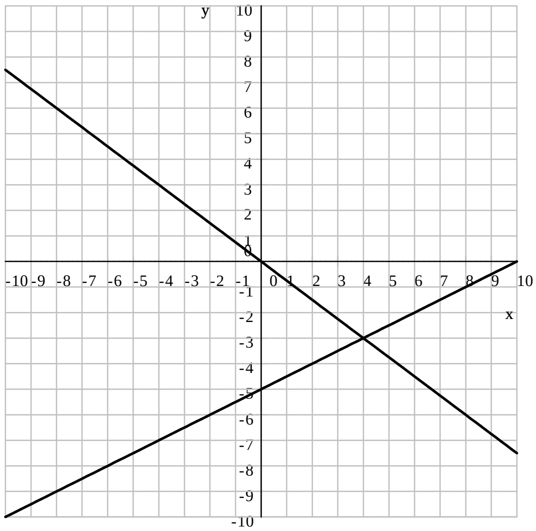
Recall that the graph of an equation is the set of all points whose coordinates form a solution to the equation. Consider the lines $y = -\frac{3}{4}x$ and $x - 2y = 10$. We graph these lines in the same coordinate system. What can we say about the point where the two lines intersect each other? In this particular case, we can read the coordinates of this point: $(4, -3)$.



This point is on the line $y = -\frac{3}{4}x$ if and only if its coordinates form a solution of the equation. We check:

$$\text{LHS} = -3 \quad \text{and} \quad \text{RHS} = -\frac{3}{4}(4) = -3 \checkmark$$

Therefore, this point is on the line $y = -\frac{3}{4}x$.

Similarly, this point is on the line $x - 2y = 10$ if and only if its coordinates form a solution of the equation.

$$\text{LHS} = 4 - 2(-3) = 4 + 6 = 10 \quad \text{and} \quad \text{RHS} = 10 \checkmark$$

Therefore, this point is also on the line $x - 2y = 10$.

Two different lines cannot have more than one point in common. Two points uniquely determine a line. Therefore, if two lines have two points in common, they are really the same line. In this sense, the intersection point is special. Algebraically, the intersection point is the unique point whose coordinates are a solution for the equations of both lines.

Definition: Two equations in x and y form a **system of equations**. The **solution**(s) of the system are the point(s) whose coordinates form a solution of both equations. To **solve** a system means to find all solutions of it.

In our example above, $(-4, 3)$ is the only solution of the system $\begin{cases} y = -\frac{3}{4}x \\ x - 2y = 10 \end{cases}$.

In this case, the intersection was a point whose both coordiantes happen to be integers. (Such points are called lattice points.) In case we are less lucky, we will need more precise tools for solving than graphing the two equations in the same cordinate system.

There are several algebraic methods to solve a system of linear equations.

Solving Linear Systems Using Elimination.

This method is sometimes called the addition method. We will refer to it as elimination. The basic principle behind this method is the following. We can multiply both sides of an equation by the same non-zero number- and this step will not change the solution set. We can also add equations: If $x = y$ and $A = B$, then $x + A = y + B$.

Example 1. Solve the given system of linear equations using elimination. $\begin{cases} 3x - 5y = 11 \\ 2x + 3y = 20 \end{cases}$

Solution: We will multiply both sides of one or both equations with the goal to end up with one unknown with opposite coefficients. This is possible for both x and y. To elimininate y, we will multiply the first equation by 3 and the second equation by 5. That way the coeffcients of y will be 15 and -15. Once we add the two equations, y will be eliminated.

$$\begin{cases} 3x - 5y = 11 & \text{multiply by } 3 \\ 2x + 3y = 20 & \text{multiply by } 5 \end{cases} \implies \begin{cases} 9x - 15y = 33 \\ 10x + 15y = 100 \end{cases}$$

We will eliminate y by adding the two equations (i.e adding the left-hand side to left-hand side and right-hand side to right-hand side.) Then the equation will become an equation in only x, so we can solve for it.

$$\begin{array}{rl} & 9x - 15y = 33 \\ + & 10x + 15y = 100 \\ \hline & 19x = 133 \quad \text{divide by } 19 \\ & x = 7 \end{array}$$

