{3x^2 - 2x - 15}$

Practice Problems

1. Compute each of the following limits.

a) $\lim\limits_{x\to\infty} \left(-\dfrac{3}{8}x^{15}\right)$ c) $\lim\limits_{x\to\infty} \dfrac{1}{3}x^8$ e) $\lim\limits_{x\to\infty} 4x^9$ g) $\lim\limits_{x\to\infty} \left(-7x^{10}\right)$

b) $\lim\limits_{x\to-\infty} \left(-\dfrac{3}{8}x^{15}\right)$ d) $\lim\limits_{x\to-\infty} \dfrac{1}{3}x^8$ f) $\lim\limits_{x\to-\infty} 4x^9$ h) $\lim\limits_{x\to-\infty} \left(-7x^{10}\right)$

2. Compute each of the following limits.

a) $\displaystyle\lim_{x\to\infty}\frac{2^{3x-1}}{5^{x-1}}$ c) $\displaystyle\lim_{x\to\infty}\frac{2^{2x+3}}{5^{x-1}}$ e) $\displaystyle\lim_{x\to\infty}\frac{2^{2x+3}}{4^{x-1}}$ g) $\displaystyle\lim_{x\to\infty}\frac{2^{x+3}\cdot 3^{x-1}}{7^{x-2}}$

b) $\displaystyle\lim_{x\to-\infty}\frac{2^{3x-1}}{5^{x-1}}$ d) $\displaystyle\lim_{x\to-\infty}\frac{2^{2x+3}}{5^{x-1}}$ f) $\displaystyle\lim_{x\to-\infty}\frac{2^{2x+3}}{4^{x-1}}$ h) $\displaystyle\lim_{x\to\infty}\frac{2^{2x+3}\cdot 3^{x-1}}{7^{x-2}}$

3. Compute each of the following limits.

a) $\displaystyle\lim_{x\to\infty}\frac{3}{x^5}$ g) $\displaystyle\lim_{x\to\infty}\left(5x-\frac{2}{x+3}\right)$ m) $\displaystyle\lim_{x\to\infty}\frac{-3x^5+2x-5}{x^2}$

b) $\displaystyle\lim_{x\to-\infty}\frac{3}{x^5}$ h) $\displaystyle\lim_{x\to-\infty}\left(5x-\frac{2}{x+3}\right)$ n) $\displaystyle\lim_{x\to-\infty}\frac{-3x^5+2x-5}{x^2}$

c) $\displaystyle\lim_{x\to\infty}\left(1-\frac{2}{x}+\frac{5}{3x^4}\right)$ i) $\displaystyle\lim_{x\to\infty}\frac{5x-3}{x}$ o) $\displaystyle\lim_{x\to\infty}\frac{-4x^8+x^3-x+7}{x^4}$

d) $\displaystyle\lim_{x\to-\infty}\left(1-\frac{2}{x}+\frac{5}{3x^4}\right)$ j) $\displaystyle\lim_{x\to-\infty}\frac{5x-3}{x}$ p) $\displaystyle\lim_{x\to-\infty}\frac{-4x^8+x^3-x+7}{x^4}$

e) $\displaystyle\lim_{x\to\infty}\left(3+\frac{5}{x^3}-\frac{7}{6x}\right)$ k) $\displaystyle\lim_{x\to\infty}\frac{1-3x}{2x}$

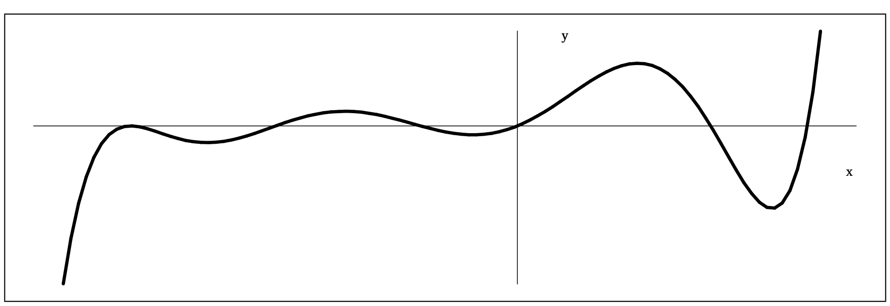
f) $\displaystyle\lim_{x\to-\infty}\left(3+\frac{5}{x^3}-\frac{7}{6x}\right)$ l) $\displaystyle\lim_{x\to-\infty}\frac{1-3x}{2x}$

