Definition: **The symbol** $\log_a \boldsymbol{b}$ **represents the number that if we write as an exponent of** a**, we achieve** b**.** This expression is only meaningful if both a and b are positive numbers and $a \neq 1$.

Every logarithmic statement can be re-written as an exponential statement.

$$\log_a b = x \text{ is the same as } a^x = b$$

Rules of Logarithms

1. $\log_a (a^x) = x$
2. $a^{\log_a x} = x$
3. When both sides exist, $\log_a b + \log_a c = \log_a bc$
4. When both sides exist, $\log_a b - \log_a c = \log_a \left(\dfrac{b}{c}\right)$
5. When both sides exist, $x \log_a b = \log_a (b^x)$
6. The change-base theorem: $\log_a b = \dfrac{\log_c b}{\log_c a}$

Sample Problems

1. Simplify each of the following expressions.

a) $\log_6 4 + \log_6 54$

b) $1 + 2\log_2 3 - \log_2 36$

c) $2\log_{10}(2x) + \log_{10} 25x$

d) $\log 21 - \dfrac{1}{2}\log 28 - \log 15 - \log\sqrt{700}$

e) $2\log_2\left(2x^5\right) - \log_4\left(144x^8\right) + \dfrac{1}{3}\log_2\left(216x^6\right)$

f) $\log_a\left(\left(3 - \dfrac{3a-2}{a+1}\right)\cdot\dfrac{a^2+a}{5}\right)$

g) $\log_9\sqrt{27}$

h) $\log_{\sqrt{m}}\sqrt[3]{m^7}$

i) $e^{-3\ln 5}$

j) $8^{\log_2 x}$

k) $3^{\log_9 x}$

l) $(\log_3 4)(\log_4 5)(\log_5 6)(\log_6 7)(\log_7 8)(\log_8 9)$

2. Which of the following is NOT equivalent to $\log_8\left(\dfrac{50}{3}\right)$?

A) $\dfrac{\ln\left(\dfrac{50}{3}\right)}{\ln 8}$ B) $\dfrac{\ln 50 - \ln 3}{\ln 8}$ C) $\dfrac{\ln 50 - \ln 3}{3\ln 2}$ D) $\dfrac{2\ln 5 + \ln 2 - \ln 3}{3\ln 2}$ E) $\dfrac{2\ln 5 - \ln 3}{3}$

3. Prove that $\log_{(8/15)}\left(\dfrac{24}{25}\right) = \dfrac{3\ln 2 + \ln 3 - 2\ln 5}{3\ln 2 - \ln 3 - \ln 5}$.

4. Let $x = \log_3 2$. Express each of the following in terms of x.

a) $\log_3 6$

b) $\log_3 18$

c) $\log_3 12$

d) $\log_3 24$

e) $\log_3 72$

f) $\log_2 3$

g) $\log_{12} 24$

h) $\log_3\left(\dfrac{2}{3}\right)$

i) $\log_3\left(\dfrac{9}{8}\right)$

j) $\log_{72} 24$

5. a) Suppose that $\log_2 6 = a$ and $\log_8 5 = b$. Express $\log_{10} 144$ in terms of a and b.

b) Let $a = \log_3 75$ and $b = \log_2 27$. Express $\log_3 10$ in terms of a and b.

6. a) Simplify $\dfrac{\log_3 90}{\log_{30} 3} - \dfrac{\log_3 270}{\log_{10} 3}$

b) Write $\log_2 5 - \log_4 10$ as a single logarithm.

7. a) Prove that $\log_{\left(a^k\right)}\left(b^k\right) = \log_a b$.

b) Prove that $\log_{a/b}\left(\dfrac{c}{d}\right) = \log_{b/a}\left(\dfrac{d}{c}\right)$.

8. Find the domain of each of the following expressions.

a) $\log_3\left(x^2 - 16\right)$

b) $\log_3(x+4) + \log_3(x-4)$

c) $\dfrac{1}{\ln(x-3)}$

d) $\log_3\left(x^2+1\right)$

e) $\dfrac{1}{\log_3(2x-1) - 4}$

f) $\dfrac{3}{\log_{10}\left(2x - x^2\right)}$

9. Solve each of the following equations.

a) $\log_2(x-3)(x+1)=5$

b) $\log_2(x-3)+\log_2(x+1)=5$

c) $\log_2(x+29)-\log_2(x-3)=1$

d) $\log_6 2+\log_6(2x-5)+\log_6(x-5)=2$

e) $\log_2(x-3)-\log_2(x+1)=1$

f) $\left[64^{\frac{2}{3}}\cdot 3^{-\log_{27}8}\right]^{\frac{1}{3}}+\log_2 x^3=14$

g) $\log_{64}x+\log_x 64=\dfrac{13}{6}$

10. (Enrichment) Solve each of the following equations.

a) $\log_{2x}16+\log_{4x}8=\log_x 8$

b) $x(1-\log_{21}3)=\log_{21}30-\log_{21}(7^x+1)$

c) $\log_x(x-3)\cdot\log_{x-3}(x+20)=2$

Practice Problems

1. Simplify each of the following expressions.

a) $\log_{10}5+\log_{10}2$

b) $\log_4 320-\log_4 5$

c) $\log_2(40a)-\log_2 5a$

d) $\log_{10}0.0002+\log_{10}5$

e) $4^{\log_2 a}$

f) $2^{\log_4 y}$

g) $2\ln\sqrt{x^2-1}-\ln(x+1)$

h) $\log_6\sqrt{12}+\log_6\sqrt{18}$

i) $\log_5(3x)+\log_5(15x^2)-2\log_5 3$

j) $2\ln\sqrt{m}+3\ln\sqrt[3]{m}$

k) $2\log_3(2A^5)-\log_9(144A^8)$

l) $\log_3(12b^2)-2\log_3(2b)$

m) $\log\sqrt{52}+3\log 2+\log 125+\log\sqrt{325}-\log 13$

2. Which of the following is NOT equivalent to $\log_9\left(\dfrac{36}{25}\right)$?

A) $\dfrac{\ln\left(\frac{36}{25}\right)}{\ln 9}$ B) $\dfrac{\ln\left(\frac{6}{5}\right)}{2\ln 3}$ C) $\dfrac{\ln 36-\ln 25}{2\ln 3}$ D) $\dfrac{\ln 36-\ln 25}{\ln 9}$ E) $\dfrac{\ln 6-\ln 5}{\ln 3}$