Now that we know the value of x, we can use either one of the two equations to find the value of y. The second equation, in its original form, will be transformed from $2x + 3y = 20$ to $2 \cdot 7 + 3y = 20$. Now we can easily solve for y.

$$\begin{aligned} 2 \cdot 7 + 3y &= 20 \\ 14 + 3y &= 20 \quad \text{subtract} \\ 3y &= 6 \\ y &= 2 \end{aligned}$$

Thus, the solution of this system is $x = 7$ and $y = 2$, or, in short, $(7, 2)$. We check: the solution of a system is a simultaneous solution of both equations.

Checking $3x - 5y = 11$

$$\begin{aligned} \text{LHS} &= 3 \cdot 7 - 5 \cdot 2 = 21 - 10 = 11 \\ \text{RHS} &= 11 \checkmark \end{aligned}$$

Checking $2x + 3y = 20$

$$\begin{aligned} \text{LHS} &= 2 \cdot 7 + 3 \cdot 2 = 14 + 6 = 20 \\ \text{RHS} &= 20 \checkmark \end{aligned}$$

Therefore, our solution, $\boxed{(7, 2)}$ is correct.

Example 2. Solve the given system of linear equations using elimination. $\begin{cases} 2x - y = -19 \\ -x + 3y = 12 \end{cases}$

Solution: We will multiply both sides of one or both equations with the goal to end up with one unknown with opposite coefficients. This is possible for both x and y. To elimininate x, we will leave the first equation as it is, and multiply the second equation by 2. This way the coeffcients of x will be 2 and 2. Then, when we add the two equations, x will be eliminated, and we can solve the equation for y.

$$\begin{cases} 2x - y = -19 & \\ -x + 3y = 12 & \text{multiply by } 2 \end{cases} \implies \begin{cases} 2x - y = -19 \\ -2x + 6y = 24 \end{cases}$$

We now add the two equations:

$$\begin{cases} \quad 2x - y \;=\; -19 \\ + \quad -2x + 6y = 24 \\ \hline \end{cases}$$

$$5y = 5 \quad \text{divide by } 5$$

$$y = 1$$

Now that we know the value of y, we can use either one of the two equations to find the value of x. The first equation, in its original form, will be transformed from $2x - y = -19$ to $2x - 1 = -19$. Now we can easily solve for x.

$$\begin{aligned} 2x - 1 &= -19 && \text{add } 1 \\ 2x &= -18 && \text{divide by } 2 \\ x &= -9 \end{aligned}$$

Thus, the solution of this system is $x = -9$ and $y = 1$, or, in short, $(-9, 1)$. We check: the solution of a system is a simultaneous solution of both equations.

Checking $2x - y = -19$

$$\begin{aligned} \text{LHS} &= 2(-9) - 1 = -18 - 1 = -19 \\ \text{RHS} &= -19 \checkmark \end{aligned}$$

Checking $-x + 3y = 12$

$$\begin{aligned} \text{LHS} &= -(-9) + 3 \cdot 1 = 9 + 3 = 12 \\ \text{RHS} &= 12 \checkmark \end{aligned}$$

Therefore, our solution, $\boxed{(-9, 1)}$ is correct.

Most real-world problems boil down to systems of equations. In this sense, solving systems of equations is one of the most important tasks in problem solving.

Example 3. There is an animal farm where chickens and cows live. All together, there are 53 heads and 174 legs. How many chickens and how many cows are there on the farm?