4. Compute each of the following limits.

a) $\displaystyle\lim_{x\to-\infty}\left(-7x^5+x^3\right)$ c) $\displaystyle\lim_{x\to-\infty}\left(120x^5-\frac{1}{4}x^6\right)$ e) $\displaystyle\lim_{x\to-\infty}\left(8x^4-3x^3-\frac{1}{5}x+2\right)$

b) $\displaystyle\lim_{x\to\infty}\left(-7x^5+x^3\right)$ d) $\displaystyle\lim_{x\to\infty}\left(120x^5-\frac{1}{4}x^6\right)$ f) $\displaystyle\lim_{x\to-\infty}\left(8x^4-3x^3-\frac{1}{5}x+2\right)$

5. The graph of a polynomial function is shown on the picture below. What can we state about this polynomial based on its end-behavior?



6. Compute each of the following limits.

a) $\displaystyle\lim_{x\to-\infty}\frac{1}{x}$ c) $\displaystyle\lim_{x\to-\infty}\left(2-\frac{5}{x^3}\right)$ e) $\displaystyle\lim_{x\to\infty}\frac{2x^2+3x+1}{3x^2-5x+2}$ g) $\displaystyle\lim_{x\to-\infty}\frac{3x^2-1}{5x^2-3x+2}$

b) $\displaystyle\lim_{x\to-\infty}\frac{-5}{2x^3}$ d) $\displaystyle\lim_{x\to\infty}\left(3-\frac{2}{x}+\frac{11}{x^4}\right)$ f) $\displaystyle\lim_{x\to-\infty}\frac{-3x^3+2x+1}{5x-3}$ h) $\displaystyle\lim_{x\to-\infty}\frac{20x-2x^2-42}{5x^3-20x^2-105x}$

Sample Problems - Answers

1. a) ∞ b) ∞ c) $-\infty$ d) ∞ e) $-\infty$ f) $-\infty$ g) ∞ h) $-\infty$

2. a) ∞ b) 0 c) 0 d) ∞ e) 0 f) ∞ g) ∞ h) 0

3. a) 0 b) 0 c) 0 d) -7 e) $-\infty$ f) 3 g) $-\infty$ h) -5 i) 0

4. a) ∞ b) $-\infty$ c) ∞ d) ∞

5. a) 0 b) $\frac{1}{2}$ c) $-\infty$

Practice Problems - Answers

1. a) $-\infty$ b) ∞ c) ∞ d) ∞ e) ∞ f) $-\infty$ g) $-\infty$ h) $-\infty$

2. a) ∞ b) 0 c) 0 d) ∞ e) 32 f) 32 g) 0 h) ∞

3. a) 0 b) 0 c) 1 d) 1 e) 3 f) 3 g) ∞ h) $-\infty$ i) 5 j) 5 k) $-\frac{3}{2}$ l) $-\frac{3}{2}$
   m) $-\infty$ n) ∞ o) $-\infty$ p) $-\infty$

4. a) ∞ b) $-\infty$ c) $-\infty$ d) $-\infty$ e) ∞ f) ∞

5. Since $\lim\limits_{x\to-\infty} f(x) = -\infty$ and $\lim\limits_{x\to\infty} f(x) = \infty$, the polynomial is of odd degree and has a positive leading coefficient.