3. Let $x=\log_2 5$. Express each of the following in terms of x.

a) $\log_2 125$ c) $\log_2 1000$ e) $\log_2\left(\dfrac{5}{2}\right)$ g) $\log_{10}2$ i) $\log_{50}80$ k) $\log_2\left(\dfrac{16}{25}\right)$

b) $\log_2 10$ d) $\log_2 80$ f) $\log_5 2$ h) $\log_5 10$ j) $\log_{100}10$ l) $\log_5\left(\dfrac{16}{25}\right)$

4. Let $p=\log_2 5$ and $q=\log_5 3$. Express each of the following in terms of p and q.

a) $\log_2 10$ c) $\log_5 10$ e) $\log_5 30$ g) $\log_2 24$

b) $\log_5 45$ d) $\log_5 6$ f) $\log_2 3$ h) $\log_{24}30$

5. Find the domain of each of the following expressions.

a) $\log_5(-x^2+10x-23)$

b) $\log_2(x^2-6x+8)$

c) $\dfrac{1}{\log_2(x^2-6x+8)}$

d) $\dfrac{1}{\log_2(x-2)+\log_2(x-4)}$

e) $\ln\left(\dfrac{x^2-6x}{4-x^2}\right)$

f) $\ln(x^2-6x)-\ln(4-x^2)$

g) $\dfrac{\ln\left(x^2-6x\right)}{\ln\left(4-x^2\right)}$

6. Solve each of the following equations.

a) $\log_6(8-x)+\log_6(x+12)=2$

b) $\log_4(3m+5)-\log_4(m+7)=\dfrac{1}{2}$

c) $\log_2(3x-5)+\log_2(x-6)=4$

d) $\log_2(2-y)+\log_2(10-y)=7$

e) $\log_x(12-x)=2$

f) $\log_{x-1}(x+2)+\log_{x-1}(x-2)=2$

g) $\log_2 x+\log_2(x-4)=5$

h) $\log_4(x-1)+\log_4(x+3)=\dfrac{5}{2}$

i) $\log_6 x+\log_6(2x+1)=2$

j) $\log_2(x-5)+\log_2(x+11)=9$

Sample Problems - Answers

1.) a) 3 b) -1 c) $2+3\log_{10}x$ d) -2 e) $1+8\log_2 x$ f) 1 g) $\dfrac{3}{4}$ h) $\dfrac{14}{3}$ i) $\dfrac{1}{125}$
j) x^3 k) $\sqrt{x}$ l) 2 2.) E 3.) see solutions

4.) a) $x+1$ b) $x+2$ c) $2x+1$ d) $3x+1$ e) $3x+2$ f) $\dfrac{1}{x}$ g) $\dfrac{3x+1}{2x+1}$
h) $x-1$ i) $2-3x$ j) $\dfrac{3x+1}{3x+2}$

5.) a) $\dfrac{2(a+1)}{3b+1}$ b) $\dfrac{3}{b}+\dfrac{a-1}{2}=\dfrac{ab-b+6}{2b}$ 6.) a) 3 b) $\log_2\left(\dfrac{\sqrt{10}}{2}\right)$ 7.) see solutions

8.) a) $\{x|x<-4 \text{ or } x>4\}$ in interval notation: $(-\infty,-4)\cup(4,\infty)$
b) $\{x|x>4\}$ in interval notation: $(4,\infty)$ c) $\{x|x>3 \text{ but } x\neq 4\}$ in interval notation: $(3,\infty)\setminus\{4\}$
d) $\mathbb{R}$ e) $\left\{x|x>\dfrac{1}{2} \text{ but } x\neq 41\right\}$ in interval notation: $\left(\dfrac{1}{2},\infty\right)\setminus\{41\}$
f) $\{x|\ 0<x<2 \text{ but } x\neq 1\}$ in interval notation: $(0,2)\setminus\{1\}$

9.) a) $-5,\ 7$ b) 7 c) 35 d) 7 e) no solution f) 16 g) $16,\ 512$ 10.) a) $\dfrac{1}{\sqrt{8}},2$ b) $\log_7 5$ c) 5

Practice Problems - Answers

1.) a) 1 b) 3 c) 3 d) -3 e) a^2 f) $\sqrt{y}$ g) $x-1$ h) $\dfrac{3}{2}$ i) $1+3\log_5 x$
j) $2\ln m$ k) $-1+6\log_3 A$ l) 1 m) 4 2.) B

3.) a) $3x$ b) $x+1$ c) $3x+3$ d) $x+4$ e) $x-1$ f) $\dfrac{1}{x}$ g) $\dfrac{1}{x+1}$ h) $\dfrac{x+1}{x}$
i) $\dfrac{x+4}{2x+1}$ j) $\dfrac{1}{2}$ k) $4-2x$ l) $\dfrac{4-2x}{x}=\dfrac{4}{x}-2$

4.) a) $p+1$ b) $2q+1$ c) $\dfrac{1}{p}+1$ d) $\dfrac{1}{p}+q$ e) $\dfrac{1}{p}+q+1$ f) pq g) $pq+3$

h) $\dfrac{\dfrac{1}{p}+q+1}{\dfrac{3}{p}+q}=\dfrac{p+pq+1}{pq+3}$

5.) a) $\left\{x \mid 5-\sqrt{2}<x<5+\sqrt{2}\right\}$ - in interval notation: $\left(5-\sqrt{2}, 5+\sqrt{2}\right)$

b) $\{x \mid x<2 \text{ or } x>4\}$ - in interval notation: $(-\infty, 2) \cup (4, \infty)$

c) $\left\{x<2 \text{ and } x \neq 3-\sqrt{2} \text{ or } x>4 \text{ and } x \neq 3+\sqrt{2}\right\}$

in interval notation: $(-\infty, 2) \cup (4, \infty) \setminus \left\{3-\sqrt{2}, 3+\sqrt{2}\right\}$

d) $\left\{x>4 \text{ and } x \neq 3+\sqrt{2}\right\}$ - in interval notation: $(4, \infty) \setminus \left\{3+\sqrt{2}\right\}$

e) $\{x \mid -2<x<0 \text{ or } 2<x<6\}$ -in interval notation: $(-2,0) \cup (2,6)$

f) $\{x \mid -2<x<0\}$ in interval notation: $(-2,0)$

g) $\left\{-2<x<0 \text{ and } x \neq -\sqrt{3}\right\}$ in interval notation: $(-2,0) \setminus \left\{-\sqrt{3}\right\}$