Solution: We will denote the number of chickens by x and the number of cows by y. The first equation will express the number of heads. x many chickens come with x many heads, and y many cows come with y many heads. The second equation will express the number of legs. x many chickens come with $2x$ many legs, and y many cows come with $4y$ many heads.

$$\begin{cases} x + y = 53 \\ 2x + 4y = 174 \end{cases}$$

Before we start solving the system, let us notice that we can simplify the second equation by dividing both sides by 2. We can often make our life easier with simplifications such as this one.

$$\begin{cases} x + y = 53 \\ x + 2y = 87 \end{cases}$$

To eliminate x, we will multiply the first equation by -1 leave the second equation as is. Then we add the two equations.

$$\begin{cases} \quad -x - y = -53 \\ + \quad x + 2y = 87 \\ \hline \end{cases}$$

$$y = 34$$

Now that we know the value of y, we use the first equation to find x.

$$\begin{aligned} x + 34 &= 53 \quad \text{subtract } 34 \\ x &= 19 \qquad \Longrightarrow x = 19, \; y = 34 \end{aligned}$$

Thus we have $\boxed{\text{19 chickens and 34 cows}}$. We check: the number of heads is $19 + 34 = 53$, and the number of legs is $2 \cdot 19 + 4 \cdot 34 = 38 + 136 = 174$. So our solution is correct.

Example 4. We invested $\$10\,000$ into two bank accounts. One account earns 14% per year, the other account earns 8% per year. How much did we invest into each account if after the first year, the combined interest from the two accounts is $\$1238$?

Solution: Let us denote the amount invested at 14% by x and the amount invested at 8% by y. The two equations will express the total amount invested, and the total interest earned.

$$\begin{aligned} x + y &= 10\,000 \qquad \text{the amounts invested add up to } \$10\,000 \\ 0.14x + 0.08y &= 1238 \qquad \text{the interests earned add up to } \$1238 \end{aligned}$$

We solve the system of equation by elimination. But let us first make the second equation simpler:

$$\begin{aligned} 0.14x + 0.08y &= 1238 \qquad \text{multiply by } 100 \\ 14x + 8y &= 123\,800 \quad \text{divide by } 2 \\ 7x + 4y &= 61\,900 \end{aligned}$$

We now have

$$\begin{aligned} x + y &= 10\,000 \\ 7x + 4y &= 61\,900 \end{aligned}$$

We will multiply the first equation by -4 to eliminate y. Then we add the two equations.

$$\begin{array}{r} -4x - 4y = -40\,000 \\ \underline{7x + 4y = \;\;61\,900} \\ 3x = 21\,900 \quad \text{divide by } 3 \\ x = 7300 \end{array}$$

Thus we invested $\$7300$ at 14%. The other amount can be found using the first equation:

$$\begin{aligned} 7300 + y &= 10\,000 \\ y &= 2700 \end{aligned}$$

We invested $\boxed{\$7300 \text{ at } 14\% \text{ and } \$2700 \text{ at } 8\%}$. We check: the amounts add up to $\$7300 + \$2700 = \$10\,000$. The interest from the accounts are:

$$14\% \;\text{ of }\; 7300 \text{ is } \; 0.14\,(7300) = 1022 \;\text{ and }\; 8\% \;\text{ of }\; 2700 \text{ is } \; 0.08\,(2700) = 216$$

Since $1022 + 216 = 1238$, our solution is correct.

Example 5. We have a jar of coins, all pennies and dimes. All together, we have 372 coins, and the total value of all coins in the jar is $\$20.91$. How many pennies are there in the jar?

Solution: Let us denote the number of pennies by x and the number of dimes by y. The first equation will express the number of the coins. This equation is therefore $x + y = 372$. To express the value of all coins, x many pennies are worth $0.01x$ and y many dimes are worth $0.1y$. The total value of all coins is then

$$0.01x + 0.1y = 20.91$$

In order to clear the decimals, we may multiply both sides by 100. Then we have

$$x + 10y = 2091$$

Let us notice that this is the same equation that we would obtain if we expressed the value of all coins in cents and not in dollars. So our system is now

$$\begin{aligned} x + y &= 372 \\ x + 10y &= 2091 \end{aligned}$$

We will eliminate x by multiplying the first equation by -1 and leaving the second equation as is. Then we add the two equations and solve for y.

$$\begin{aligned} -x - y &= -372 \\ x + 10y &= 2091 \\ \hline 9y &= 1719 \qquad \text{divide by } 9 \\ y &= 191 \end{aligned}$$

Thus we have 191 dimes. The number of pennies can be found using any of the two equations. We will use the simplest one, the original first equation.

$$\begin{aligned} x + 191 &= 372 \qquad \text{subtract } 191 \\ x &= 181 \end{aligned}$$

Thus we have $\boxed{\text{181 pennies and 191 dimes}}$. We check: the number of all coins is $181 + 191 = 372$, and the value of the coins is $0.01 \cdot 181 + 0.1 \cdot 191 = 1.81 + 19.1 = 20.91$. Thus our solution is correct.