6. a) 0 b) 0 c) 2 d) 3 e) $\frac{2}{3}$ f) $-\infty$ g) $\frac{3}{5}$ h) 0

Sample Problems - Solutions

1. Compute each of the following limits.

a) $\lim\limits_{x\to\infty} 3x^4$

Solution: Since the limit we are asked for is as x approaches infinity, we should think of x as a very large positive number. Then $3x^4$ is very large, and also positive because it is the product of five positive numbers.

$$3x^4 = \underset{\text{positive}}{3} \cdot \underset{\text{positive}}{x} \cdot \underset{\text{positive}}{x} \cdot \underset{\text{positive}}{x} \cdot \underset{\text{positive}}{x}$$

So the answer is ∞. We state the answer: $\lim\limits_{x\to\infty} 3x^4 = \infty$.

b) $\lim\limits_{x\to-\infty} 3x^4$

Solution: Since the limit we are asked for is as x approaches negative infinity, we should think of x as a very large negative number. Then $3x^4$ is very large, and also positive because it is the product of one positive and four negative numbers.

$$3x^4 = \underset{\text{positive}}{3} \cdot \underset{\text{negative}}{x} \cdot \underset{\text{negative}}{x} \cdot \underset{\text{negative}}{x} \cdot \underset{\text{negative}}{x}$$

So the answer is ∞. We state the answer: $\lim\limits_{x\to-\infty} 3x^4 = \infty$

c) $\lim\limits_{x\to\infty} \left(-2x^5\right)$

Solution: Since the limit we are asked for is as x approaches infinity, we should think of x as a very large positive number. Then $-2x^5$ is very large, and also negative because it is the product of one negative and five positive numbers.

$$-2x^5 = \underset{\text{negative}}{-2} \cdot \underset{\text{positive}}{x} \cdot \underset{\text{positive}}{x} \cdot \underset{\text{positive}}{x} \cdot \underset{\text{positive}}{x} \cdot \underset{\text{positive}}{x}$$

So the answer is $-\infty$. We state the answer: $\lim\limits_{x\to\infty} \left(-2x^5\right) = -\infty$

d) $\lim\limits_{x\to-\infty} \left(-2x^5\right)$

Solution: Since the limit we are asked for is as x approaches negative infinity, we should think of x as a very large negative number. Then $-2x^5$ is very large, and also positive because it is the product of six negative numbers.

$$-2x^5 = \underset{\text{negative}}{-2} \cdot \underset{\text{negative}}{x} \cdot \underset{\text{negative}}{x} \cdot \underset{\text{negative}}{x} \cdot \underset{\text{negative}}{x} \cdot \underset{\text{negative}}{x}$$

So the answer is ∞. We state the answer: $\lim\limits_{x\to-\infty} \left(-2x^5\right) = \infty$

e) $\lim\limits_{x\to\infty} \left(-\frac{2}{3}x^6\right)$

Solution: Since the limit we are asked for is as x approaches infinity, we should think of x as a very large positive number. Then $-\frac{2}{3}x^6$ is very large, and also negative because it is the product of one negative and six positive numbers.

$$-\frac{2}{3}x^6 = \underset{\text{negative}}{-\frac{2}{3}} \cdot \underset{\text{positive}}{x} \cdot \underset{\text{positive}}{x} \cdot \underset{\text{positive}}{x} \cdot \underset{\text{positive}}{x} \cdot \underset{\text{positive}}{x} \cdot \underset{\text{positive}}{x}$$

So the answer is $-\infty$. We state the answer: $\lim\limits_{x\to\infty}\left(-\frac{2}{3}x^6\right)=-\infty$

f) $\lim\limits_{x\to-\infty}\left(-\frac{2}{3}x^6\right)$

Solution: Since the limit we are asked for is as x approaches negative infinity, we should think of x as a very large negative number. Then $-\frac{2}{3}x^6$ is very large, and also negative because it is the product of seven negative numbers.

$$-\frac{2}{3}x^6=\underset{\text{negative}}{-\frac{2}{3}}\cdot\underset{\text{negative}}{x}\cdot\underset{\text{negative}}{x}\cdot\underset{\text{negative}}{x}\cdot\underset{\text{negative}}{x}\cdot\underset{\text{negative}}{x}\cdot\underset{\text{negative}}{x}$$