6.) a) $-10, 6$ b) 9 c) 7 d) -6 e) 3 f) $\dfrac{5}{2}$ g) 8 h) 5 i) 4 j) 21

Sample Problems - Solutions

1. Simplify each of the following expressions.

a) $\log_6 4 + \log_6 54 = \log_6 (4 \cdot 54) = \log_6 216 = 3$

b) $1 + 2\log_2 3 - \log_2 36$

Solution: We re-write each expression as a single base 2 logarithm. We will use by the rule $n \log_a b = \log_a (b^n)$

$$1 = \log_2 2 \ \text{ and } 2\log_2 3 = \log_2 3^2 = \log_2 9$$

$$1 + 2\log_2 3 - \log_2 36 = \log_2 2 + \log_2 9 - \log_2 36 = \log_2 \left(\frac{2 \cdot 9}{36}\right) = \log_2 \left(\frac{1}{2}\right) = -1$$

c) $2\log_{10} (2x) + \log_{10} (25x)$

Solution: by the rule $n \log_a b = \log_a (b^n)$, we have $2\log_{10} (2x) = \log_{10} \left[(2x)^2\right] = \log_{10} \left(4x^2\right)$

$$\begin{aligned} 2\log_{10} (2x) + \log_{10} (25x) &= \log_{10} \left(4x^2\right) + \log_{10} (25x) = \log_{10} \left(4x^2 \cdot 25x\right) = \log_{10} \left(100x^3\right) \\ &= \log_{10} 100 + \log_{10} \left(x^3\right) = 2 + 3\log_{10} x \end{aligned}$$

d) $\log 21 - \frac{1}{2}\log 28 - \log 15 - \log \sqrt{700}$

Solution: Note that $\log 21$ is the same as $\log_{10} 21$

$$\begin{aligned} E &= \log 21 - \frac{1}{2}\log 28 - \log 15 - \log \sqrt{700} = \log 21 - \log \sqrt{28} - \log 15 - \log \sqrt{700} \\ &= \log 21 - \left(\log \sqrt{28} + \log 15 + \log \sqrt{700}\right) = \log 21 - \left(\log \sqrt{28} \cdot 15 \cdot \sqrt{700}\right) \\ &= \log \frac{21}{\sqrt{28} \cdot 15 \cdot \sqrt{700}} = \log \frac{21}{2\sqrt{7} \cdot 15 \cdot 10\sqrt{7}} = \log \frac{3 \cdot 7}{2 \cdot 7 \cdot 15 \cdot 10} = \log \frac{3}{300} = \log \frac{1}{100} = -2 \end{aligned}$$

e) $2\log_2 \left(2x^5\right) - \log_4 \left(144x^8\right) + \frac{1}{3}\log_2 \left(216x^6\right)$

Solution: We can combine the expressions only if they are simple logarithms of the same base. Recall the rule $n \log_a b = \log_a (b^n)$

$$2\log_2 \left(2x^5\right) = \log_2 \left(2x^5\right)^2 = \log_2 \left(4x^{10}\right)$$

We change the second expression to base 2.

$$\log_4 \left(144x^8\right) = \frac{\log_2 \left(144x^8\right)}{\log_2 4} = \frac{\log_2 \left(144x^8\right)}{2} = \frac{1}{2}\log_2 \left(144x^8\right)$$

and we use the rule $n \log_a b = \log_a (b^n)$ to get rid of the coefficient

$$\frac{1}{2}\log_2 \left(144x^8\right) = \log_2 \left[\left(144x^8\right)^{1/2}\right] = \log_2 \sqrt{144x^8} = \log_2 \left(12x^4\right)$$

We similarly get rid of $\frac{1}{3}$ in the third expression:

$$\frac{1}{3}\log_2 \left(216x^6\right) = \log_2 \left[\left(216x^6\right)^{1/3}\right] = \log_2 \sqrt[3]{216x^6} = \log_2 \left(6x^2\right)$$

We are now ready to simplify the expression:

$$\begin{aligned} E &= 2\log_2 \left(2x^5\right) - \log_4 \left(144x^8\right) + \frac{1}{3}\log_2 \left(216x^6\right) \\ &= \log_2 \left(4x^{10}\right) - \log_2 \left(12x^4\right) + \log_2 \left(6x^2\right) \end{aligned}$$

And now we use $\log_a b - \log_a c = \log_a \left(\frac{b}{c}\right)$

$$= \log_2 \left(\frac{4x^{10}}{12x^4}\right) + \log_2 \left(6x^2\right) = \log_2 \left(\frac{x^6}{3}\right) + \log_2 \left(6x^2\right)$$

And now we use $\log_a b + \log_a c = \log_a (bc)$

$$= \log_2 \left(\frac{x^6}{3}\right)\left(6x^2\right) = \log_2 \frac{x^6\left(6x^2\right)}{3} = \log_2 2x^8$$

Now we use $\log_a b + \log_a c = \log_a (bc)$ and $n \log_a b = \log_a b^n$ again.

$$\log_2 2x^8 = \log_2 2 + \log_2 x^8 = 1 + 8\log_2 x$$

f) $\log_a \left(\left(3 - \frac{3a-2}{a+1}\right) \cdot \frac{a^2+a}{5}\right)$

Solution:

$$\begin{aligned} E &= \log_a \left(\left(3 - \frac{3a-2}{a+1}\right) \cdot \frac{a^2+a}{5}\right) = \log_a \left(\left(\frac{3(a+1)}{a+1} - \frac{3a-2}{a+1}\right) \cdot \frac{a^2+a}{5}\right) \\ &= \log_a \left(\frac{3(a+1)-(3a-2)}{a+1} \cdot \frac{a(a+1)}{5}\right) = \log_a \left(\frac{3a+3-3a+2}{1} \cdot \frac{a}{5}\right) = \log_a \left(5 \cdot \frac{a}{5}\right) = \log_a a = 1 \end{aligned}$$

g) $\log_9 \sqrt{27}$

Solution: We have seen problems like this in the previous logarithms lecture notes (logarithms 1) but the change base theorem makes solving it much easier. We simply switch to base 3.

$$\log_9 \sqrt{27} = \frac{\log_3 \sqrt{27}}{\log_3 9} = \frac{\frac{3}{2}}{2} = \frac{3}{4}$$

h) $\log_{\sqrt{m}} \sqrt[3]{m^7}$

Solution: We will switch to base m.

