

## Sample Problems

Solve each of the following equations.

1.)  $\sin x = -\frac{\sqrt{3}}{2}$     2.)  $\cos x = -\frac{1}{2}$     3.)  $\tan x = -1$     4.)  $\cos x = 2$     5.)  $\sin x = -1$

## Practice Problems

1. Solve each of the following equations.

a)  $\tan x = -\frac{1}{\sqrt{3}}$     d)  $\sin x = -\frac{1}{2}$     c)  $\cos x = 1$

b)  $\sin x = 0$     e)  $\sin x = -1.3$     f)  $\cos x = \frac{\sqrt{3}}{2}$

2. Find the domain of each of the following functions.

a)  $f(x) = \tan x$     b)  $g(x) = \cot x$     c)  $t(x) = \frac{1}{\sin^2 x + \cos^2 x}$

## Sample Problems- Answers

- 1.)  $-\frac{\pi}{3} + 2k\pi$  and  $-\frac{2\pi}{3} + 2k\pi$  where  $k \in \mathbb{Z}$       2.)  $x = \pm\frac{2\pi}{3} + 2k\pi$        $k \in \mathbb{Z}$   
 3.)  $x = -\frac{\pi}{4} + k\pi$ , where  $k \in \mathbb{Z}$       4.) no solution      5.)  $-\frac{\pi}{2} + 2k\pi$  where  $k \in \mathbb{Z}$

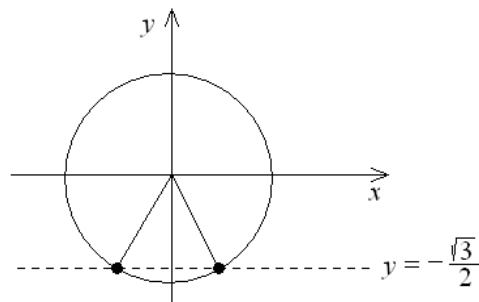
## Practice Problems - Answers

1. a)  $-\frac{1}{6}\pi + k\pi$        $k \in \mathbb{Z}$       b)  $k\pi$        $k \in \mathbb{Z}$       c)  $2k\pi$        $k \in \mathbb{Z}$   
 d)  $-\frac{1}{6}\pi + 2k_1\pi$  and  $\frac{7}{6}\pi + 2k_2\pi$        $k_1, k_2 \in \mathbb{Z}$       e) no solution      f)  $\pm\frac{\pi}{6} + 2k\pi$        $k \in \mathbb{Z}$   
 2. a)  $x \neq \frac{\pi}{2} + k\pi$        $k \in \mathbb{Z}$       b)  $x \neq k\pi$        $k \in \mathbb{Z}$       c)  $\mathbb{R}$

## Sample Problems - Solutions

- 1.) Solve the equation  $\sin x = -\frac{\sqrt{3}}{2}$ .

Step 1. We draw a unit circle to find all solutions on it first.  $\sin x = -\frac{\sqrt{3}}{2}$  means that we are looking for all points on it with second coordinate  $-\frac{\sqrt{3}}{2}$ . The two solutions on the unit circle are the intersections of the unit circle and the line  $y = -\frac{\sqrt{3}}{2}$ .



Step 2. We translate the solutions to two angles. In this case,  $-60^\circ$  and  $-120^\circ$  are two reasonable choices. ( $240^\circ$  and  $300^\circ$  are also correct).

Step 3. We algebraically state the result, which are the previously found two angles, and all co-terminal angles.

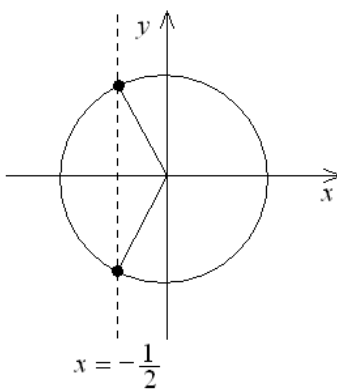
$$x = -60^\circ + k \cdot 360^\circ \text{ or } x = -120^\circ + k \cdot 360^\circ \text{ where } k = 0, 1, -1, 2, -2, 3, -3, \dots$$

Step 4. Convert the solutions to radians. Remember,  $180^\circ = \pi$  radians.

$$x = -\frac{\pi}{3} + 2k\pi \text{ or } x = -\frac{2\pi}{3} + 2k\pi \text{ where } k = 0, 1, -1, 2, -2, 3, -3, \dots$$

2.) Solve the equation  $\cos x = -\frac{1}{2}$ .