So the answer is $-\infty$. We state the answer: $\lim\limits_{x\to-\infty}-\frac{2}{3}x^6=-\infty$

g) $\lim\limits_{x\to\infty}4x^3$

Solution: Since the limit we are asked for is as x approaches infinity, we should think of x as a very large positive number. Then $4x^3$ is very large, and also positive because it is the product of four positive numbers.

$$4x^3=\underset{\text{positive}}{4}\cdot\underset{\text{positive}}{x}\cdot\underset{\text{positive}}{x}\cdot\underset{\text{positive}}{x}$$

So the answer is ∞. We state the answer: $\lim\limits_{x\to\infty}4x^3=\infty$

h) $\lim\limits_{x\to-\infty}4x^3$

Solution: Since the limit we are asked for is as x approaches negative infinity, we should think of x as a very large negative number. Then $4x^3$ is very large, and also negative because it is the product of one positive and three negative numbers.

$$4x^3=\underset{\text{positive}}{4}\cdot\underset{\text{negative}}{x}\cdot\underset{\text{negative}}{x}\cdot\underset{\text{negative}}{x}$$

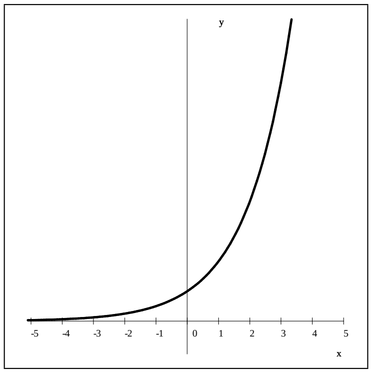
So the answer is $-\infty$. We state the answer: $\lim\limits_{x\to-\infty}4x^3=-\infty$

2. Compute each of the following limits.

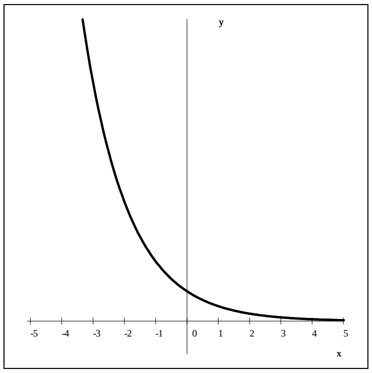
Let $a>0$. Then the limit of the exponential function $f(x)=a^x$ is as follows.

Case 1. If $a>1$, then $\lim\limits_{x\to\infty}a^x=\infty$ and $\lim\limits_{x\to-\infty}a^x=0$

Case 2. If $0<a<1$, then $\lim\limits_{x\to\infty}a^x=0$ and $\lim\limits_{x\to-\infty}a^x=\infty$



$a>1$



$0<a<1$

a) $\lim_{x\to\infty} 2^x$ and b) $\lim_{x\to-\infty} 2^x$
Solution: Since $2>1$, these limits are ∞ and 0, i.e. $\lim_{x\to\infty} 2^x = \infty$ and $\lim_{x\to-\infty} 2^x = 0$.

c) $\lim_{x\to\infty} \left(\frac{2}{3}\right)^x$ and d) $\lim_{x\to-\infty} \left(\frac{2}{3}\right)^x$
Solution: Since $\frac{2}{3} < 1$, these limits are 0 and ∞, i.e. $\lim_{x\to\infty} \left(\frac{2}{3}\right)^x = 0$ and $\lim_{x\to-\infty} \left(\frac{2}{3}\right)^x = \infty$.

e) $\lim_{x\to\infty} \frac{2^{x+3}}{3^{x+1}}$
Solution: We start by re-writing the exponential expressions. The goal is to bring it into a form where there is only one exponential expression involving x.