$$\log_{\sqrt{m}} \sqrt[3]{m^7} = \frac{\log_m \sqrt[3]{m^7}}{\log_m \sqrt{m}} = \frac{\log_m \left(m^{7/3}\right)}{\log_m \left(m^{1/2}\right)} = \frac{\frac{7}{3}}{\frac{1}{2}} = \frac{7}{3} \cdot \frac{2}{1} = \frac{14}{3}$$

i) $e^{-3\ln 5}$

Solution: Recall that $a^{\log_a b} = b$. Thus $e^{\ln x} = x$.

$$e^{-3\ln 5} = \left(e^{\ln 5}\right)^{-3} = 5^{-3} = \frac{1}{125}$$

j) $8^{\log_2 x}$

Solution: Recall that $a^{\log_a b} = b$. Thus $2^{\log_2 x} = x$

$$8^{\log_2 x} = \left(2^3\right)^{\log_2 x} = 2^{3\log_2 x} = \left(2^{\log_2 x}\right)^3 = x^3$$

This proble is about matching the base of the exponentiation with the base of the logarithm. There is another way of solving this problem now that we have the switch-base theorem. We can switch to base 8.

$$\log_2 x = \frac{\log_8 x}{\log_8 2} = \frac{\log_8 x}{\frac{1}{3}} = 3\log_8 x \quad \text{and so} \quad 8^{\log_2 x} = 8^{3\log_8 x} = \left(8^{\log_8 x}\right)^3 = x^3$$

k) $3^{\log_9 x} = \left(9^{1/2}\right)^{\log_9 x} = (9)^{\frac{1}{2}\log_9 x} = \left(9^{\log_9 x}\right)^{\frac{1}{2}} = x^{\frac{1}{2}} = \sqrt{x}$

Solution: We can either change the base of exponentiation

$$3^{\log_9 x} = \left(9^{1/2}\right)^{\log_9 x} = (9)^{\frac{1}{2}\log_9 x} = \left(9^{\log_9 x}\right)^{\frac{1}{2}} = x^{\frac{1}{2}} = \sqrt{x}$$

or change the base of the logarithm:

$$\log_9 x = \frac{\log_3 x}{\log_3 9} = \frac{\log_3 x}{2} = \frac{1}{2}\log_3 x$$

$$3^{\log_9 x} = 3^{(1/2)\log_3 x} = \left(3^{\log_3 x}\right)^{1/2} = x^{\frac{1}{2}} = \sqrt{x}$$

l) $(\log_3 4)(\log_4 5)(\log_5 6)(\log_6 7)(\log_7 8)(\log_8 9)$

Solution: We use the change-base formula for logarithms to re-write the expression

$$\log_a b = \frac{\ln b}{\ln a}$$

$$\begin{aligned} E &= (\log_3 4)(\log_4 5)(\log_5 6)(\log_6 7)(\log_7 8)(\log_8 9) \\ &= \frac{\ln 4}{\ln 3}\cdot\frac{\ln 5}{\ln 4}\cdot\frac{\ln 6}{\ln 5}\cdot\frac{\ln 7}{\ln 6}\cdot\frac{\ln 8}{\ln 7}\cdot\frac{\ln 9}{\ln 8} = \frac{\ln 9}{\ln 3} = \frac{\ln 3^2}{\ln 3} = \frac{2\ln 3}{\ln 3} = 2 \end{aligned}$$

2. Which of the following is NOT equivalent to $\log_8\left(\frac{50}{3}\right)$?

A) $\dfrac{\ln\left(\frac{50}{3}\right)}{\ln 8}$ B) $\dfrac{\ln 50 - \ln 3}{\ln 8}$ C) $\dfrac{\ln 50 - \ln 3}{3\ln 2}$ D) $\dfrac{2\ln 5 + \ln 2 - \ln 3}{3\ln 2}$ E) $\dfrac{2\ln 5 - \ln 3}{3}$

Solution: Let us use the change base theorem to change to natural base.

$$\log_8\left(\frac{50}{3}\right) = \frac{\ln\left(\frac{50}{3}\right)}{\ln 8} \qquad \text{which is choice A}$$

Now we use the rule $\log_a b - \log_a c = \log_a\left(\frac{b}{c}\right)$ in the numerator.

$$\frac{\ln\left(\frac{50}{3}\right)}{\ln 8} = \frac{\ln 50 - \ln 3}{\ln 8} \qquad \text{which is choice B}$$

Now we use the rule $n\log_a b = \log_a\left(b^n\right)$ in the denominator.

$$\frac{\ln 50 - \ln 3}{\ln 8} = \frac{\ln 50 - \ln 3}{\ln 2^3} = \frac{\ln 50 - \ln 3}{3\ln 2} \qquad \text{which is choice C}$$

Now we apply the rules $n\log_a b = \log_a(b^n)$ and $\log_a b + \log_a c = \log_a(bc)$ in the denominator to re-write $\ln 50$.

$$\begin{aligned}\ln 50 &= \ln\left(2\cdot 5^2\right) = \ln 2 + \ln\left(5^2\right) = \ln 2 + 2\ln 5\\ \frac{\ln 50 - \ln 3}{3\ln 2} &= \frac{2\ln 5 + \ln 2 - \ln 3}{3\ln 2} \quad \text{which is choice D}\end{aligned}$$

At this point, choice E is the only expression possibly not equaivalent to $\log_8\left(\dfrac{50}{3}\right)$. Indeed, choice E represents a serious algebraic error in simplifying our expression in D. Since

$$\frac{a+b-c}{3b} \neq \frac{a-c}{3}$$

E is NOT equivalent to the other expressions. One way to verify this is to enter these expressions into the calculator and see that the decimal approximations are all the same for $\log_8\left(\dfrac{50}{3}\right)$ and choices A, B, C, and D, but different for E.

