Sample Problems

Solve each of the non-linear systems of equations.

1. $\begin{cases} y = x^2 + 6x - 10 \\ y + 5 = 2x \end{cases}$

2. $\begin{cases} y = 2x^2 - 8x + 1 \\ y = -4x - 1 \end{cases}$

3. $\begin{cases} y = -x^2 + 3x - 5 \\ y = x + 1 \end{cases}$

4. $\begin{cases} (x-4)^2 + (y+1)^2 = 20 \\ x + 3y = 11 \end{cases}$

5. $\begin{cases} (x+5)^2 + (y-6)^2 = 10 \\ y = -\frac{1}{3}x + 1 \end{cases}$

6. $\begin{cases} (x-3)^2 + (y-1)^2 = 20 \\ x - y = -5 \end{cases}$

7. $\begin{cases} (x+1)^2 + (y-2)^2 = 25 \\ 4y = 3x + 11 \end{cases}$

8. $\begin{cases} x + y = -1 \\ xy = -12 \end{cases}$

Practice Problems

Solve each of the non-linear systems of equations.

1. $\begin{cases} y = x^2 + 4x - 18 \\ y + 3 = 2x \end{cases}$

2. $\begin{cases} y = x^2 + 4x - 12 \\ y = -2x - 21 \end{cases}$

3. $\begin{cases} y = -x^2 + 3x + 1 \\ y = 7x + 9 \end{cases}$

4. $\begin{cases} y = x^2 - 5x - 4 \\ y = x - 12 \end{cases}$

5. $\begin{cases} y = -\frac{1}{2}x^2 + 3x + 4 \\ y = 5x + 6 \end{cases}$

6. $\begin{cases} y = -x^2 - 6x + 1 \\ y = -2x + 9 \end{cases}$

7. $\begin{cases} x^2 + (y+4)^2 = 20 \\ x - 3y = 2 \end{cases}$

8. $\begin{cases} (x-2)^2 + (y+5)^2 = 10 \\ 3y - x = -7 \end{cases}$

9. $\begin{cases} (x-3)^2 + (y+1)^2 = 16 \\ y + x = 12 \end{cases}$

10. $\begin{cases} (x-4)^2 + (y-3)^2 = 25 \\ 3x + 4y = 24 \end{cases}$

11. $\begin{cases} (x-2)^2 + (y+1)^2 = 20 \\ x - 2y = 19 \end{cases}$

12. $\begin{cases} (x+1)^2 + (y-3)^2 = 50 \\ x - 7y + 72 = 0 \end{cases}$

13. $\begin{cases} (x-1)^2 + (y+7)^2 = 25 \\ y = -\frac{1}{2}x - \frac{3}{2} \end{cases}$

14. $\begin{cases} (x-2)^2 + (y-1)^2 = 25 \\ 3y = 4x - 30 \end{cases}$

15. $\begin{cases} (x+1)^2 + (y-4)^2 = 17 \\ x + 4y = 15 \end{cases}$

16. $\begin{cases} x + y = 12 \\ xy = 35 \end{cases}$

17. $\begin{cases} x^2 + y^2 = 68 \\ x + y = -6 \end{cases}$

18. $\begin{cases} x - y = 4 \\ \frac{1}{x} + \frac{1}{y} = \frac{2}{3} \end{cases}$

Sample Problems - Answers

1.) $(-5,-15)$ and $(1,-3)$ 2.) $(1,-5)$ 3.) no solution 4.) $(8,1)$ and $(2,3)$ 5.) $(-6,3)$

6.) no solution 7.) $(3,5)$ and $(-5,-1)$ 8.) $(3,-4)$ and $(-4,3)$

Practice Problems - Answers

1.) $(3,3)$ and $(-5,-13)$ 2.) $(-3,-15)$ 3.) no solution 4.) $(2,-10)$ and $(4,-8)$ 5.) $(-2,-4)$

6.) no solution 7.) $(4,-2)$ and $(2,0)$ 8.) $(1,-2)$ 9.) no solution 10.) $(0,6)$ and $(8,0)$

11.) no solution 12.) $(-2,10)$ 13.) $(1,-2)$ and $(5,-4)$ 14.) $(6,-2)$ 15.) $(3,3)$ and $(-5,5)$

16.) $(5,7)$ and $(7,5)$ 17.) $(-8,2)$ and $(2,-8)$ 18.) $(6,2)$ and $(1,-3)$

Sample Problems - Solutions

Solve each of the non-linear systems of equations.