$$\frac{2^{x+3}}{3^{x+1}} = \frac{2^x \cdot 2^3}{3^x \cdot 3^1} = \frac{2^x \cdot 8}{3^x \cdot 3} = \frac{8}{3}\left(\frac{2}{3}\right)^x$$

$$\text{Thus} \quad \lim_{x\to\infty} \frac{2^{x+3}}{3^{x+1}} = \lim_{x\to\infty} \frac{8}{3}\left(\frac{2}{3}\right)^x = \frac{8}{3}\lim_{x\to\infty}\left(\frac{2}{3}\right)^x = 0 \qquad \text{since } \frac{2}{3} < 1$$

f) $\lim_{x\to-\infty} \frac{2^{x+3}}{3^{x+1}}$
Solution: $\lim_{x\to-\infty} \frac{2^{x+3}}{3^{x+1}} = \lim_{x\to-\infty} \frac{2^{x+3}}{3^{x+1}} = \lim_{x\to-\infty} \frac{8}{3}\left(\frac{2}{3}\right)^x = \frac{8}{3}\lim_{x\to-\infty}\left(\frac{2}{3}\right)^x = \infty$

g) $\lim_{x\to\infty} \frac{2^{2x+1}}{3^{x-1}}$
Solution: We start by re-writing the exponential expressions. The goal is to bring it into a form where there is only one exponential expression involving x.

$$\frac{2^{2x+1}}{3^{x-1}} = \frac{2^{2x}\cdot 2^1}{\frac{3^x}{3^1}} = \frac{\left(2^2\right)^x \cdot 2}{3^x \cdot \frac{1}{3}} = \frac{4^x \cdot 6}{3^x} = 6\left(\frac{4}{3}\right)^x$$

$$\text{Thus} \quad \lim_{x\to\infty} \frac{2^{2x+1}}{3^{x-1}} = \lim_{x\to\infty} 6\left(\frac{4}{3}\right)^x = 6\lim_{x\to\infty}\left(\frac{4}{3}\right)^x = \infty \quad \text{since } \frac{4}{3} > 1$$

h) $\lim_{x\to-\infty} \frac{2^{2x+1}}{3^{x-1}}$
Solution: $\lim_{x\to-\infty} \frac{2^{2x+1}}{3^{x-1}} = \lim_{x\to-\infty} 6\left(\frac{4}{3}\right)^x = 6\lim_{x\to-\infty}\left(\frac{4}{3}\right)^x = 0$

3. Compute each of the following limits.

a) $\lim_{x\to\infty} \frac{1}{x}$

Solution: This is a very important limit. Since the limit we are asked for is as x approaches infinity, we should think of x as a very large positive number. The reciprocal of a very large positive number is a very small positive number. This limit is 0.

b) $\lim_{x\to-\infty} \frac{1}{x}$

Solution: Since the limit we are asked for is as x approaches negative infinity, we should think of x as a very large negative number. The reciprocal of a very large negative number is a very small negative number. This limit is 0.

c) $\displaystyle\lim_{x\to\infty}\frac{-5}{2x^3}$

Solution: Since the limit we are asked for is as x approaches infinity, we should think of x as a very large positive number. We divide -5 by a very large positive number. This limit is 0.

d) $\displaystyle\lim_{x\to-\infty}\left(\frac{-5}{2x^3}-7+\frac{8}{x}\right)$

Solution: This limit is -7 since the other two terms aproach zero as x approaches negative infinity. Using mathematical notation,

$$\lim_{x\to-\infty}\frac{-5}{2x^3}-7+\frac{8}{x}=\lim_{x\to-\infty}\frac{-5}{2x^3}+\lim_{x\to-\infty}-7+\lim_{x\to-\infty}\frac{8}{x}=0-7+0=-7$$

e) $\displaystyle\lim_{x\to\infty}\left(-2x^3+1-\frac{5}{x}+\frac{12}{x^4}\right)$

Solution: This limit is $-\infty$ since the first term approaches negative infinity, the second term approaches 1 and the other two terms aproach zero as x approaches infinity. Using mathematical notation,

$$\lim_{x\to\infty}\left(-2x^3+1-\frac{5}{x}+\frac{12}{x^4}\right)=\lim_{x\to\infty}\left(-2x^3\right)+\lim_{x\to\infty}1+\lim_{x\to\infty}\left(-\frac{5}{x}\right)+\lim_{x\to\infty}\left(\frac{12}{x^4}\right)=-\infty+1+0+0=-\infty$$

f) $\displaystyle\lim_{x\to-\infty}\frac{3x-2}{x}$

Solution: This problem is similar to the previous problems after a bit of algebra. We simply divide by x and then the limit becomes familiar.