3. Prove that $\log_{(8/15)}\left(\dfrac{24}{25}\right) = \dfrac{3\ln 2 + \ln 3 - 2\ln 5}{3\ln 2 - \ln 3 - \ln 5}$.

Solution: We first switch to natural logarithm.

$$\log_{(8/15)}\left(\frac{24}{25}\right) = \frac{\ln\left(\frac{24}{25}\right)}{\ln\left(\frac{8}{15}\right)} = \frac{\ln 24 - \ln 25}{\ln 8 - \ln 15} = \frac{\ln\left(2^3\cdot 3\right) - \ln\left(5^2\right)}{\ln\left(2^3\right) - \ln(3\cdot 5)} = \frac{\ln\left(2^3\right) + \ln 3 - 2\ln 5}{3\ln 2 - (\ln 3 + \ln 5)} = \frac{3\ln 2 + \ln 3 - 2\ln 5}{3\ln 2 - \ln 3 - \ln 5}$$

4. Let $x = \log_3 2$. Express each of the following in terms of x.

a) $\log_3 6$

Solution: Recall that $\log_a(bc) = \log_a b + \log_a c$

$$\log_3 6 = \log_3(3\cdot 2) = \log_3 3 + \log_3 2 = 1 + x = \boxed{x+1}$$

b) $\log_3 18$

Solution: Recall that $\log_a(bc) = \log_a b + \log_a c$

$$\log_3 18 = \log_3(9\cdot 2) = \log_3 9 + \log_3 2 = 2 + x = \boxed{x+2}$$

c) $\log_3 12$

Solution: Recall that $\log_a(bc) = \log_a b + \log_a c$ and $\log_a(b^n) = n\log_a b$

$$\log_3 12 = \log_3(3\cdot 4) = \log_3\left(3\cdot 2^2\right) = \log_3 3 + \log_3\left(2^2\right) = 1 + 2\log_3 2 = 1 + 2x = \boxed{2x+1}$$

d) $\log_3 24$

Solution: Recall that $\log_a(bc) = \log_a b + \log_a c$ and $\log_a(b^n) = n\log_a b$

$$\log_3 24 = \log_3(3\cdot 8) = \log_3\left(3\cdot 2^3\right) = \log_3 3 + \log_3\left(2^3\right) = 1 + 3\log_3 2 = 1 + 3x = \boxed{3x+1}$$

e) $\log_3 72$

Solution: Recall that $\log_a(bc) = \log_a b + \log_a c$ and $\log_a(b^n) = n\log_a b$

$$\log_3 72 = \log_3(9\cdot 8) = \log_3\left(3^2\cdot 2^3\right) = \log_3\left(3^2\right) + \log_3\left(2^3\right) = 2 + 3\log_3 2 = 2 + 3x = \boxed{3x+2}$$

f) $\log_2 3$

Solution: We will change base to 3 using the change base formula. Recall that $\log_a b = \dfrac{\log_c b}{\log_c a}$.

$$\log_2 3 = \frac{\log_3 3}{\log_3 2} = \frac{1}{\log_3 2} = \boxed{\frac{1}{x}}$$

Note that this is a useful piece of information: if we swap the two numbers in a logarithm, we obtain the opposite of the original logarithm. In short, $\log_x y$ and $\log_y x$ are reciprocals.

g) $\log_{12} 24 = \dfrac{\log_3 24}{\log_3 12} = \dfrac{3x+1}{2x+1}$

Solution: We will change base to 3 using the change base formula. Recall that $\log_a b = \dfrac{\log_c b}{\log_c a}$. Also note that the expressions $\log_3 24$ and $\log_3 12$ were already worked out in previous problems.

$$\log_{12} 24 = \frac{\log_3 24}{\log_3 12} = \boxed{\frac{3x+1}{2x+1}}$$

h) $\log_3 \left(\dfrac{2}{3}\right)$

Solution: Recall that $\log_a \left(\dfrac{b}{c}\right) = \log_a b - \log_a c$

$$\log_3 \left(\frac{2}{3}\right) = \log_3 2 - \log_3 3 = \boxed{x-1}$$

i) $\log_3 \left(\dfrac{9}{8}\right)$

$$\log_3 \left(\frac{9}{8}\right) = \log_3 9 - \log_3 8 = 2 - \log_3 \left(2^3\right) = 2 - 3\log_3 2 = 2 - 3x = \boxed{-3x+2}$$

Solution: Recall that $\log_a \left(\dfrac{b}{c}\right) = \log_a b - \log_a c$ and $\log_a \left(b^n\right) = n \log_a b$

j) $\log_{72} 24 = \dfrac{\log_3 24}{\log_3 72} = \dfrac{3x+1}{3x+2}$

Solution: We will change base to 3 using the change base formula. Recall that $\log_a b = \dfrac{\log_c b}{\log_c a}$. Also note that the expressions $\log_3 24$ and $\log_3 72$ were already worked out in previous problems.

$$\log_{72} 24 = \frac{\log_3 24}{\log_3 72} = \boxed{\frac{3x+1}{3x+2}}$$

5. a) Suppose that $\log_2 6 = a$ and $\log_8 5 = b$. Express $\log_{10} 144$ in terms of a and b.
Solution: We will only need to find $\log_2 3$ and $\log_2 5$ in terms of a and b.

$$\begin{aligned} a &= \log_2 6 = \log_2 (2 \cdot 3) = \log_2 2 + \log_2 3 = 1 + \log_2 3 \\ a &= \log_2 3 + 1 \quad \Longrightarrow \quad \log_2 3 = a - 1 \\ b &= \log_8 5 = \frac{\log_2 5}{\log_2 8} = \frac{\log_2 5}{3} \quad \Longrightarrow \quad \log_2 5 = 3b \end{aligned}$$

$$\begin{aligned}\log_{10} 144 &= \frac{\log_2 144}{\log_2 10} = \frac{\log_2 (16\cdot 9)}{\log_2 (2\cdot 5)} = \frac{\log_2 16 + \log_2 9}{\log_2 2 + \log_2 5} = \frac{4+\log_2 3^2}{1+\log_2 5} = \frac{4+2\log_2 3}{1+\log_2 5}\\ &= \frac{4+2(a-1)}{1+3b} = \frac{4+2a-2}{3b+1} = \frac{2a+2}{3b+1}\end{aligned}$$

b) Let $a=\log_3 75$ and $b=\log_2 27$. Express $\log_3 10$ in terms of a and b.
Solution:

$$a=\log_3 75 = \log_3(3\cdot 25) = \log_3 3 + \log_3 25 = 1+\log_3 5^2 = 1+2\log_3 5$$

$$\begin{aligned} a &= 1+2\log_3 5\\ a-1 &= 2\log_3 5 \quad\Longrightarrow\quad \log_3 5 = \frac{a-1}{2}\end{aligned}$$

$$b=\log_2 27 = \frac{\log_3 27}{\log_3 2} = \frac{3}{\log_3 2} \quad\Longrightarrow\quad b=\frac{3}{\log_3 2} \quad\Longrightarrow\quad b\log_3 2 = 3 \quad\Longrightarrow\quad \log_3 2 = \frac{3}{b}$$

$$\log_3 10 = \log_3 2 + \log_3 5 = \frac{3}{b}+\frac{a-1}{2} = \frac{6}{2b}+\frac{(a-1)\,b}{2b} = \frac{6+ab-b}{2b}$$

6. a) Simplify $\dfrac{\log_3 90}{\log_{30} 3} - \dfrac{\log_3 270}{\log_{10} 3}$.