1. $\begin{cases} y = x^2 + 6x - 10 \\ y + 5 = 2x \end{cases}$

Solution: We will use substitution. We solve for y in the second equation: $y = 2x - 5$. We substitute this into the first equation and solve for x.

$$\begin{aligned} 2x - 5 &= x^2 + 6x - 10 \\ 0 &= x^2 + 4x - 5 \\ 0 &= (x+5)(x-1) \qquad \Longrightarrow \qquad x_1 = -5 \text{ and } x_2 = 1 \end{aligned}$$

We can find the value of y by using either of the equations, so let us use the easier one which is linear: $y = 2x - 5$.

$$y_1 = 2x_1 - 5 = 2(-5) - 5 = -15 \quad \text{and} \quad y_2 = 2x_2 - 5 = 2(1) - 5 = -3$$

Thus the solutions are $x_1 = -5$ and $y_1 = -15$ or $x_2 = 1$ and $y_2 = -3$. Or, using a more compact notation, the solutions are $(-5, -15)$ and $(1, -3)$.

We check: for the solution $(-5, -15)$

$$\begin{aligned} y &= x^2 + 6x - 10: \qquad \text{RHS} = x^2 + 6x - 10 = (-5)^2 + 6(-5) - 10 = 25 - 30 - 10 = -15 = \text{LHS} \\ y &= 2x - 5: \qquad \text{RHS} = 2(-5) - 5 = -15 = \text{LHS} \end{aligned}$$

Thus $(-5, -15)$ is indeed a solution. We also check $(1, -3)$

$$\begin{aligned} y &= x^2 + 6x - 10: \qquad \text{RHS} = x^2 + 6x - 10 = (1)^2 + 6 \cdot 1 - 10 = 1 + 6 - 10 = -3 = \text{LHS} \\ y &= 2x - 5: \qquad \text{RHS} = 2(1) - 5 = -3 = \text{LHS} \end{aligned}$$

Thus $(1, -3)$ is also a solution, and so both solutions are correct.

2. $\begin{cases} y = 2x^2 - 8x + 1 \\ y = -4x - 1 \end{cases}$

Solution: We will use substitution. We substitute $y = -4x - 1$ into the first equation and solve for x.

$$\begin{aligned} -4x - 1 &= 2x^2 - 8x + 1 \\ 0 &= 2x^2 - 4x + 2 \\ 0 &= 2\left(x^2 - 2x + 1\right) \\ 0 &= 2(x-1)^2 \qquad \Longrightarrow \qquad x = 1 \end{aligned}$$

There is one solution for x. We can find the value of y by using either of the equations, so let us use the simpler one, $y = -4x - 1 = -4(1) - 1 = -5$. Thus the solution is $(1, -5)$. We check: $(-1, 5)$ must be the solution of both equations.

$$\begin{aligned} y &= 2x^2 - 8x + 1: \qquad \text{RHS} = 2(1)^2 - 8 \cdot 1 + 1 = 2 - 8 + 1 = -5 = \text{LHS} \\ y &= -4x - 1: \qquad \text{RHS} = -4 \cdot 1 - 1 = -5 = \text{LHS} \end{aligned}$$

Thus $(1, -5)$ is indeed a solution.

3. $\begin{cases} y = -x^2 + 3x - 5 \\ y = x + 1 \end{cases}$

Solution: We will use substitution. We substitute $y = x + 1$ into the first equation and solve for x.

$$\begin{aligned} x + 1 &= -x^2 + 3x - 5 \\ 0 &= -x^2 + 2x - 6 \\ 0 &= -\left(x^2 - 2x + 6\right) \\ 0 &= -\left[(x-1)^2 + 5\right] \quad \Longrightarrow \quad \text{no real solution for } x \end{aligned}$$

The fact that the equation has no real solution for x indicates that the system has no solution among the pairs of real numbers.