Solving Linear Systems using Substitution

The basic idea of this method is to absorb the information of one equation and to substitute that into the other equation, thereby reducing the number of unknowns to one.

Example 6. Solve the given system of linear equations using substitution. $\begin{cases} 2x - y = -19 \\ -x + 3y = 12 \end{cases}$

Solution: We first inspect the two equations and look for coefficients such as 1 or -1. In this case, the coefficient of y is -1 in the first equation. We solve for y in this equation. We can't solve for y and obtain a number, we can only solve for it in terms of x.

$$\begin{aligned} 2x - y &= -19 && \text{add } y \\ 2x &= y - 19 && \text{add } 19 \\ 2x + 19 &= y \end{aligned}$$

The information from the first equation can be expressed as $\boxed{y = 2x + 19}$. This is going to be what we substitute into the other equation by substituting $2x + 19$ into y. This way, the equation $x + 3y = 12$ will become $-x + 3(2x + 19) = 12$. This is now an equation in only one variable for which we can solve.

$$\begin{aligned} -x + 3(2x + 19) &= 12 \\ -x + 6x + 57 &= 12 \\ 5x + 57 &= 12 && \text{subtract } 57 \\ 5x &= -45 && \text{divide by } 5 \\ x &= -9 \end{aligned}$$

Now that we know the value of x, we return to what we used for substitution and get the value of the other unknown.

$$y = 2x + 19 = 2(-9) + 19 = -18 + 19 = 1$$

Therefore, the solution of this system is $x = -9$ and $y = 1$, or, in short, $(-9, 1)$. We check: the solution of a system is a simultaneous solution of both equations.

Checking $2x - y = -19$

$$\begin{aligned} \text{LHS} &= 2(-9) - 1 = -18 - 1 = -19 \\ \text{RHS} &= -19 \checkmark \end{aligned}$$

Checking $-x + 3y = 12$

$$\begin{aligned} \text{LHS} &= -(-9) + 3 \cdot 1 = 9 + 3 = 12 \\ \text{RHS} &= 12 \checkmark \end{aligned}$$

Therefore, our solution, $\boxed{(-9, 1)}$ is correct.

Of course, not all linear systems contain easy coefficients such as 1 or -1.

Example 7. Solve the given system of linear equations. $\begin{cases} 3x - 5y = 11 \\ 2x + 3y = 20 \end{cases}$

Solution: We will solve for x in the second equation and substitute the information into the first equation. First, we solve for x in $2x + 3y = 20$.

$$\begin{aligned} 2x + 3y &= 20 && \text{subtract } 3y \\ 2x &= -3y + 20 && \text{divide by } 2 \\ x &= \frac{-3y + 20}{2} \end{aligned}$$

This is the information we will substitute into the first equation. $3x - 5y = 11$ will become $3\left(\frac{-3y+20}{2}\right) - 5y = 11$. We solve this equation for y.

$$\begin{aligned} 3\left(\frac{-3y+20}{2}\right) - 5y &= 11 && \text{multiply by } 2 \\ 3(-3y + 20) - 10y &= 22 && \text{distribute } 3 \\ -9y + 60 - 10y &= 22 && \text{combine like terms} \\ -19y + 60 &= 22 && \text{subtract } 60 \\ -19y &= -38 && \text{divide by } -19 \\ y &= 2 \end{aligned}$$

Now that we know that y is 2, we find x using the expression we used for the substitution.

$$x = \frac{-3y + 20}{2} = \frac{-3 \cdot 2 + 20}{2} = \frac{-6 + 20}{2} = \frac{14}{2} = 7$$

Thus, the solution of this system is $x = 7$ and $y = 2$, or, in short, $(7, 2)$. We check: the solution of a system is a simultaneous solution of both equations.