Solution: We first switch to base 3 and simplify the logarithms as much as possible.

$$\begin{aligned}\frac{\log_3 90}{\log_{30} 3} - \frac{\log_3 270}{\log_{10} 3} &= \frac{\log_3 90}{\dfrac{\log_3 3}{\log_3 30}} - \frac{\log_3 270}{\dfrac{\log_3 3}{\log_3 10}} = \frac{\log_3 9 + \log_3 10}{\dfrac{1}{\log_3 3+\log_3 10}} - \frac{\log_3 27+\log_3 10}{\dfrac{1}{\log_3 10}}\\ &= \frac{2+\log_3 10}{\dfrac{1}{1+\log_3 10}} - \frac{3+\log_3 10}{\dfrac{1}{\log_3 10}} = (2+\log_3 10)(1+\log_3 10) - (3+\log_3 10)\log_3 10\\ &= 2+2\log_3 10 + \log_3 10 + (\log_3 10)^2 - 3\log_3 10 - (\log_3 10)^2\\ &= 2+3\log_3 10 - 3\log_3 10 = 2\end{aligned}$$

b) Write $\log_2 5 - \log_4 10$ as a single logarithm.

Solution: We first switch to base 2.

$$\begin{aligned}\log_2 5 - \log_4 10 &= \log_2 5 - \frac{\log_2 10}{\log_2 4} = \log_2 5 - \frac{\log_2 10}{2} = \log_2 5 - \frac{1}{2}\log_2 10 = \log_2 5 - \log_2 (10)^{1/2} = \log_2 5 - \log_2 \sqrt{10}\\ &= \log_2\left(\frac{5}{\sqrt{10}}\right) = \log_2\left(\frac{5\sqrt{10}}{10}\right) = \log_2\left(\frac{\sqrt{10}}{2}\right)\end{aligned}$$

7. a) Prove that $\log_{(a^k)}\left(b^k\right) = \log_a b$.

Prove: We use the conversion formula for logarithms, to switch to base a.

$$\log_{(a^k)}\left(b^k\right) = \frac{\log_a\left(b^k\right)}{\log_a\left(a^k\right)} = \frac{k\log_a b}{k} = \log_a b$$

b) Prove that $\log_{a/b}\left(\dfrac{c}{d}\right) = \log_{b/a}\left(\dfrac{d}{c}\right)$.

Prove: We use the conversion formula for logarithms, to switch to the natural logarithm and then back. Also, we will use the following fact: $\dfrac{x-y}{z-w} = \dfrac{y-x}{w-z}$. This is true because

$$\frac{x-y}{z-w} = \frac{-1(-x+y)}{-1(-z+w)} = \frac{y-x}{w-z}$$

$$\log_{a/b}\left(\frac{c}{d}\right)=\frac{\ln\left(\frac{c}{d}\right)}{\ln\left(\frac{a}{b}\right)}=\frac{\ln c-\ln d}{\ln a-\ln b}=\frac{\ln d-\ln c}{\ln b-\ln a}=\frac{\ln\left(\frac{d}{c}\right)}{\ln\left(\frac{b}{a}\right)}=\log_{b/a}\left(\frac{d}{c}\right)$$

8. Find the domain of each of the following expressions.

a) $\log_3\left(x^2-16\right)$
Solution: We need to solve the inequality $x^2-16>0$. (If you need to review these, see Quadratic Inequalities.) The solution is $\{x|x<-4 \text{ or } x>4\}$ or in interval notation, $(-\infty,-4)\cup(4,\infty)$.

b) $\log_3(x+4)+\log_3(x-4)$
Solution: We need to solve the inequalities $x+4>0$ and $x-4>0$.

$$\begin{aligned} x+4 &> 0 \text{ and } x-4>0 \\ x &> -4 \text{ and } x>4 \quad\Longrightarrow\quad x>4 \end{aligned}$$

Thus the domain is $\{x|x>4\}$ or in interval notation, $(4,\infty)$

c) $\dfrac{1}{\ln(x-3)}$
Solution: for the expression $\ln(x-3)$ to be defined, we need that $x-3>0$, thus $x>3$. Now if x is greater than 3, $\ln(x-3)$ is defined but we still need to worry about division by zero. We have to rule out all values of x for which $\ln(x-3)=0$. So we solve the equation

$$\begin{aligned} \ln(x-3) &= 0 & 1&=x-3 \\ e^0 &= x-3 & 4&=x \end{aligned}$$

Thus the domain is: $\{x|x>3 \text{ but } x\neq 4\}$ or in interval notation, $(3,\infty)\setminus\{4\}$

d) $\log_3\left(x^2+1\right)$
Solution: for this logarithm to be defined, $x^2+1>0$ needs to be true. Since this inequality is true for all real numbers, this expression's domain is the set of all real numbers, $\mathbb{R}$.

e) $\dfrac{1}{\log_3(2x-1)-4}$
Solution: for $\log_3(2x-1)$ to be defined, $2x-1>0$ needs to be true. We solve this inequality and get that $x>\frac{1}{2}$. Even if the logarithm is defined, we still need to worry about division by zero. We have to rule out all values of x for which $\log_3(2x-1)-4=0$. We solve the equation

$$\begin{aligned} \log_3(2x-1)-4 &= 0 & 2x-1&=81 \\ \log_3(2x-1) &= 4 & 2x&=82 \\ 2x-1 &= 3^4 & x&=41 \end{aligned}$$

So the domain of this expression is $\left\{x|x>\frac{1}{2} \text{ but } x\neq 41\right\}$ or in interval notation, $\left(\frac{1}{2},\infty\right)\setminus\{41\}$.

f) $\dfrac{3}{\log_{10}\left(2x-x^2\right)}$
Solution: for $\log_{10}\left(2x-x^2\right)$ to be defined, $2x-x^2>0$ needs to be true. We solve this inequality and get that $0<x<2$. Even if the logarithm is defined, we still need to worry about division by zero. We have to rule out all values of x for which $\log_{10}\left(2x-x^2\right)=0$. We solve the equation

$$\begin{aligned} \log_{10}\left(2x-x^2\right) &= 0 & 0&=x^2-2x+1 \\ 2x-x^2 &= 10^0 & 0&=(x-1)^2 \\ 2x-x^2 &= 1 & x&=1 \end{aligned}$$

So the domain of this expression is $\{x|\ 0<x<2 \text{ but } x\neq 1\}$ or in interval notation, $(0,2)\setminus\{1\}$.