$$\lim_{x\to-\infty}\frac{3x-2}{x}=\lim_{x\to-\infty}\left(\frac{3x}{x}-\frac{2}{x}\right)=\lim_{x\to-\infty}\left(3-\frac{2}{x}\right)=3$$

g) $\displaystyle\lim_{x\to\infty}\frac{-5x^3-2x+4}{x^2}$

Solution:

$$\begin{aligned}\lim_{x\to\infty}\frac{-5x^3-2x+4}{x^2}&=\lim_{x\to\infty}\left(\frac{-5x^3}{x^2}+\frac{-2x}{x^2}+\frac{4}{x^2}\right)=\lim_{x\to\infty}\left(-5x-\frac{2}{x}+\frac{4}{x^2}\right)\\&=\lim_{x\to\infty}(-5x)+\lim_{x\to\infty}\left(-\frac{2}{x}\right)+\lim_{x\to\infty}\left(\frac{4}{x^2}\right)=-\infty+0+0=-\infty\end{aligned}$$

h) $\displaystyle\lim_{x\to\infty}\frac{-5x^3-2x+4}{x^3}$

Solution:

$$\lim_{x\to\infty}\frac{-5x^3-2x+4}{x^3}=\lim_{x\to\infty}\left(\frac{-5x^3}{x^3}+\frac{-2x}{x^3}+\frac{4}{x^3}\right)=\lim_{x\to\infty}\left(-5-\frac{2}{x^2}+\frac{4}{x^3}\right)=-5$$

i) $\displaystyle\lim_{x\to\infty}\frac{-5x^3-2x+4}{x^4}$

Solution:

$$\lim_{x\to\infty}\frac{-5x^3-2x+4}{x^4}=\lim_{x\to\infty}\left(\frac{-5x^3}{x^4}+\frac{-2x}{x^4}+\frac{4}{x^4}\right)=\lim_{x\to\infty}\left(\frac{-5}{x}-\frac{2}{x^3}+\frac{4}{x^4}\right)=0$$

4. Compute each of the following limits.

a) $\displaystyle\lim_{x\to-\infty}\left(-2x^5-8x^4+7x^3-10\right)$

Solution: The first term, $-2x^5$ approaches infinity and the secomd term, $-8x^4$ approaches negative infinity. This does not give us enough information about the entire polynomial. A limit like this is called an **indeterminate**. We will bring this expression to a form that is not an indeterminate. In this case, factoring out the first term does the trick.

In case of a polynomial, the limits at infinity and negative infinity are completely determined by its leading term. Recall that the leading term is the highest degree term.

$$\lim_{x\to-\infty}\left(-2x^5-8x^4+7x^3-10\right)=\lim_{x\to-\infty}\left(-2x^5\right)$$

Here is the computation showing why this is true. We first factor out the entire leading term.

$$\begin{aligned}\lim_{x\to-\infty}\left(-2x^5-8x^4+7x^3-10\right) &= \lim_{x\to-\infty}\left(-2x^5\right)\left(1+\frac{4}{x}-\frac{7}{x^2}+\frac{5}{x^5}\right)\\ &= \lim_{x\to-\infty}\left(-2x^5\right)\cdot\lim_{x\to-\infty}\left(1+\frac{4}{x}-\frac{7}{x^2}+\frac{5}{x^5}\right)\\ &= \lim_{x\to-\infty}\left(-2x^5\right)\cdot 1=\lim_{x\to-\infty}\left(-2x^5\right)\end{aligned}$$

We can now easily determine that this limit is ∞. (See problem number 1.)

b) $\displaystyle\lim_{x\to\infty}\left(-2x^5-8x^4+7x^3-10\right)$

Solution: **In case of a polynomial, the limits at infinity and negative infinity are completely determined by its leading term.** Recall that the leading term is the highest degree term.