Checking $3x - 5y = 11$

$$\begin{aligned} \text{LHS} &= 3 \cdot 7 - 5 \cdot 2 = 21 - 10 = 11 \\ \text{RHS} &= 11 \checkmark \end{aligned}$$

Checking $2x + 3y = 20$

$$\begin{aligned} \text{LHS} &= 2 \cdot 7 + 3 \cdot 2 = 14 + 6 = 20 \\ \text{RHS} &= 20 \checkmark \end{aligned}$$

Therefore, our solution, $\boxed{(7, 2)}$ is correct.

Most real-world problems boil down to systems of equations. In this sense, solving systems of equations is one of the most important tasks in problem solving.

Example 8. There is an animal farm where chickens and cows live. All together, there are 53 heads and 174 legs. How many chickens and how many cows are there on the farm?

Solution: We will denote the number of chickens by x and the number of cows by y. The first equation will express the number of heads. x many chickens come with x many heads, and y many cows come with y many heads. The second equation will express the number of legs. x many chickens come with $2x$ many legs, and y many cows come with $4y$ many heads.

$$\begin{cases} x + y = 53 \\ 2x + 4y = 174 \end{cases}$$

Before we start solving the system, let us notice that we can simplify the second equation by dividing both sides by 2. We can often make our life easier with simplifications such as this one.

$$\begin{cases} x + y = 53 \\ x + 2y = 87 \end{cases}$$

We will solve for x in the first equation and substitute that expression into the second equation.

$$x = 53 - y \qquad \Longrightarrow \qquad (53 - y) + 2y = 87$$

We solve this equation for y.

$$\begin{aligned} 53 - y + 2y &= 87 \\ 53 + y &= 87 \qquad \text{subtract 53} \\ y &= 34 \end{aligned}$$

Now that we know the value of y, we can easily find x.

$$x = 53 - 34 = 19 \qquad \Longrightarrow x = 19, \ \ y = 34$$

Thus we have $\boxed{\text{19 chickens and 34 cows}}$. We check: the number of heads is $19 + 34 = 53$, and the number of legs is $2 \cdot 19 + 4 \cdot 34 = 38 + 136 = 174$. So our solution is correct.

Example 9. We invested \$10 000 into two bank accounts. One account earns 14% per year, the other account earns 8% per year. How much did we invest into each account if after the first year, the combined interest from the two accounts is \$1238?

Solution: Let us denote the amount invested at 14% by x and the amount invested at 8% by y. The two equations will express the total amount invested, and the total interest earned. Then the interest earned form the first account is 14% of x, and that of the second account is 8% of y. Recall that 14% of x can be written as $0.14x$ and 8% of y as $0.08y$.

$$\begin{cases} x + y = 10\,000 & \text{the amounts invested add up to \$10 000} \\ 0.14x + 0.08y = 1238 & \text{the interests earned add up to \$1238} \end{cases}$$

We solve the system of equation by substitution, but let us first make the second equation simpler:

$$\begin{aligned} 0.14x + 0.08y &= 1238 \qquad \text{multiply by 100} \\ 14x + 8y &= 123\,800 \qquad \text{divide by 2} \\ 7x + 4y &= 61\,900 \end{aligned}$$

We now have

$$\begin{cases} x+y=10\,000 \\ 7x+4y=61\,900 \end{cases}$$

We will solve for y in the first equation and substitute the result into the second equation.

$$x+y=10\,000 \quad \Longrightarrow \quad y=10\,000-x$$

Now the equation $7x+4y=61\,900$ becomes $7x+4\,(10\,000-x)=61\,900$. We can solve this equation for x.

$$\begin{aligned} 7x+4\,(10\,000-x) &= 61\,900 && \text{distribute } 4 \\ 7x+40\,000-4x &= 61\,900 && \text{combine like terms} \\ 3x+40\,000 &= 61\,900 && \text{subtract } 40\,000 \\ 3x &= 21\,900 && \text{divide by } 3 \\ x &= 7300 \end{aligned}$$

