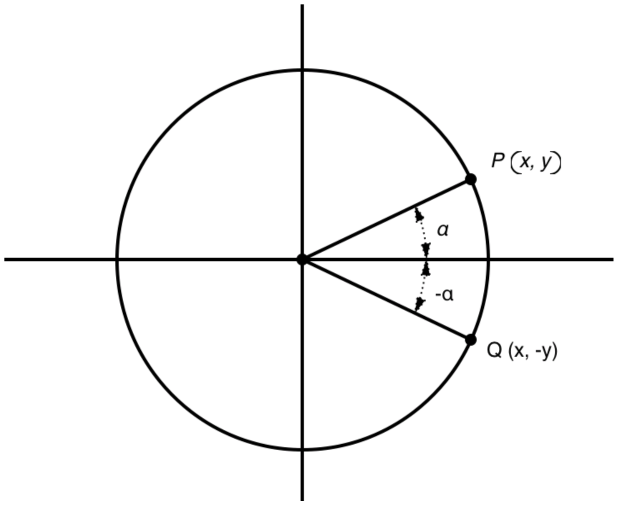
Let α be an angle denoted by $P(x,y)$ on the unit circle. If α is measured from the positive part of the $x-$axis, counterclockwise, then by definition,

$$\cos\alpha = x \text{ and } \sin\alpha = y$$

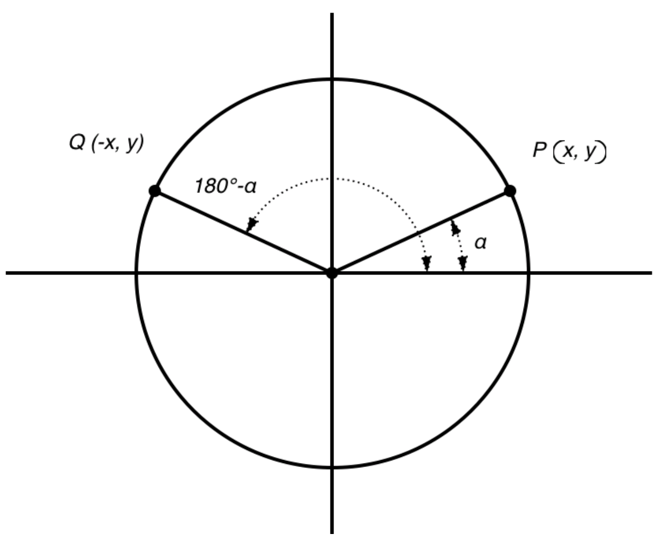
Case 1. Reflect P to the $x-$axis to obtain point $Q(x,-y)$. Then the angle associated by Q is $-\alpha$.

$$\begin{aligned} \sin(-\alpha) &= -\sin\alpha \\ \cos(-\alpha) &= \cos\alpha \\ \tan(-\alpha) &= \frac{\sin(-\alpha)}{\cos(-\alpha)} = \frac{-\sin\alpha}{\cos\alpha} = -\tan\alpha \end{aligned}$$



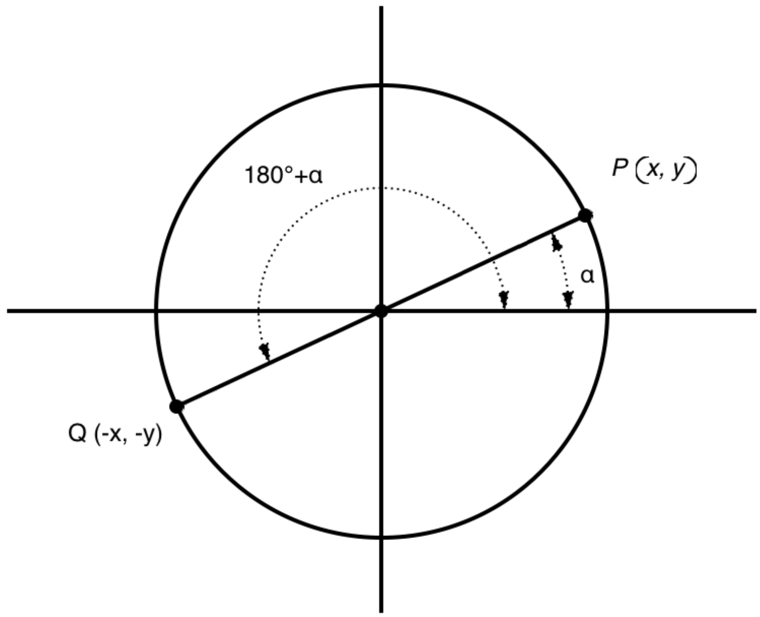
Case 2. Reflect P to the $y-$axis to obtain point $Q(-x,y)$. Then the angle associated by Q is $180° - \alpha$. In other words, these two angles are supplements.

$$\begin{aligned} \sin(180° - \alpha) &= \sin\alpha \\ \cos(180° - \alpha) &= -\cos\alpha \\ \tan(180° - \alpha) &= \frac{\sin(180° - \alpha)}{\cos(180° - \alpha)} = \frac{\sin\alpha}{-\cos\alpha} = -\tan\alpha \end{aligned}$$



Case 3. Reflect P to the origin to obtain point $Q(-x,-y)$. Then the angle associated by Q is $\alpha+180°$.

$$\begin{aligned}\sin(\alpha+180°) &= -\sin\alpha\\ \cos(\alpha+180°) &= -\cos\alpha\\ \tan(\alpha+180°) &= \frac{\sin(\alpha+180°)}{\cos(\alpha+180°)}=\frac{-\sin\alpha}{-\cos\alpha}=\tan\alpha\end{aligned}$$



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