9. a) $\log_2 (x-3)(x+1) = 5$
Solution: We re-write the logarithmic statement as an exponential statement and then solve for x.

$$\begin{aligned} (x-3)(x+1) &= 2^5 \\ (x-3)(x+1) &= 32 \\ x^2-2x-3 &= 32 \end{aligned} \qquad \begin{aligned} x^2-2x-35 = 0 \\ (x+5)(x-7) = 0 \\ x_1 = -5 \qquad x_2 = 7 \end{aligned}$$

We check: If $x = -5$, then

$$\text{LHS} = \log_2 (-5-3)(-5+1) = \log_2 (-8)(-4) = \log_2 32 = 5 = \text{RHS}$$

and if $x = 7$, then

$$\text{LHS} = \log_2 (7-3)(7+1) = \log_2 (4)(8) = \log_2 32 = 5 = \text{RHS}$$

b) $\log_2 (x-3) + \log_2 (x+1) = 5$
Solution: $\log_2 (x-3) + \log_2 (x+1) = \log_2 (x-3)(x+1)$ and so this equation appears to be identical to the previous one. But it is not. Let's check:If $x = -5$, then

$$\text{LHS} = \log_2 (-5-3) + \log_2 (-5+1) = \log_2 (-8) + \log_2 (-4) = \text{undefined}$$

and if $x = 7$, then

$$\text{LHS} = \log_2 (7-3) + \log_2 (7+1) = \log_2 4 + \log_2 8 = 2 + 3 = 5 = \text{RHS}$$

and so this equation has only one solution, $x = 7$. So, it is very important to check.

c) $\log_2 (x+29) - \log_2 (x-3) = 1$
Solution:

$$\begin{aligned} \log_2 (x+29) - \log_2 (x-3) &= 1 \\ \log_2 \frac{x+29}{x-3} &= 1 \\ \frac{x+29}{x-3} &= 2^1 \end{aligned} \qquad \begin{aligned} x+29 &= 2(x-3) \\ x+29 &= 2x-6 \\ 35 &= x \end{aligned}$$

We check: if $x = 35$, then

$$\text{LHS} = \log_2 (35+29) - \log_2 (35-3) = \log_2 64 - \log_2 32 = 6 - 5 = 1 = \text{RHS}$$

d) $\log_6 2 + \log_6 (2x-5) + \log_6 (x-5) = 2$
Solution:

$$\begin{aligned} \log_6 2 + \log_6 (2x-5) + \log_6 (x-5) &= 2 \\ \log_6 2(2x-5)(x-5) &= 2 && \text{re-write as exponential} \\ 6^2 &= 2(2x-5)(x-5) \\ 36 &= 2(2x-5)(x-5) && \text{divide by 2} \\ 18 &= (2x-5)(x-5) \\ 18 &= 2x^2-15x+25 \\ 0 &= 2x^2-15x+7 && \text{factor} \\ 0 &= (2x-1)(x-7) && \Longrightarrow \quad x_1 = \frac{1}{2} \text{ and } x_2 = 7 \end{aligned}$$

We check: if $x = \frac{1}{2}$, then

$$\text{LHS} = \log_6 2 + \log_6 \left(2\left(\frac{1}{2}\right) - 5\right) + \log_6 \left(\frac{1}{2} - 5\right) = \log_6 2 + \log_6 (-4) + \log_6 (-4.5)$$

The left-hand side is undefined because the logarithm of negative numbers is NOT defined. Thus $x = \frac{1}{2}$ is NOT a solution. If $x = 7$, then

$$\text{LHS} = \log_6 2 + \log_6 (2 \cdot 7 - 5) + \log_6 (7 - 5) = \log_6 2 + \log_6 9 + \log_6 2 = \log_6 (2 \cdot 9 \cdot 2) = \log_6 36 = 2 = \text{RHS}$$

Thus $x = 7$ is the only solution.

e) $\log_2 (x - 3) - \log_2 (x + 1) = 1$
Solution:

$$\begin{aligned} \log_2 (x - 3) - \log_2 (x + 1) &= 1 \\ \log_2 \frac{x-3}{x+1} &= 1 \\ \frac{x-3}{x+1} &= 2 \end{aligned} \qquad \begin{aligned} x - 3 &= 2(x + 1) \\ x - 3 &= 2x + 2 \\ -5 &= x \end{aligned}$$

We check: if $x = -5$, then

$$\text{LHS} = \log_2 (-5 - 3) - \log_2 (-5 + 1) = \log_2 (-8) - \log_2 (-4) = \text{undefined}$$

The only number, -5 that could work with this equation, doesn't and so this equation has no solution.

f) $\left[64^{\frac{2}{3}} \cdot 3^{-\log_{27} 8}\right]^{\frac{1}{3}} + \log_2 x^3 = 14$
Solution:

$$\begin{aligned} 64^{\frac{2}{3}} &= \left(\sqrt[3]{64}\right)^2 = 4^2 = 16 \\ \log_{27} 8 &= \frac{\ln 8}{\ln 27} = \frac{\ln 2^3}{\ln 3^3} = \frac{3 \ln 2}{3 \ln 3} = \frac{\ln 2}{\ln 3} = \log_3 2 \\ 3^{-\log_{27} 8} &= 3^{-\log_3 2} = \frac{1}{3^{\log_3 2}} = \frac{1}{3^{\log_3 2}} = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \left(16 \cdot \frac{1}{2}\right)^{\frac{1}{3}} + \log_2 x^3 &= 14 \\ 8^{\frac{1}{3}} + 3 \log_2 x &= 14 \\ 2 + 3 \log_2 x &= 14 \end{aligned} \qquad \begin{aligned} 3 \log_2 x &= 12 \\ \log_2 x &= 4 \\ x &= 16 \end{aligned}$$

g) $\log_{64} x + \log_x 64 = \frac{13}{6}$
Solution: The trick is to realize that $\log_{64} x$ and $\log_x 64$ are reciprocals since

$$\log_{64} x = \frac{\log_2 x}{\log_2 64} = \frac{\log_2 x}{6} \quad \text{and} \quad \log_x 64 = \frac{\log_2 64}{\log_2 x} = \frac{6}{\log_2 x}$$