We can now easily find y using $y=10\,000-x$.

$$y=10\,000-x=y=10\,000-7300=2700$$

Our solution, $x=7300$ and $y=2700$ means that we invested $\boxed{\$7300 \text{ at } 14\% \text{ and } \$2700 \text{ at } 8\%}$. We check: the amounts add up to $\$7300+\$2700=\$10\,000$. The interest from the accounts are:

$$14\% \text{ of } 7300 \text{ is } 0.14\,(7300)=1022 \text{ and } 8\% \text{ of } 2700 \text{ is } 0.08\,(2700)=216$$

Since $1022+216=1238$, our solution is correct.

Example 10. We have a jar of coins, all pennies and dimes. All together, we have 372 coins, and the total value of all coins in the jar is \$20.91. How many pennies are there in the jar?

Solution: Let us denote the number of pennies by x and the number of dimes by y. The first equation will express the number of the coins. This equation is therefore $x+y=372$. To express the value of all coins, x many pennies are worth $0.01x$ and y many dimes are worth $0.1y$. The total value of all coins is then

$$0.01x+0.1y=20.91$$

In order to clear the decimals, we may multiply both sides by 100. Then we have

$$x+10y=2091$$

Let us notice that this is the same equation that we would obtain if we expressed the value of all coins in cents and not in dollars. So our system is now

$$\begin{cases} x+y=372 \\ x+10y=2091 \end{cases}$$

We solve for x in the first equation and substitute that into the second equation.

$$x+y=372 \quad \Longrightarrow \quad x=372-y$$

Now the other equation, $x + 10y = 2091$ becomes

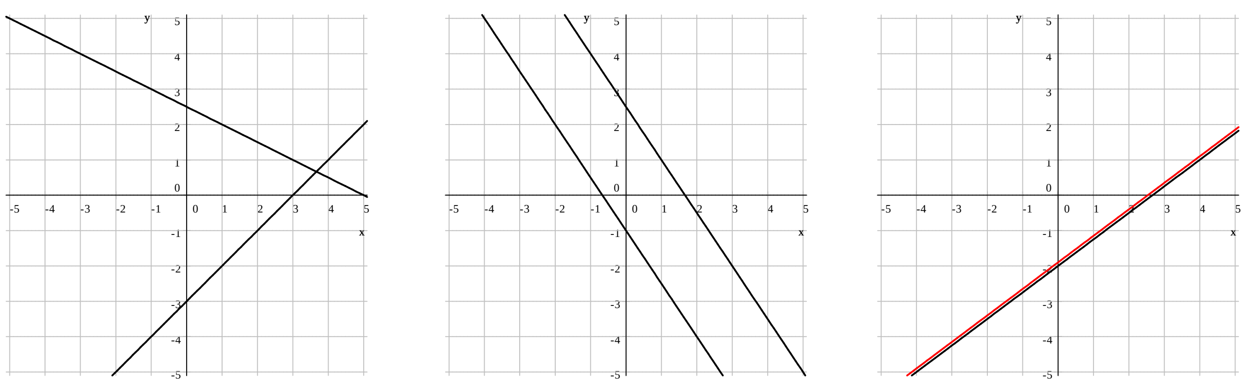
$$\begin{aligned} 372 - y + 10y &= 2091 && \text{combine like terms} \\ 9y + 372 &= 2091 && \text{subtract } 372 \\ 9y &= 1719 && \text{divide by } 9 \\ y &= 191 && \Longrightarrow \quad x = 372 - y = 372 - 191 = 181 \end{aligned}$$

The solution $x = 181$, $y = 191$ means that we have 181 pennies and 191 dimes. We check: the number of all coins is $181 + 191 = 372$, and the value of the coins is $0.01 \cdot 181 + 0.1 \cdot 191 = 1.81 + 19.1 = 20.91$. Thus our solution is correct.

Most systems of linear equations have a unique solution (x, y). This is not always the case.

If we think of a system of linear equations as two lines, it is clear that geometrically, there are three distinct ways two lines in a plane can behave.