$$\lim_{x\to\infty}\left(-2x^5-8x^4+7x^3-10\right)=\lim_{x\to\infty}\left(-2x^5\right)$$

Here is the computation showing why this is true. We first factor out the entire leading term.

$$\begin{aligned}\lim_{x\to\infty}\left(-2x^5-8x^4+7x^3-10\right) &= \lim_{x\to\infty}\left(-2x^5\right)\left(1+\frac{4}{x}-\frac{7}{2x^2}+\frac{5}{x^5}\right)\\ &= \lim_{x\to\infty}\left(-2x^5\right)\cdot\lim_{x\to\infty}\left(1+\frac{4}{x}-\frac{7}{2x^2}+\frac{5}{x^5}\right)\\ &= \lim_{x\to\infty}\left(-2x^5\right)\cdot 1=\lim_{x\to\infty}\left(-2x^5\right)\end{aligned}$$

We can now easily determine that this limit is $-\infty$. (See problem number 1.)

c) $\displaystyle\lim_{x\to-\infty}\left(-2x^5+8x^6\right)$

Solution: **In case of a polynomial, the limits at infinity and negative infinity are completely determined by its leading term.**

$$\begin{aligned}\lim_{x\to-\infty}\left(-2x^5+8x^6\right) &= \lim_{x\to-\infty}8x^6=\infty \quad \text{because}\\ \lim_{x\to-\infty}\left(-2x^5+8x^6\right) &= \lim_{x\to-\infty}\left(8x^6-2x^5\right)=\lim_{x\to-\infty}\left(8x^6\right)\left(1-\frac{1}{4x}\right)=\lim_{x\to-\infty}\left(8x^6\right)\cdot\lim_{x\to-\infty}\left(1-\frac{1}{4x}\right)\\ &= \lim_{x\to-\infty}\left(8x^6\right)\cdot 1=\lim_{x\to-\infty}8x^6\end{aligned}$$

We can now easily determine that this limit is ∞. (See problem number 1.)

d) $\displaystyle\lim_{x\to\infty}\left(-2x^5+8x^6\right)$

Solution: **In case of a polynomial, the limits at infinity and negative infinity are completely determined by its leading term.**

$$\begin{aligned}
\lim_{x\to\infty}\left(-2x^5+8x^6\right) &= \lim_{x\to\infty} 8x^6=\infty \quad \text{because}\\
\lim_{x\to\infty}\left(-2x^5+8x^6\right) &= \lim_{x\to\infty}\left(8x^6-2x^5\right)=\lim_{x\to\infty}\left(8x^6\right)\left(1-\frac{1}{4x}\right)=\lim_{x\to\infty}\left(8x^6\right)\cdot\lim_{x\to\infty}\left(1-\frac{1}{4x}\right)\\
&= \lim_{x\to\infty}\left(8x^6\right)\cdot 1=\infty
\end{aligned}$$

We can now easily determine that this limit is ∞. (See problem number 1.)

5. Compute each of the following limits.

a) $\displaystyle\lim_{x\to-\infty}\frac{x+x^2-6}{6x+5x^2+2x^3}=0$

Solution: The numerator approaches infinity and the denominator approaches negative infinity. This does not give us enough information about the quotient. A limit like this is called an **indeterminate**. We will bring this expression to a form that is not an indeterminate. Let us rearrange the polynomials in the rational function given. Then we will factor out the leading term in the numerator and denominator.

$$\lim_{x\to-\infty}\frac{x^2+x-6}{2x^3+5x^2+6x}=\lim_{x\to-\infty}\frac{x^2\left(1+\dfrac{1}{x}-\dfrac{6}{x^2}\right)}{2x^3\left(1+\dfrac{5}{2x}+\dfrac{3}{x^2}\right)}$$