If we denote $a = \log_{64} x$, then we have $a + \frac{1}{a} = \frac{13}{6}$ where $a \neq 0$. We solve this equation using the quadratic formula (completing the square would also work).

$$\begin{aligned} a + \frac{1}{a} &= \frac{13}{6} \\ a^2 - \frac{13}{6} a + 1 &= 0 \\ 6a^2 - 13a + 6 &= 0 \end{aligned}$$

$$a_{1,2} = \frac{13 \pm \sqrt{(-13)^2 - 4 \cdot 6 \cdot 6}}{2 \cdot 6} = \frac{13 \pm \sqrt{169 - 144}}{12} = \frac{13 \pm \sqrt{25}}{12} = \frac{13 \pm 5}{12} = \frac{2}{3} \quad \text{or} \quad \frac{3}{2}$$

We check:

$$\text{If } x = \frac{2}{3}, \text{ LHS} = \frac{2}{3} + \frac{1}{\left(\frac{2}{3}\right)} = \frac{2}{3} + \frac{3}{2} = \frac{4}{6} + \frac{9}{6} = \frac{13}{6} = \text{RHS}$$

$$\text{If } x = \frac{3}{2}, \text{ LHS} = \frac{3}{2} + \frac{1}{\left(\frac{3}{2}\right)} = \frac{3}{2} + \frac{2}{3} = \frac{9}{6} + \frac{4}{6} = \frac{13}{6} = \text{RHS}$$

So now we have $a = \frac{2}{3}$ or $\frac{3}{2}$. Since $a = \log_{64} x$, we have: if $a = \frac{2}{3}$

$$\log_{64} x = \frac{2}{3} \implies 64^{2/3} = x \implies x = 64^{2/3} = \left(\sqrt[3]{64}\right)^2 = 4^2 = 16$$

Or, if $a = \frac{3}{2}$, then

$$\log_{64} x = \frac{3}{2} \implies 64^{3/2} = x \implies x = 64^{3/2} = \left(\sqrt{64}\right)^3 = 8^3 = 512$$

10. (Enrichment) Solve each of the following equations.

a) $\log_{2x} 16 + \log_{4x} 8 = \log_x 8$

Solution:

$$\begin{aligned}
\log_{2x} 16 + \log_{4x} 8 &= \log_x 8 && \text{switch to base 2} \\
\frac{\log_2 16}{\log_2 2x} + \frac{\log_2 8}{\log_2 4x} &= \frac{\log_2 8}{\log_2 x} \\
\frac{4}{1 + \log_2 x} + \frac{3}{2 + \log_2 x} &= \frac{3}{\log_2 x} && \text{Let } a \text{ denote } \log_2 x \\
\frac{4}{a+1} + \frac{3}{a+2} &= \frac{3}{a} && \text{multiply by } a(a+1)(a+2) \\
4a(a+2) + 3a(a+1) &= 3(a+1)(a+2) \\
4a^2 + 8a + 3a^2 + 3a &= 3\left(a^2 + 3a + 2\right) \\
7a^2 + 11a &= 3a^2 + 9a + 6 \\
4a^2 + 2a - 6 &= 0 \\
2a^2 + a - 3 &= 0 \\
(2a+3)(a-1) &= 0
\end{aligned}$$

$$a_1 = -\frac{3}{2} \qquad a_2 = 1$$

So $x_1 = 2^{-3/2} = \frac{1}{\sqrt{8}}$ and $x_2 = 2$

b) $x(1-\log_{21}3) = \log_{21}30 - \log_{21}(7^x+1)$

Solution:

$$\begin{aligned}
x(1-\log_{21}3) &= \log_{21}30 - \log_{21}(7^x+1)\\
x(\log_{21}21-\log_{21}3) &= \log_{21}30 - \log_{21}(7^x+1)\\
x\log_{21}\frac{21}{3} &= \log_{21}30 - \log_{21}(7^x+1)\\
x\log_{21}7 &= \log_{21}30 - \log_{21}(7^x+1)\\
\log_{21}(7^x) &= \log_{21}\left(\frac{30}{7^x+1}\right)
\end{aligned}$$

Since $f(x) = \log_{21}x$ is a one-to-one function, we can conclude that

$$\begin{aligned}
7^x &= \frac{30}{7^x+1}\\
7^x(7^x+1) &= 30
\end{aligned}$$

Let $a = 7^x$

$$\begin{aligned}
a(a+1) &= 30\\
a^2+a-30 &= 0\\
(a+6)(a-5) &= 0\\
a_1 = -6 \quad &\text{and} \quad a_2 = 5
\end{aligned}$$

If $a=-6$, then we have $7^x=-6$. This equation has no solution. If $a=5$, then we have $7^x=5$ and so $x=\log_7 5$.

c) $\log_x(x-3)\cdot\log_{x-3}(x+20) = 2$

Solution: We will first change the base of the second logarithm to x.

$$\begin{aligned}
\log_x(x-3)\cdot\log_{x-3}(x+20) &= 2\\
\log_x(x-3)\cdot\frac{\log_x(x+20)}{\log_x(x-3)} &= 2 \qquad \text{cancel out } \log_x(x-3)\\
\log_x(x+20) &= 2\\
x^2 &= x+20\\
x^2-x-20 &= 0\\
(x-5)(x+4) &= 0\\
x_1 = 5 \qquad &x_2 = -4
\end{aligned}$$

Since x is the base of a logarithm, and also $x-3$ is the argument of a logarithm, $x=-4$ clearly does not work. We check the other solution and find that it does work.

$$\text{LHS} = \log_5(5-3)\cdot\log_{5-3}(5+20) = \log_5 2\cdot\log_2 25 = \log_5 2\cdot(2\log_2 5) = 2(\log_5 2\cdot\log_2 5) = 2\cdot 1 = 2$$

So $x=5$.

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