One intersection point | No intersection point | All points on the line are intersection points



Until now, we have only seen linear systems with a unique solution, corresponding to two lines intersecting each other in a uinque point. However, there are linear systems that end up with different results.

Example 11. Solve the given system of linear equations. $\begin{cases} x - 2y = 6 \\ y = \dfrac{1}{2}x + 1 \end{cases}$

Solution: Since the second equation is already solved for y, we will use substitution, but elimination would also work fine. We substitute $y = \dfrac{1}{2}x + 1$ into the first equation and solve the linear equation for x.

$$\begin{aligned} x - 2\left(\frac{1}{2}x + 1\right) &= 6 && \text{distribute } 2 \\ x - x - 2 &= 6 && \text{combine like terms} \\ -2 &= 6 \end{aligned}$$

As we solve the equation for x, it disappears from the equation and we are left with an unconditionally false equation. We have seen this before when we solved linear equations. An equation such as $-2 = 6$ is called a contradiction, and it has no solution. What does this result mean for a system?

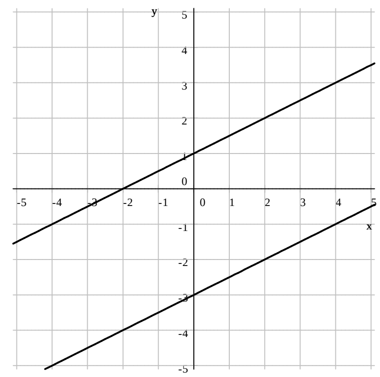
Let us solve the first equation for y.

$$\begin{aligned} x - 2y &= 6 && \text{add } 2y \\ x &= 2y + 6 && \text{subtract } 6 \\ x - 6 &= 2y && \text{divide by } 2 \\ \frac{x-6}{2} &= y \implies && y = \frac{x}{2} - \frac{6}{2} = \frac{1}{2}x - 3 \end{aligned}$$

Let us look again at the system, but this time from a geometric point of view. Both equations represent a straight line.

$$\begin{cases} y = \dfrac{1}{2}x - 3 \\ y = \dfrac{1}{2}x + 1 \end{cases}$$

These lines are parallel because they have the same slope, $m = \dfrac{1}{2}$, and parallel lines have no intersection points.



In case of such a system, the last line is an unconditionally false equation, a contradiction. Such a system is called an **inconsistent system**, and there is no solution of it.

Example 12. Solve the given system of linear equations. $\begin{cases} 2x + 6y = -12 \\ y = -\dfrac{1}{3}x - 2 \end{cases}$

Solution: Since the second equation is already solved for y, we will use substitution, but elimination would also work fine. We substitute $y = -\dfrac{1}{3}x - 2$ into the first equation and solve the linear equation for x.

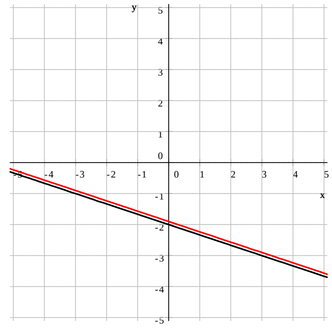
$$\begin{aligned} 2x + 6\left(-\frac{1}{3}x - 2\right) &= -12 && \text{distribute } 6 \\ 2x - 2x - 12 &= -12 && \text{combine like terms} \\ -12 &= -12 \end{aligned}$$

As we solve the equation for x, it disappears from the equation again, but this time we are left with an unconditionally true equation. We have seen this before when we solved linear equations. Such an eqaution is called an identity, and all real numbers are solutions of it.

Just as before, we solve the first equation for y.

$$\begin{aligned} 2x + 6y &= -12 && \text{subtract } 2x \\ 6y &= -2x - 12 && \text{divide by } 6 \\ y &= \frac{-2x-12}{6} && \text{divide by } 2 \\ y &= \frac{-2x-12}{6} = -\frac{2x}{6} - \frac{12}{6} = -\frac{1}{3}x - 2 \end{aligned}$$

So, the linear system now look like this: $\left\{\begin{array}{l} y=-\dfrac{1}{3}x-2 \\ \\ y=-\dfrac{1}{3}x-2 \end{array}\right.$



These lines are identical and so every point on the line is a solution. We can express this as the set of points $\left(x,-\dfrac{1}{3}x-2\right)$.

