

Sample Problems

1. Divide $x^4 + x^3 - 5x^2 + 26x - 21$ by $x^2 + 3x - 4$

Practice Problems

Perform each of the following divisions with remainder.

1. $(3x^5 - 8x^4 + 11x^2 + 2x - 7) \div (x - 2)$
2. $(3x^5 - 8x^4 + 11x^2 + 2x - 7) \div (x^2 + x)$
3. $(3x^5 - 8x^4 + 11x^2 + 2x - 7) \div (x^3 - 3x)$
4. $(3x^5 - 8x^4 + 11x^2 + 2x - 7) \div (x^2 + 1)$
5. $(5x^5 - 24x^3 - 7x^2 - 5x + 35) \div (x - 1)$
6. $(5x^5 - 24x^3 - 7x^2 - 5x + 35) \div (x^3 + 2x - 1)$
7. $(5x^5 - 24x^3 - 7x^2 - 5x + 35) \div (x^2 - 5)$
8. $(5x^5 - 24x^3 - 7x^2 - 5x + 35) \div (x + 3)$
9. $(x^6 - 1) \div (x - 1)$
10. $(x^6 - 1) \div (x + 2)$
11. $(x^6 - 1) \div (x^2 - 4)$
12. $(x^2 - 7x + 16) \div (x - 3)$
13. $(x^3 - 5x^2 + 8x - 1) \div (x - 3)$
14. $(x^4 - 5x^3 + 4x^2 + x + 8) \div (x - 1)$
15. $(3x^4 - x^3 + 5x - 4) \div (x + 1)$
16. $(x^5 - 2) \div (x - 2)$
17. $(5x^3 - 2x^2 + 5x - 7) \div (x^2 - 3)$
18. $(x^4 - 3x^3 + x^2 - 8x - 2) \div (x + 2)$
19. $(x^4 - 4x^3 + 10x^2 - 7) \div (x + 1)$
20. $(x^4 - 4x^3 + 10x^2 - 7) \div (x - 2)$

Answers - Sample Problems

1. $x^2 - 2x + 5$ R $3x - 1$

Answers - Practice Problems

1. $3x^4 - 2x^3 - 4x^2 + 3x + 8$ R 9

2. $3x^3 - 11x^2 + 11x$ R $2x - 7$

3. $3x^2 - 8x + 9$ R $-13x^2 + 29x - 7$

4. $3x^3 - 8x^2 - 3x + 19$ R $5x - 26$

5. $5x^4 + 5x^3 - 19x^2 - 26x - 31$ R 4

6. $5x^2 - 34$ R $-2x^2 + 63x + 1$

7. $5x^3 + x - 7$

8. $5x^4 - 15x^3 + 21x^2 - 70x + 205$ R -580

9. $x^5 + x^4 + x^3 + x^2 + x + 1$

10. $x^5 - 2x^4 + 4x^3 - 8x^2 + 16x - 32$ R 63

11. $x^4 + 4x^2 + 16$ R 63

12. $x - 4$ R 4

13. $x^2 - 2x + 2$ R 5

14. $x^3 - 4x^2 + 1$ R 9

15. $3x^3 - 4x^2 + 4x + 1$ R -5

16. $x^4 + 2x^3 + 4x^2 + 8x + 16$ R 30

17. $5x - 2$ R $20x - 13$

18. $x^3 - 5x^2 + 11x - 30$ R $\frac{58}{x+2}$

19. $x^3 - 5x^2 + 15x - 15$ R 8

20. $x^3 - 2x^2 + 6x + 12$ R 17

Sample Problems - Solutions

1. Divide $x^4 + x^3 - 5x^2 + 26x - 21$ by $x^2 + 3x - 4$

Step 1: Divide the first term by the first term: $\frac{x^4}{x^2} = x^2$

x^2 is the first term of the quotient; we write it above the line.