We now express the limit of the product as the product of two limits

$$\lim_{x\to-\infty}\frac{x^2\left(1+\dfrac{1}{x}-\dfrac{6}{x^2}\right)}{2x^3\left(1+\dfrac{5}{2x}+\dfrac{3}{x^2}\right)}=\lim_{x\to-\infty}\frac{x^2}{2x^3}\cdot\lim_{x\to-\infty}\frac{\left(1+\dfrac{1}{x}-\dfrac{6}{x^2}\right)}{\left(1+\dfrac{5}{2x}+\dfrac{3}{x^2}\right)}$$

The first expression can be simplified and thus has a limit we can easily determine its limit. The second expression, although looks unfriendly, is always going to approach 1.

$$\lim_{x\to-\infty}\frac{x^2}{2x^3}\cdot\lim_{x\to-\infty}\frac{\left(1+\dfrac{1}{x}-\dfrac{6}{x^2}\right)}{\left(1+\dfrac{5}{2x}+\dfrac{3}{x^2}\right)}=\lim_{x\to-\infty}\frac{1}{2x}\cdot 1=0\cdot 1=0$$

The entire computation should look like this:

$$\begin{aligned}
\lim_{x\to-\infty}\frac{x^2+x-6}{2x^3+5x^2+6x} &= \lim_{x\to-\infty}\frac{x^2\left(1+\dfrac{1}{x}-\dfrac{6}{x^2}\right)}{2x^3\left(1+\dfrac{5}{2x}+\dfrac{3}{x^2}\right)}=\lim_{x\to-\infty}\frac{x^2}{2x^3}\cdot\lim_{x\to-\infty}\left(\frac{1+\dfrac{1}{x}-\dfrac{6}{x^2}}{1+\dfrac{5}{2x}+\dfrac{3}{x^2}}\right)\\
&= \lim_{x\to-\infty}\frac{1}{2x}\cdot 1=0\cdot 1=0
\end{aligned}$$

b) $\lim_{x\to\infty} \dfrac{x^2+9}{5x+2x^2-3} = \dfrac{1}{2}$

Solution: Both numerator and denominator approach infinity. This does not give us enough information about the quotient. A limit like this is called an **indeterminate**. We will bring this expression to a form that is not an indeterminate. Let us rearrange the polynomials in the rational function given. Then we will factor out the leading term in the numerator and denominator.

$$\begin{aligned}
\lim_{x\to\infty} \frac{x^2+9}{2x^2+5x-3} &= \lim_{x\to\infty} \frac{x^2\left(1+\dfrac{9}{x^2}\right)}{2x^2\left(1+\dfrac{5}{2x}-\dfrac{3}{2x^2}\right)} = \lim_{x\to\infty} \frac{x^2}{2x^2} \cdot \lim_{x\to\infty} \frac{1+\dfrac{9}{x^2}}{1+\dfrac{5}{2x}-\dfrac{3}{2x^2}} \\
&= \lim_{x\to\infty} \frac{1}{2} \cdot \lim_{x\to\infty} \frac{1+\dfrac{9}{x^2}}{1+\dfrac{5}{2x}-\dfrac{3}{2x^2}} = \frac{1}{2}\cdot 1 = \frac{1}{2}
\end{aligned}$$

c) $\lim_{x\to-\infty} \dfrac{x^3-9x+1}{3x^2-2x-15} = -\infty$

Solution:

$$\begin{aligned}
\lim_{x\to-\infty} \frac{x^3-9x+1}{3x^2-2x-15} &= \lim_{x\to-\infty} \frac{x^3\left(1-\dfrac{9}{x^2}+\dfrac{1}{x^3}\right)}{3x^2\left(1-\dfrac{2}{3x}-\dfrac{5}{x^2}\right)} = \lim_{x\to-\infty} \frac{x^3}{3x^2} \cdot \lim_{x\to-\infty} \frac{1-\dfrac{9}{x^2}+\dfrac{1}{x^3}}{\left(1-\dfrac{2}{3x}-\dfrac{5}{x^2}\right)} \\
&= \left(\lim_{x\to-\infty} \frac{x^3}{3x^2}\right)\cdot 1 = \left(\lim_{x\to-\infty} \frac{x}{3}\right)\cdot 1 = -\infty\cdot 1 = -\infty
\end{aligned}$$