In case of such a system, the last line is an unconditionally true equation, an identity. Such a system is called a **dependent system**, and all points on the line are solutions.

Practice Problems

1. Solve each of the following system of linear equations.

   a) $\left\{\begin{array}{c} 2x-y=-8 \\ x+2y=-9 \end{array}\right.$

   b) $\left\{\begin{array}{c} 2(p-1)-3(q-1)=24 \\ p+q=-6 \end{array}\right.$

   c) $\left\{\begin{array}{c} a+3b=10 \\ b=\dfrac{-a-10}{3} \end{array}\right.$

   d) $\left\{\begin{array}{c} 3x-y=10 \\ \dfrac{1}{3}x-y=2 \end{array}\right.$

   e) $\left\{\begin{array}{c} \dfrac{1}{2}x+\dfrac{1}{4}y=-1 \\ \dfrac{1}{2}y-\dfrac{1}{3}x=6 \end{array}\right.$

   f) $\left\{\begin{array}{c} 2x+3y=4 \\ 4x=-6y+8 \end{array}\right.$

   g) $\left\{\begin{array}{c} 3x-y=4 \\ 2(y-3)=-2(x+1) \end{array}\right.$

   h) $\left\{\begin{array}{c} 2a+b=17 \\ a+b=5 \end{array}\right.$

   i) $\left\{\begin{array}{c} 3x-2y=2 \\ 2x+3y=5 \end{array}\right.$

   j) $\left\{\begin{array}{c} x-2y=5 \\ 2x+3y=10 \end{array}\right.$

   k) $\left\{\begin{array}{c} 0.5x-1.2y=-1.21 \\ x+3.2y=2.06 \end{array}\right.$

2. Given the equations of two straight lines, find both coordinates of all intersection points.

   a) $2x-5y=-41$ and $x+y=4$

   b) $x+y=-5$ and $2y=-2x-10$

   c) $y=\dfrac{3}{4}x-2$ and $3x-4y=-24$

   d) $5x-y=-35$ and $y=-\dfrac{3}{4}x+\dfrac{1}{2}$

   e) $y=-\dfrac{2}{3}x+2$ and $2x+3y=6$

3. There is an animal farm where chickens and cows live. All together, there are 60 heads and 164 legs. How many chickens and how many cows are there on the farm?

4. We invested \$6000 into two bank accounts. One account earns 7% per year, the other account earns 11% per year. How much did we invest into each account if after the first year, the combined interest from the two accounts is \$520?

5. We have 51 coins, all dimes and quarters, in the total value of \$7.05. How many quarters and how many dimes are there?

6. We invested \$7600 in two bank accounts. One account earns 9% per year, the other account earns 13% per year. How much did we invest into each account if after the first year we have a total of \$8508 in the accounts?

Answers

Practice Problems

1. a) $x = -5, y = -2$ b) $p = 1, q = -7$ c) there is no solution d) $x = 3, y = -1$ e) $x = -6, y = 8$

   f) there are infintely many solutions; x can be any number and then $y = \dfrac{4 - 2x}{3}$ g) $x = \dfrac{3}{2}, y = \dfrac{1}{2}$

   h) $a = 12, b = -7$ i) $x = \dfrac{16}{13}, y = \dfrac{11}{13}$ j) $x = 5, y = 0$ k) $x = -0.5, y = 0.8$

2. a) $(-3, 7)$ b) all points on $y = -x - 5$ are common; the two lines given are identical

   c) no common points; the two lines given are parallel d) $(-6, 5)$

   e) all points on $y = -\dfrac{2}{3}x + 2$ are common; the two lines given are identical.

3. 38 chickens, 22 cows 4. \$3500 at 7% and \$2500 at 11%

5. 38 dimes and 13 quarters 6. \$2000 at 9% and \$5600 at 13%