We multiply x^2 and what we are dividing by: $x^2(x^2 + 3x - 4) = x^4 + 3x^3 - 4x^2$

We take the opposite of the result: $-(x^4 + 3x^3 - 4x^2) = -x^4 - 3x^3 + 4x^2$

We add that to the original polynomial shown above. This is now the new polynomial to divide, and the process begins again.

$$\begin{array}{r} x^2 + 3x - 4 \overline{) x^4 + x^3 - 5x^2 + 26x - 21} \\ \underline{-x^4 - 3x^3 + 4x^2} \\ -2x^3 - x^2 + 26x - 21 \end{array}$$

Step 2: We now need to divide $-2x^3 - x^2 + 26x - 21$ by $x^2 + 3x - 4$. Divide the first term by the first term: $\frac{-2x^3}{x^2} = -2x$

$-2x$ is the second term of the quotient; we write it above the line.

We multiply $-2x$ and what we are dividing by: $-2x(x^2 + 3x - 4) = -2x^3 - 6x^2 + 8x$

We take the opposite of the result: $-1(-2x^3 - 6x^2 + 8x) = 2x^3 + 6x^2 - 8x$

We add that to the polynomial to be divided. This sum is now the new polynomial to divide, and the process begins again.

$$\begin{array}{r} x^2 + 3x - 4 \overline{) x^4 + x^3 - 5x^2 + 26x - 21} \\ \underline{-x^4 - 3x^3 + 4x^2} \\ -2x^3 - x^2 + 26x - 21 \\ \underline{2x^3 + 6x^2 - 8x} \\ 5x^2 + 18x - 21 \end{array}$$

Step 3: Divide the first term by the first term: $\frac{5x^2}{x^2} = 5$

5 is the third term of the quotient; we write it above the line.

We multiply 5 and what we are dividing by: $5(x^2 + 3x - 4) = 5x^2 + 15x - 20$

We take the opposite of the result: $-1(5x^2 + 15x - 20) = -5x^2 - 15x + 20$

We add that to the polynomial to be divided.

$$\begin{array}{r} x^2 + 3x - 4 \overline{) x^4 + x^3 - 5x^2 + 26x - 21} \\ \underline{-x^4 - 3x^3 + 4x^2} \\ -2x^3 - x^2 + 26x - 21 \\ \underline{2x^3 + 6x^2 - 8x} \\ 5x^2 + 18x - 21 \\ \underline{-5x^2 - 15x + 20} \\ 3x - 1 \end{array}$$

For the next step, we would have to divide $3x - 1$ by $x^2 + 3x - 4$. The division of the first terms $\frac{3x}{x^2} = \frac{3}{x}$ is not allowed: this is where we must stop. The quotient is $x^2 - 2x + 5$ and the remainder is $3x - 1$. The result can be presented in two different ways, very much like the division $38 \div 5$ can be represented in two ways:

$$\frac{38}{5} = 7\frac{3}{5} \quad \text{or} \quad 38 \div 5 = 7 \text{ R } 3$$

our result is

$$\frac{x^4 + x^3 - 5x^2 + 26x - 21}{x^2 + 3x - 4} = \boxed{x^2 - 2x + 5 + \frac{3x - 1}{x^2 + 3x - 4}}$$

or

$$(x^4 + x^3 - 5x^2 + 26x - 21) \div (x^2 + 3x - 4) = \boxed{x^2 - 2x + 5 \text{ R } 3x - 1}$$

We can check our division by multiplying back:

$$\begin{aligned} P &= (x^2 + 3x - 4) \left(x^2 - 2x + 5 + \frac{3x - 1}{x^2 + 3x - 4} \right) \\ &= (x^2 + 3x - 4)(x^2 - 2x + 5) + (x^2 + 3x - 4) \frac{3x - 1}{x^2 + 3x - 4} \\ &= x^2(x^2 - 2x + 5) + 3x(x^2 - 2x + 5) - 4(x^2 - 2x + 5) + 3x - 1 \\ &= x^4 - 2x^3 + 5x^2 + 3x^3 - 6x^2 + 15x - 4x^2 + 8x - 20 + 3x - 1 = x^4 + x^3 - 5x^2 + 26x - 21 \end{aligned}$$

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