Step 1. We draw a unit circle to find all solutions on it first.  $\cos x = -\frac{1}{2}$  means that we are looking for all points on it with first coordinate  $-\frac{1}{2}$ . The two solutions on the unit circle are the intersections of the unit circle and the line  $x = -\frac{1}{2}$ .



Step 2. We translate the solutions to two angles. In this case,  $120^\circ$  and  $-120^\circ$  are two reasonable choices. ( $120^\circ$  and  $240^\circ$  are also correct).

Step 3. We algebraically state the result, which are the previously found two angles, and all co-terminal angles.

$$x = 120^\circ + k \cdot 360^\circ \text{ or } x = -120^\circ + k \cdot 360^\circ \text{ where } k \in \mathbb{Z}$$

Notice that the new notation,  $k \in \mathbb{Z}$  is a shorter way to write  $k = 0, 1, -1, 2, -2, 3, -3, \dots$ . In the same spirit, the result (unlike with  $\sin x$ ) can be written as a single line, as

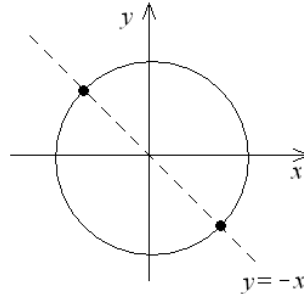
$$x = \pm 120^\circ + k \cdot 360^\circ \text{ where } k \in \mathbb{Z}$$

Step 4. Convert the solutions to radians. Remember,  $180^\circ = \pi$  radians.

$$x = \pm \frac{2\pi}{3} + 2k\pi \quad k \in \mathbb{Z}$$

3.) Solve the equation  $\tan x = -1$ .

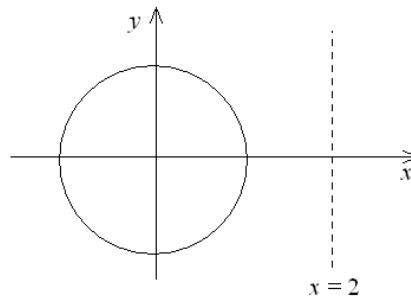
Like in the case of sine and cosine, there will also be two solutions on the unit circle. Also, they will be the intersections of a line and the unit circle.  $\tan x = -1$  is true when  $x = -45^\circ$ . The second solution on the unit circle can be found by continuing the same straight line that connected  $-45^\circ$  with the origin. The solutions are now on the line of whose **slope** is  $-1$ .



The solutions are  $x = -45^\circ + k \cdot 180^\circ$  where  $k \in \mathbb{Z}$ . Or, in radians,  $x = -\frac{\pi}{4} + k\pi$ , where  $k \in \mathbb{Z}$ .

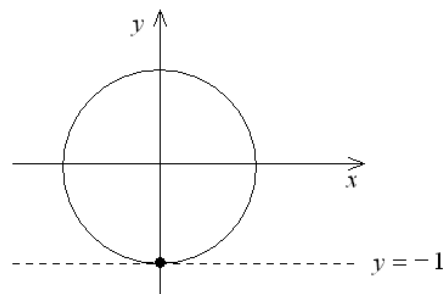
4.) Solve the equation  $\cos x = 2$ .

We draw a unit circle to find all solutions on it first.  $\cos x = 2$  means that we are looking for all points on it with first coordinate 2. The solutions would be the intersections of the unit circle and the line  $x = 2$ . Since the circle and line do not intersect, there are no solutions to this equation.



5.) Solve the equation  $\sin x = -1$ .

We draw a unit circle to find all solutions on it first.  $\sin x = -1$  means that we are looking for all points on it with second coordinate  $-1$ . There is only one solution on the unit circle because the line  $y = -1$  is a tangent line.



The intersection point corresponds to  $-90^\circ$  (or  $270^\circ$ ).

$$x = -90^\circ + k \cdot 360^\circ \quad k \in \mathbb{Z} \quad \text{or in radians:} \quad -\frac{\pi}{2} + 2k\pi \quad \text{where } k \in \mathbb{Z}$$

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