4. $\begin{cases} (x-4)^2 + (y+1)^2 = 20 \\ x + 3y = 11 \end{cases}$

Solution: We will use substitution. We solve for x in the second equation: $x = -3y + 11$. We substitute this into the first equation and solve for y.

$$\begin{aligned} (x-4)^2 + (y+1)^2 &= 20 \\ (-3y + 11 - 4)^2 + (y+1)^2 &= 20 \\ (-3y+7)^2 + (y+1)^2 &= 20 \\ 9y^2 - 42y + 49 + y^2 + 2y + 1 &= 20 \\ 10y^2 - 40y + 50 &= 20 \end{aligned}$$

$$\begin{aligned} &10y^2 - 40y + 30 = 0 \\ &10\left(y^2 - 4y + 3\right) = 0 \\ &10\,(y-1)\,(y-3) = 0 \\ &y_1 = 1 \text{ and } y_2 = 3 \end{aligned}$$

There are two solutions for y. For each, we can find the value of x by using the equation $x = -3y + 11$. At this point, it is important to use the linear equation because if we used the quadratic equation, we would end up with 4 solutions, 2 correct, 2 incorrect. So, we proceed to use the linear equation to find x.

$$x_1 = -3y_1 + 11 = -3\,(1) + 11 = 8 \text{ and } x_2 = -3y_2 + 11 = -3\,(3) + 11 = 2$$

Thus the solutions are $(8, 1)$ and $(2, 3)$.

We check: the coordinates of an intersection point must be the solution of both equations. For the $(8, 1)$, we check

$$\begin{aligned} (x-4)^2 + (y+1)^2 &= 20: \quad \text{LHS} = (8-4)^2 + (1+1)^2 = 4^2 + 2^2 = 16 + 4 = 20 = \text{RHS} \\ x + 3y &= 11: \quad \text{LHS} = 8 + 3 \cdot 1 = 11 = \text{RHS} \end{aligned}$$

Thus $(8, 1)$ is indeed a solution. We also check $(2, 3)$

$$\begin{aligned} (x-4)^2 + (y+1)^2 &= 20: \quad \text{LHS} = (2-4)^2 + (3+1)^2 = (-2)^2 + 4^2 = 4 + 16 = 20 = \text{RHS} \\ x + 3y &= 11: \quad \text{LHS} = 2 + 3 \cdot 3 = 11 = \text{RHS} \end{aligned}$$

Thus $(2, 3)$ is also a solution.

Please note that our choice to solve for x and substitute that was wise. If instead we solved for y, we would have to use the substitution $y = \dfrac{-x + 11}{3}$ and thus solve the quadratic equation

$(x-4)^2 + \left(\dfrac{-x+11}{3} + 1\right)^2 = 20$ which is much more laborious than what we did.

5. $\begin{cases} (x+5)^2+(y-6)^2=10 \\ y=-\dfrac{1}{3}x+1 \end{cases}$

Solution: We will use substitution. We solve for x in the second equation:

$$\begin{aligned} y &= -\frac{1}{3}x+1 \\ 3y &= -x+3 \end{aligned} \qquad \begin{aligned} 3y+x&=3 \\ x&=-3y+3 \end{aligned}$$

We substitute $x=-3y+3$ into the first equation and solve for y.

$$\begin{aligned} (x+5)^2+(y-6)^2 &= 10 \\ (-3y+3+5)^2+(y-6)^2 &= 10 \\ (-3y+8)^2+(y-6)^2 &= 10 \\ 9y^2-48y+64+y^2-12y+36 &= 10 \\ 10y^2-60y+100 &= 10 \end{aligned} \qquad \begin{aligned} &10y^2-60y+90=0 \\ &10\left(y^2-6y+9\right)=0 \\ &10(y-3)^2=0 \\ &y=3 \end{aligned}$$

We can find the $x-$value by using the equation $x=-3y+3$.

$$x=-3y+3=-3(3)+3=-9+3=-6$$

Thus the solution is $(-6,3)$. We check:

$$\begin{aligned} (x+5)^2+(y-6)^2 &= 10: & \text{LHS} &= (-6+5)^2+(3-6)^2=(-1)^2+(-3)^2=1+9=10=\text{RHS} \\ y &= -\frac{1}{3}x+1: & \text{RHS} &= -\frac{1}{3}(-6)+1=2+1=3=\text{LHS} \end{aligned}$$

Thus $(-6,3)$ is indeed a solution.

6. $\begin{cases} (x-3)^2+(y-1)^2=20 \\ x-y=-5 \end{cases}$

Solution: We will use substitution. We solve for y in the second equation: $y=x+5$. We substitute this into the first equation and solve for y.

$$\begin{aligned} (x-3)^2+(y-1)^2 &= 20 \\ (x-3)^2+(x+5-1)^2 &= 20 \\ (x-3)^2+(x+4)^2 &= 20 \\ x^2-6x+9+x^2+8x+16 &= 20 \end{aligned} \qquad \begin{aligned} &2x^2+2x+25=20 \\ &2x^2+2x+5=0 \\ &2\left(x^2+x+\frac{5}{2}\right)=0 \\ &2\left(\left(x+\frac{1}{2}\right)^2+\frac{9}{4}\right)=0 \end{aligned}$$

There is no real solution of this equation. This indicates that there is no real solution for this system.

7. $\begin{cases} (x+1)^2 + (y-2)^2 = 25 \\ 4y = 3x + 11 \end{cases}$

Solution: We will use substitution. This problem is different from the previous problems because we can not solve for x or y without division, and so the algebra will be slightly more laborous. There are a few tricks to simplify computation in such a situation. We solve for y in the second equation: $y = \dfrac{3x+11}{4}$. We substitute this into the first equation and solve for y.

$$\begin{aligned}
(x+1)^2 + (y-2)^2 &= 25 \\
(x+1)^2 + \left(\frac{3x+11}{4} - 2\right)^2 &= 25 \\
(x+1)^2 + \left(\frac{3x+11}{4} - \frac{8}{4}\right)^2 &= 25 \\
(x+1)^2 + \left(\frac{3x+3}{4}\right)^2 &= 25 \\
(x+1)^2 + \frac{(3x+3)^2}{4^2} &= 25 \\
(x+1)^2 + \frac{[3(x+1)]^2}{16} &= 25 \\
(x+1)^2 + \frac{9(x+1)^2}{16} &= 25
\end{aligned}$$

$$\begin{aligned}
(x+1)^2 + \frac{9}{16}(x+1)^2 &= 25 \\
\frac{25}{16}(x+1)^2 &= 25 \\
(x+1)^2 &= 16 \\
(x+1)^2 - 16 &= 0 \\
(x+1+4)(x+1-4) &= 0 \\
(x+5)(x-3) &= 0 \\
x_1 = -5 \quad \text{and} \quad x_2 &= 3
\end{aligned}$$

We can find the $x-$coordinates using the equation
$y = \dfrac{3x+11}{4}$.

$$y_1 = \frac{3x_1+11}{4} = \frac{3(-5)+11}{4} = -1 \quad \text{and} \quad y_2 = \frac{3x_2+11}{4} = \frac{3(3)+11}{4} = 5$$

Thus the solutions are $(-5, -1)$ and $(3, 5)$.

8. $\begin{cases} x + y = -1 \\ xy = -12 \end{cases}$

Solution: We will use substitution. From the first equation, $y = -x - 1$. Then the second equation becomes

$$\begin{aligned}
xy &= -12 \\
x(-x-1) &= -12 \\
-x^2 - x &= -12 \\
0 &= x^2 + x - 12 \\
0 &= (x+4)(x-3) \qquad \Longrightarrow \qquad x_1 = -4 \quad x_2 = 3
\end{aligned}$$

If $x_1 = -4$, then $y_1 = -x_1 - 1 = -(-4) - 1 = 4 - 1 = 3$. If $x_2 = 3$, then $y_2 = -x_2 - 1 = -3 - 1 = -4$. Thus the system has two solutions: $(3, -4)$ and $(-4, 3)$.

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